Life insurance – textbook

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LIFE INSURANCE

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INTRODUCTION

The goal of the book is to give a general introduction to life, accident and health insurance (and some other areas that functionally belong here, e.g. the world of pension and health funds), and contains a possible discussion of the concept of life insurance. The concrete material of knowledge concerning the other mentioned insurance areas are contained in other textbooks, but the common basis can be found here.

Regarding life, accident and health insurance first of all we have to say that they are instruments to ward off events that can be anticipated in the life cycle and that occur in a standard way, but that cannot be anticipated on the individual’s level and have financial effects (pension, death, accident, sickness), and to eliminate uncertainty in the financial sense. Altogether we can say that life, accident and health insurances are instruments of the financial planning of the human life cycle. This way the first part of the book (chapters 1-3) discusses the financial planning of the life cycle.

One of the classical figures of sociology, Max Weber declared a hundred years ago that a tendency of capitalism compared to the former social order, feudalism is that it makes all social relations rationally calculable. Insurance is typically “capitalistic” in this respect, or at least a modern phenomenon, since it makes events with uncertain financial outcome calculable with certainty.

Insurance fundamentally handles risks in two ways, that are connected to each other:

1. Converting uncertain large losses into small, but certain ones.
2. By creating reserves to cover future needs.

The general principles of both methods will be discussed later on.

The book, that is the revised, re-edited and enlarged edition in a uniform structure of the author’s former book, primarily follows the material of the “Life Insurance” course of actuary speciality at the Corvinus University of Budapest. It complements the theoretical basis in the former edition with “practical” material. In these parts – just as in the university course – students can practice the terms and relations introduced in the sections discussing theory.

In the book the author uses the first person of plural, but where the author found it important to emphasize his own opinion on a subject, that differs from the opinion of other experts, the first person of singular is used.

I. LIFE INSURANCE BASICS

1. BASIC CONCEPTS OF DEMOGRAPHY

KEY WORDS

Average age	Size of population
Generation mortality table	Selection table
Mortality table	Life expectancy at birth
Probability of death	Probability of survival
Life table	Disability adjusted life expectancy
Age structure	Life expectancy

In order for insurance to be of help in the planning of the life cycle, the insurer must have concrete ideas and specific models in mind regarding the path of human life cycles and their most important parameters. (For example, their average length, the ratio of active and inactive stages, their distribution, the probability of death, illness, accidents, the expected extent of illness or injury from accidents, etc.) This information is usually obtained from public sources, which are mostly collected as a part of a separate social science, demography (the science of populations.) In the following sections we will get to know some demographical concepts and implications that are important with respect to insurance.

1.1. TOTAL POPULATION

In general, demography - similarly as insurance - deals with the patterns involving the „movements" of large population masses. One of the most important such indicator-systems is one that refers to the changes and composition of the population size of a regional unit (usually a country).

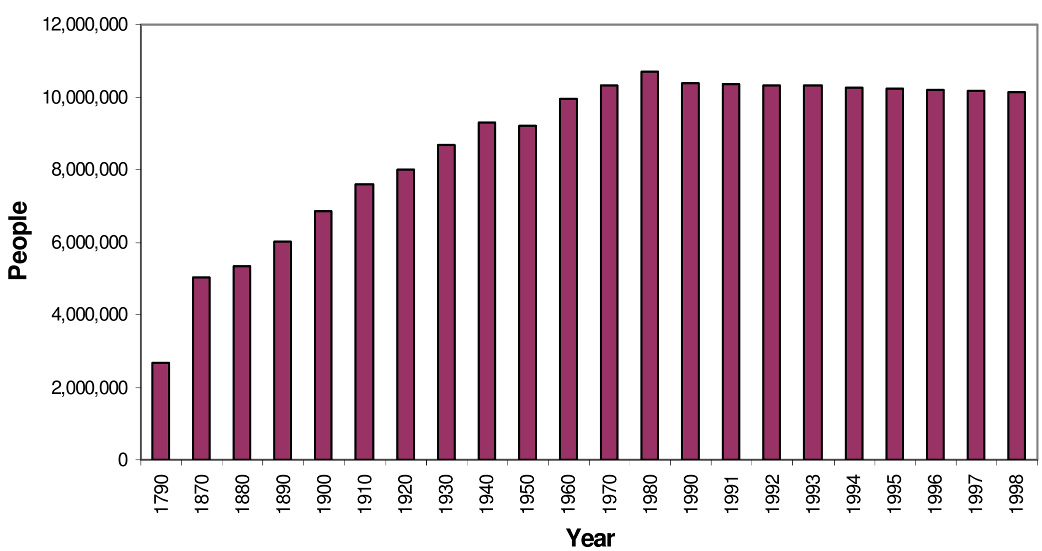

Figure 1.1.: Population size in the current area of Hungary

If we are only interested in the total population and its changes, then we can see right away that at any point in time this can be expressed as the resultant of two opposite factor-pairs. This is the balance of:

births – deaths
immigration – emigration.

If there is no immigration and emigration (as was virtually the case in Hungary in the 1970s and '80s) then the size of the population will increase if there is a greater number of births than deaths (as is the case today in most of the so-called developing countries – in Asia, Africa and Latin-America) and it will decrease if the number of births is smaller (for example in Hungary in the last two decades).

One might think that the equilibrium between births and deaths can be achieved if every single person has one offspring (or every couple has two), because this would reproduce the population. This is true in the very long term for a population that is in equilibrium in other regards as well, but this principle cannot be used to explain the seemingly paradoxical phenomena we can observe, for example, in China. Here, in the last few decades, every married couple in the city is allowed only one child, while couples from rural areas – if the first child is female - are allowed up to two children. This means that for decades the number of children for every couple has been well below two, and yet during this time the population has increased by several hundred million, and, according to forecasts, this will continue for a few more decades (while the policies regarding children will probably not change significantly).

The solution to the Chinese mystery is simple: nowadays, in China, as well as around the world, life expectancy at birth is increasing significantly. As a result, despite the fact that there are relatively few births, there are even fewer deaths, and the average age and size of the population is increasing.

The population's size and average age, and their changes over time have a very important role in certain long-run macro-level planning – for example in the planning of the welfare system (health insurance and pension system), - and through these in the opportunities of private insurance as well.

1.2. Composition of the Population by Age and Gender

We can also gain more detailed information about the population than just its total size and its changes. It is important to know, for example, how the total population is distributed among the sexes and age groups.

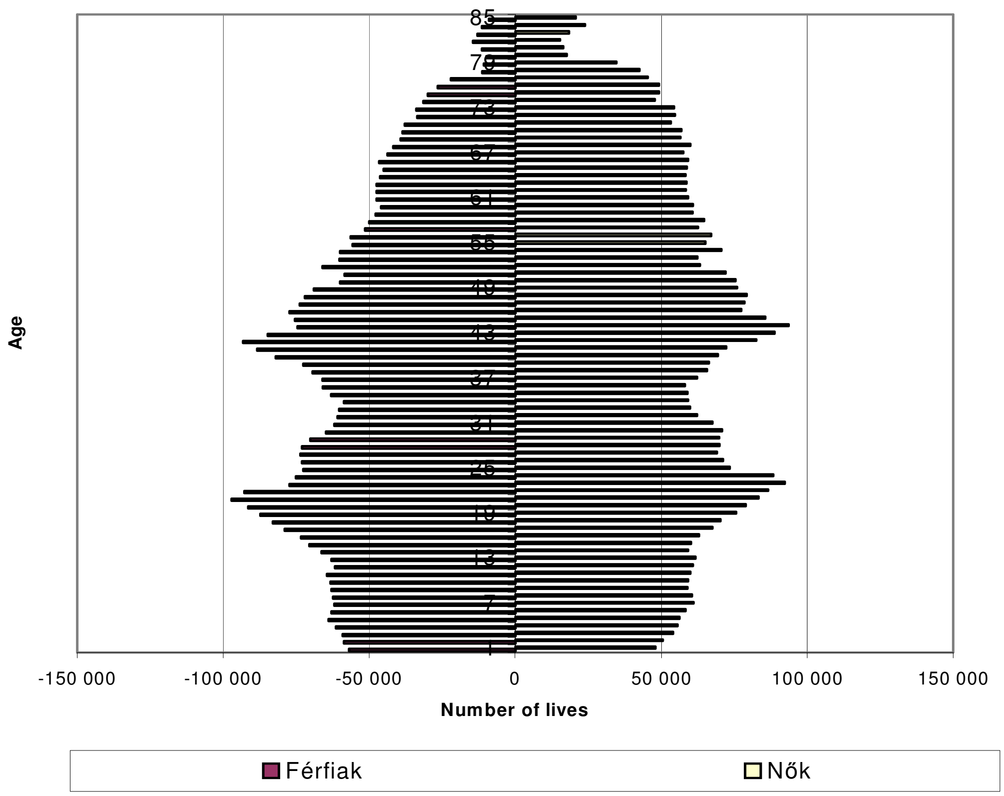

Figure 1.2.: 1998 Hungarian age structure

The above figure shows the Hungarian population's age composition up to age 84 by gender[1] (on the left is the male[2], on the right the female population). This figure gives a much more detailed view of the population then do the simple total population and average age figures and their changes, because it shows in detail its composition along certain important parameters (age and gender). It is true though that this view is static, since we do not know exactly how we got to this point, or how this figure will look in a few years' time. Of course, we can read many things about the past and future out of such a static figure, but this can be made more dynamic by placing several figures pertaining to different time periods next to each other.

Looking at these figures (and the numbers behind them) we can draw many important deductions. We can observe that the number of males at birth is significantly higher (in 1998, for example, there were 48061 females for 56943 males in Hungary), but later this difference gradually decreases, and around age 40 the population of the two sexes evens out. After this age the ratio of women gradually increases (or decreases by less than the men) and at age 84 there are more than twice as many women than men. Since we can see from the figures that there are more 1 year old children than newborns, and more 2 year olds than 1 year olds etc., we can observe that in the last few years there have been fewer and fewer children born every year. Since there is a peak on the figure around age 20, we can also observe that there was a peak in the number of births in the '70s in Hungary, and since then the number of births has decreased year after year. This is also related to the fact that there

[1] Since this figure is not based on the complete range of census data, the KSH (Hungarian Central Statistical Office) did not estimate the composition above age 85 due to its uncertainty.
[2] Whose number is, of course, non-negative, but the values are simply shown to the left of the y-axis!

is also a peak at around age 40-45 (those born between 1953-58), made up of people who are probably the parents of today's twenty-something year olds. Even from a simple figure as this one we can draw far-reaching conclusions about the necessity of certain macro-level political steps. For example, if there are fewer children, we need fewer nursery schools, kindergartens, and elementary school classrooms, teachers, etc. At the same time, there is greater demand for university openings etc. due to the peak in the number of twenty-something year olds. If the number of parents is greater than the number of children, then a few decades later the ratio of older and middle-aged people will change dramatically compared to that of today.

The above figures resemble a tree very closely; they are called the age structure of a population. The shapes of these figures have gone through significant change throughout history. Earlier it was generally more the case that a greater number of children were born, but the infant and child mortality rate was also high, and the average lifespan was also very low. These factors together create an age structure figure in the shape of a pyramid or a pine tree, which can be seen schematically in the following figure.

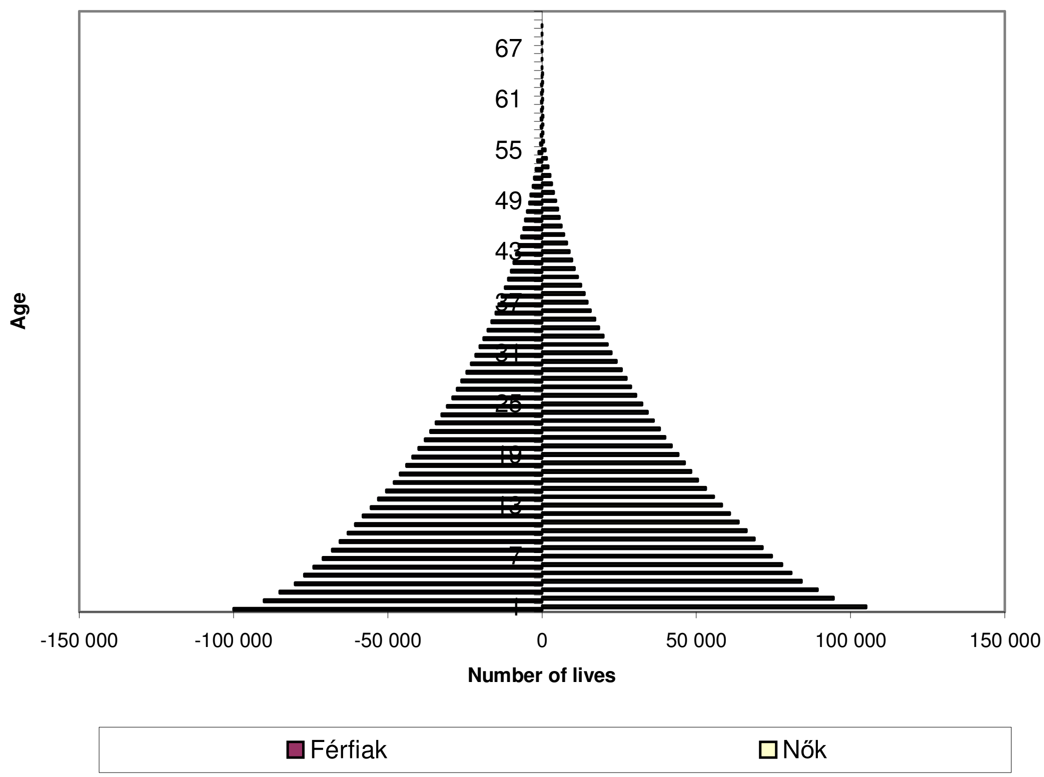

Figure 1.3.: Traditional pyramid-shaped age structure

Hungary was described by such a „traditional" age structure figure at the beginning of the 20th century, and it is still typical today in the so-called „developing" countries. In the developing countries we can see a new trend along with the high number of children typical in the western countries 100 years ago. This – similarly as in the western countries' current situation - is the increase in the life expectancy at birth. This is mostly due to the disappearance of the earlier great epidemics and some improvement in the standard of living. These two factors together result in the phenomena called the „demographic boom", which resulted in a never before seen increase in the Earth's population in the 20th century, especially in the second half, and this growth is expected to continue for at least another 50 years.

Of course it is difficult to predict the total size of the Earth's population ahead of time. Even nowadays the estimates change year by year, and this is also true for the individual countries as well. 20 years ago, for example, no one could foresee the appearance of a new, previously unknown, deadly epidemic in the southern half of Africa, which resulted in a decreasing population in some countries that previously showed increasing tendencies. Even further, AIDS – since it mostly affects young adults – has changed the age structure in a very unique way[3]; a UN study described it as a „chimney" shape.

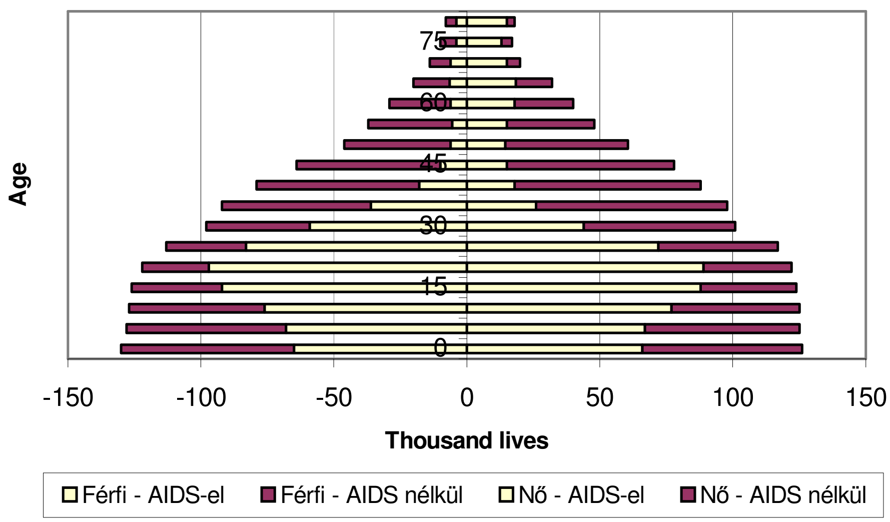

Figure 1.4.: The predicted population of Botswana in 2020 with AIDS and without

In the long-run the age structure figure changes from a pyramid shape to a tree that has an increasingly wide crown around the middle and then top, whose trunk gets gradually thinner and taller (so the number of children decreases, but the earlier generations of larger numbers live for a longer time), and finally the crown disappears completely. The number of children born will probably not decrease infinitely either, and thus in the even longer term (in a hundred years!) developed countries will have an age structure figure resembling a column, so in every year the same number of children will be born, and all who have been born will more or less live to 80 + x years old.

The well-known expert, Laszlo Hablicsek says the following about the tendencies expected in Hungary in the future:

> „The most probable demographic future for Hungary – based on the processes started by the transition – is a medium low number of children, a gradually increasing lifespan, and the development of a slight surplus in immigration. These factors together may lessen, but will not cancel out completely the decrease in population, and after a temporary pause the aging of the population will continue with new force in a second wave.
>
> The population, according to this version, was 10 million 44 thousand people at the beginning of 2000, by 2010 it will be 9.7 million people, and it could decrease to 8.0

[3] see: The Economist: 2000. July 15. issue 28. pp.91-93.: A turning-point for AIDS? In another article - 2001. February 10. issue 6. p.75.: Business and AIDS – The worst way to lose talent (South African firms are struggling to cope as AIDS spreads) – the figure in the article shows the life expectancy at birth in South-Africa in 1996 was above 60, but by 2000 this decreased to about 50, and by 2010 it is expected to decrease well below 40 (!) years.

million by 2050. Taking into account other possible tendencies, the population in 2050 can be realistically placed between 7.4 and 8.8 million. In the most extreme case Hungary's lowest population size could be 6.0 million, and the maximum population size could be 10.3 million by the middle of the 21st century.[4]

The number of people between 0-19 years old will definitely be lower than the current level. Taking the lowest fertility levels as a basis of calculations the age group could decrease by half in 50 years. The ratio of 0-19 year olds within the entire population could decrease from the current 26 percent to about 14-19 percent.

The number of working-age people (20-64 year olds) will definitely decrease significantly after 2010, since the large generations born in the 1950-s will be leaving this group. Another decrease can be expected when the generations born in the 1970s will also leave this age group. By 2050 the size of the working age group can be calculated to be about three quarters of today's size.

The size of the older population will be varying over time. The 65 and older age group could increase by one and a half by 2050. Within this time though the next decade will be very „quiet": there will not be any large changes in the size of the older population as a „preparation" for the (second) wave of aging that will begin from 2015.

These long-run manuscripts show clearly one of the possible main tendencies of population evolution that by nature threaten the systems of welfare for the elderly and their financing, the never-before seen growth of the ratio of elderly within the population.

In our basic version the ratio of the 65 and older age group could increase from 14% to 26%, so every fourth citizen will be elderly. As long as the manuscript of lower number of births and longer life spans will be realized, the ratio of elderly could reach 36 percent! Based on this version, half the population will be over 50 by the year 2050."[5]

The evolution of population size in Hungary and in western countries in general was influenced by two major tendencies in the last quarter of the 20th century:

1. The constant and still present increase in life expectancy[6]
2. The significant decrease in number of children born.

While the number of children in Mexico, Africa and Islamic countries is still very high (often 7 children born for every woman on average, which can be regarded as the theoretical maximum), in the developed western countries – and in Hungary – it has decreased well below the reproductive level (about 2 children per woman) in recent times.[7] From this point of view, Northern Italy is the negative record-holder, with a ratio of about 0.9 children/woman. The Hungarian tendency is a result of many concurrent factors, and it is impossible to tell

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[4] Of course this cannot be calculated exactly. Sometimes opposing tendencies are reported. At the beginning of 2000, for example, it was reported that „The rate of decrease of the population slowed in the first five months of the year. The size of the population at the end of May was 10 million 24 thousand, two thousand less than in April..." (Népszabadság, 2000 July 25. – Lassabban fogy a népesség), and „Between January and July not only were there more children born, than a year ago..., but the number of marriages increased as well, and fewer people died than in the same time period in 1999." But: „It is also worth noting that in the first seven months of 2000 91 out of the 10,000 children born alive died before age one, five more than last year." (Népszabadság, 2000. October 7. - Több gyerek születik az idén)

[5] Hablicsek, László: Népesedéspolitikai Ad-hoc Munkabizottság, I. munkacsoport, NÉPESSÉGCSÖKKENÉS ÉS ÖREGEDÉS – Zárójelentés, Budapest, 2000 January 10.

[6] In the last few decades not only did the life expectancy at birth increase significantly, but it can be expected that due to new discoveries (the human gene mapping, medicine based on genetics!) this will continue to increase. (The article by Demeter Polgár deals with this issue: Sose halunk meg! – Küzdelem az emberi lét végső határának kitolásáért – Népszabadság 2000. August 5.)

[7] It is important to note that there is nothing wrong with a low and stable birth rate. It is not a tragedy that the population of a country is decreasing – despite the claims of some Hungarian publicists who vision the „death of the nation" – especially if we know that one of the greatest problems of the world today is over-population. If the number of children is stable – no matter at how low a rate – then sooner or later the aging of the population will also stop (stabilize). The population – if necessary – can increase not only with births but also with immigration, as we have seen throughout history – for example even in the case of Hungary! Of course, this can be a big problem if the functioning of certain institutions (such as a pay-as-you-go pension system) were explicitly made to depend on a high birth rate, but there is no reason for this to be the only possible system of institutions.

exactly which of these will prove to be permanent in the long run. The low number of children in the 1990s, for example, can be largely explained by the economic transition, as a result of which a large number of fertile-aged women postponed giving birth until they were „in the clear" financially.

1.3. Life Expectancy, Probability of Death

Statistics pertaining to the size and composition of the entire population are very important for the purpose of politics, and the observations made from these serve as a framework for observations about the single individuals. In the planning of the life cycle (and later insurance) we are mainly interested in the statistics pertaining to the individual – and not those pertaining to the entire population. Naturally, the two are related: we take the social average as a representation of the individual.

From the above statistics of population movements we can first of all deduct the probability of death and the life expectancy of the individual.

Statistics regarding the size and the composition by gender and age of the population are collected during the census. A census including the full population takes place relatively rarely (on average every ten years). Between two such censuses changes in the population are traced by the gathering of statistics from a representative sample of the population (micro census), so we have a more or less reliable estimate of the major statistics of the population every year.

Based on the census statistics, we can compare by years of age and gender the number of those living at the beginning of the year with the number of those who died during the year, and thus calculate the raw probability of death. If these are then arranged (for example on a graph where the horizontal axis shows age), then the figure we get can be divided into a theoretical trend and a random deviation from this trend. The raw data, once it is cleaned of these random deviations, are none other than the theoretical death probabilities (q_x) pertaining to the given year, the meanings of which are:

q_x = the probability of someone dying before reaching age x+1, given that they survived to age x

Since these statistics come from a „momentary" survey of the population, every q_x marks a probability pertaining to generations born in different years living simultaneously, yet in the given moment these do give a snapshot of the mortality conditions of the entire population.

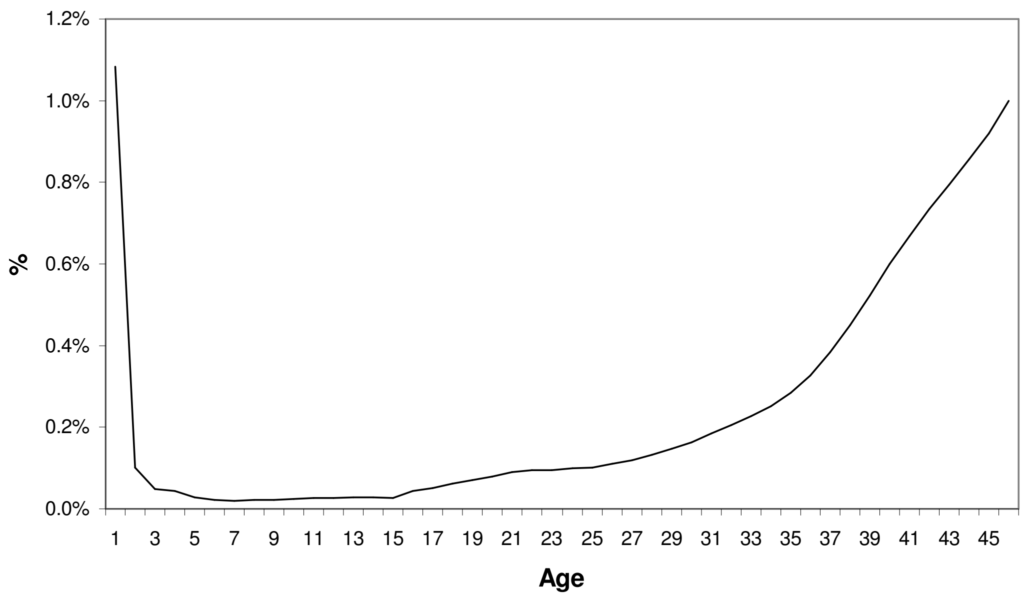

Figure 1.5.: Hungarian males' probabilities of death up to age 45 in 1998

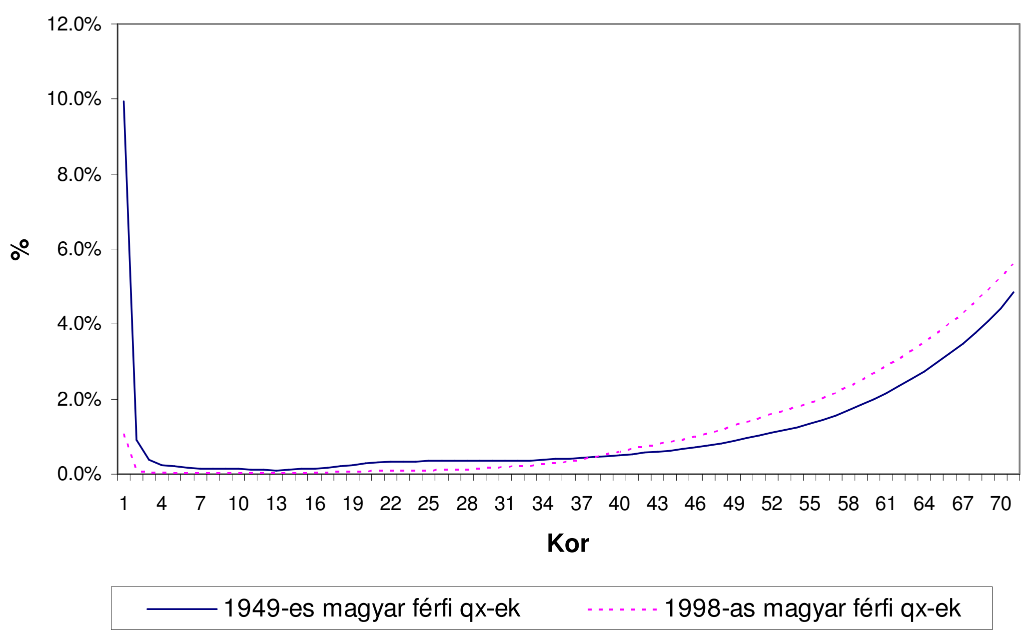

Figure 1.6.: 1998 and 1949 probabilities of death of Hungarian males up to age 70

It can be seen from the figure that in Hungary, the infant death rate decreased from 10% to a value of about 1% in fifty years following 1949,[8] but the mortality rates of those above age 40 are worse now than they were 50 years ago. It can also be seen that, disregarding infant mortality, the q_x curve is monotonically increasing with age, and this growth is at an increasing rate („exponential in nature"!).

Several other indicators can be derived from the q_x. Its complement, the probability of survival:

$p_x = 1 - q_x$ = probability of survival = the probability of someone surviving to age x+1, given that they survived to age x

It can be seen quite easily that the product of the p_x-s gives the probability of someone surviving t more years given that they lived to age x, so:

${}_tp_x = p_x * p_{x+1} * \ldots * p_{x+t-1}$ = the probability of someone surviving to age x+t, given that they survived to age x

It is obvious that:

$$ {}_1p_x = p_x $$

It is customary in statistics to mark the highest shown age level[9] with the ω symbol. This level differs from country to country (in the United States there are mortality tables that show ages up to 117!). In Hungary it is usually set at age 100, although we know about the existence of one or two Hungarian citizens above the age of 100, but the number of such individuals is very small and their „appearance" is highly variable. If we sum the ${}_tp_x$-s over t from 1 to (ω-x), (and we correct the sum by 0.5)[10] the sum is given a new meaning: the life expectancy at age x.

${}_1p_x + {}_2p_{x+1} + {}_3p_{x+2} + \ldots + {}_{\omega-x}p_x + 0{,}5 = e_x$ = life expectancy at age x

For x=0 this gives an especially significant statistic – the life expectancy at birth. This can be seen for a few countries below for the year 1999. The table clearly shows the significant difference between the life expectancy of women and men, which can be found in every country, and is especially high in Hungary.

Table 1.1.: Life expectancy at birth for some countries in 1999

Country	Female	Male
Australia	81.8	76.2
Austria	80.9	75.1
Belgium	80.8	74.4
Canada	81.7	76.3
Czech Republic	78.1	71.4
Denmark	79	74.2
Finland	81	73.8
France	82.5	75
Germany	80.7	74.7
Greece	80.6	75.5

[8] The values of infant mortality in Hungary (for both genders) per 1000 live births:

Year	1960	1970	1980	1990	2000
Infant deaths/1000 live births	47.6	35.9	23.2	14.8	9.1

The statistics of KSH were quoted in: Népszabadság, 2000. July 25.: Lassabban fogy a népesség

[9] So not the highest observed age, but the age at which there are still a „statistically significant" number of people.

[10] The correction marks the half year average lifespan beyond age x+t of those who died at age x+t.

Country	Fema le	Male
Hungary	75.1	66.3
Iceland	81.4	77.5
Ireland	79.1	73.9
Italy	81.6	75.3
Japan	84	77.1
Korea	79.2	71.7
Luxembourg	81.2	74.7
Mexico	77.3	72.8
Netherlands	80.5	75.3
New Zealand	80.8	75.7
Norway	81.1	75.6
Poland	77.5	68.8
Portugal	79.1	72
Slovakia	77	69
Spain	82.4	74.9
Sweden	81.9	77
Switzerland	82.5	76.8
Turkey	70.7	66.1
United Kingdom	79.8	75
United States	79.4	73.9

All these statistics (mortality and survival probabilities, life expectancy) are published organized by age in the so-called “mortality table” by the Hungarian Central Statistical Office. The mortality table includes a very useful technical row, the “number of survivors”. This row of numbers, that is created from the cleaned probabilities of mortality, can be regarded as a basic indicator, which shows how many people out of a starting population (usually 100,000 people) will be alive at age x if the current mortality rates of each age group apply to them at every age. The symbol of the values of the number of survivors, or the “life table” is l_x, and so – based on the above description –, l_0 = 100,000.

Almost all the statistics necessary in life insurance can be constructed very simply from the life table. In the following, this constructed chain of values will be used in the majority of the calculations (see in more detail in chapter 1.4.!).

According to what was previously said, the mortality table does not apply to a single generation, but rather it is a snapshot of several generations living simultaneously. This is also true for the life table, even though it very strongly suggests that its statistics pertain to a single generation, as if they followed life paths of 100,000 infants born at the same time until they reach age 100.

Naturally, this statistic could also be constructed, but this would require data from 100 years, and would not truly reflect very flexibly the current mortality trends of a time. For the purposes of analysis, it is still best to construct the mortality table of a generation, the so-called „generational mortality table”. This can be done most easily by taking the statistics of those born in the same year from all the different years of the survey. Since the Hungarian men’s mortality tables are available starting with 1949, we can take from every year’s table the mortality rates of those born in 1949 (so from the 1949 survey that of the 0 year olds, from the 1950 the one year olds’, ... from the 1998 the 49 year olds’), and thus construct for 1998 the generational mortality table of the 49 year olds (which by definition can only be ascertained up to age 49 in 1998).

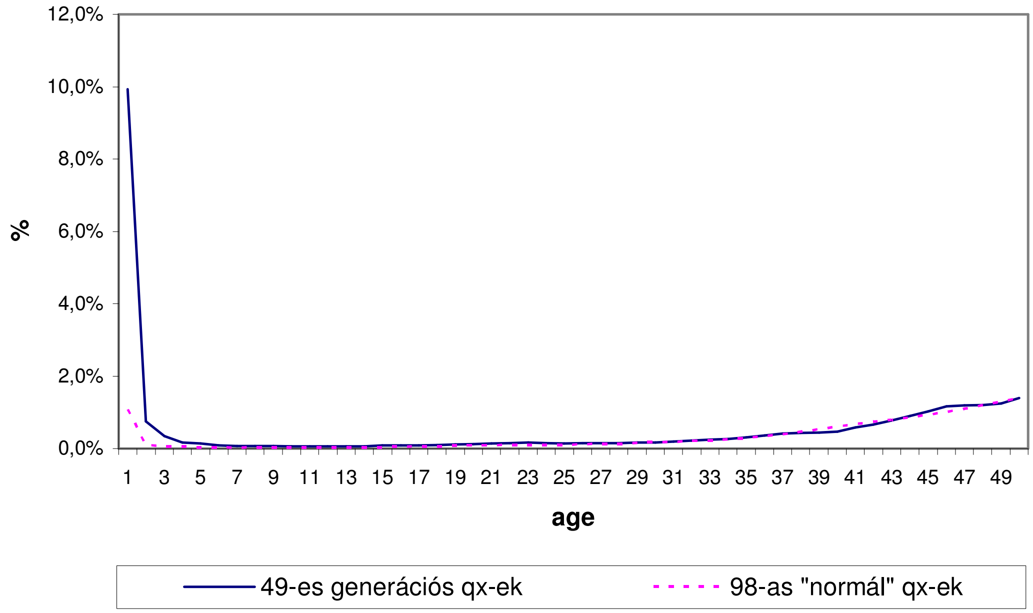

Figure 1.7.: 1949 generational and 1998 Hungarian men's mortality rates

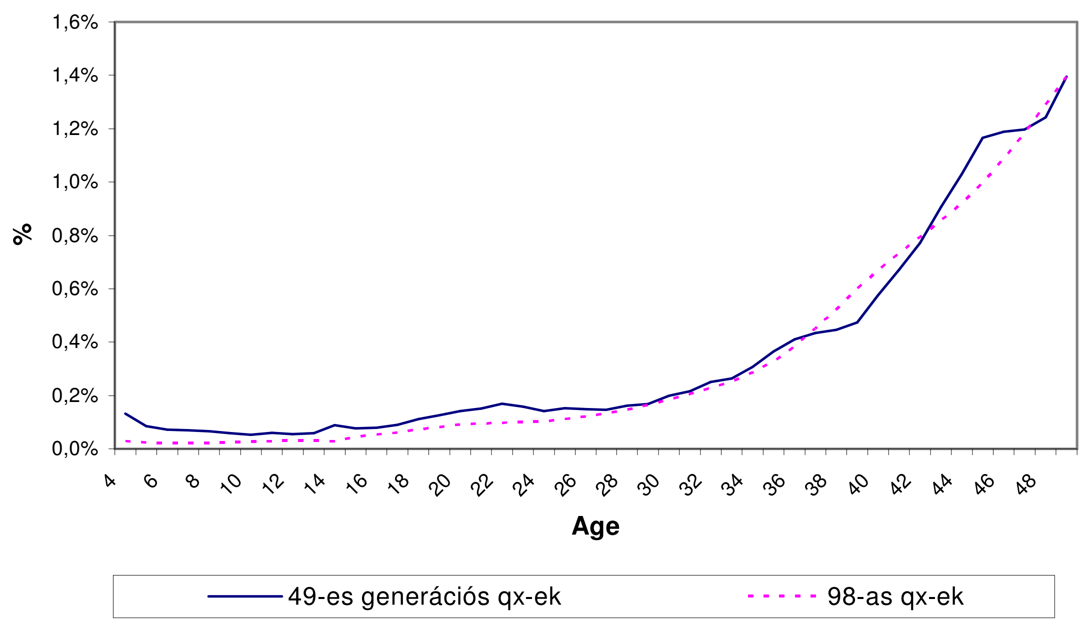

Figure 1.8.: 1949 generational and 1998 „normal" mortality rates from age 4

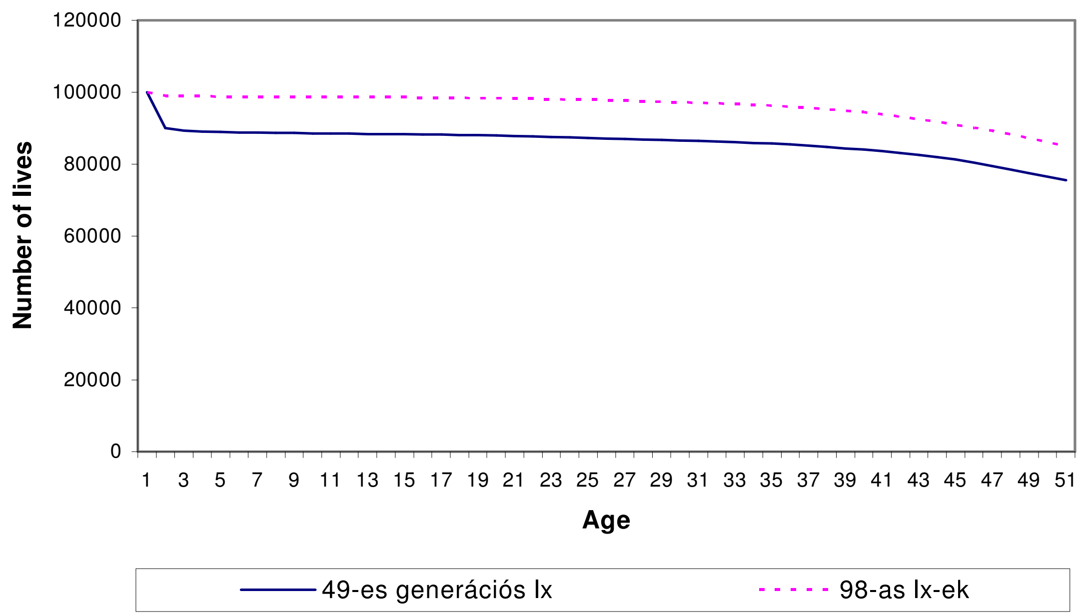

Figure 1.9.: 1949 Hungarian men's generational lx-s

One can see right away from the above figures that infant mortality has drastically, and child mortality has significantly decreased since '49, but those of the young and middle-aged have not – since the statistics of this group are shown as the statistics of the middle-aged in the 1998 data. It is also apparent that the curve of the 1998 mortality rates is much „smoother", since the raw data was previously smoothed out using statistical tools.

It should also be noted that mortality tables can be made not only for the entire population, but segments of it as well, as some of these segments have very different mortality characteristics. For example, we would surely have different life expectancies among:

- The VIII. and XII. districts of Budapest
- In Győr-Moson-Sopron and Szabolcs-Szatmár-Bereg counties
- Those working in the finance sector and miners
- The divorced, married, widowed and singles
- Smokers and non-smokers
- Those who finished only elementary school and those with university degrees
- Etc.

So we can differentiate among the groups of a population based on place of residence, education level, occupation, income level, marital status, habits, etc. At the same time, these are not such permanent characteristics as is gender (since the place of residence, marital status, etc. may change frequently, while gender cannot[11]). From time to time complete analyses are reported based on these characteristics.

Insurers also create their own mortality tables based on their specific points of view – and usually, their own data. It is especially common in the Anglo-Saxon countries to differentiate between smoking and non-smoking insured, whose rates are calculated from separate mortality tables.

[11] Nowadays one must be careful about this statement as well. It is possible that the time is near when surveys will have a category for „original gender" instead of simply „gender".

Even more common is the use of so-called selection tables. This is where they observe how the mortality profiles of those purchasing different insurance compare to each other[12]. For example, they can differentiate between the selection tables of purchasers of annuity or term insurance, since people with lower life expectancies would rather buy term insurance, than the average, and vice versa, those with high life expectancies would rather purchase annuities. The selection tables show this difference very clearly. (Unfortunately, in Hungary insurance companies have not collected sufficient data for these, though some calculate mortality based on their own data). In table 1.2. we can see what percentage of the calculations based on population mortality tables is comprised of annuity mortality rates in the USA. As we can see, there is a significant difference between the different phases of the life cycle.

Table 1.2.: U.S. annuity q_x-s compared to the population mortality tables (1990-1996)

Age	Male	Female	Age	Male	Female	Age	Male	Female
0	23%	22%						
1	123%	117%	41	35%	48%	81	60%	70%
2	101%	93%	42	38%	48%	82	60%	71%
3	99%	92%	43	41%	49%	83	61%	72%
4	111%	98%	44	44%	49%	84	62%	73%
5	110%	89%	45	46%	49%	85	63%	74%
6	107%	78%	46	49%	49%	86	64%	76%
7	106%	70%	47	50%	49%	87	64%	77%
8	132%	76%	48	52%	49%	88	65%	78%
9	163%	84%	49	52%	49%	89	66%	79%
10	195%	95%	50	52%	49%	90	66%	80%
11	198%	99%	51	52%	48%	91	67%	81%
12	156%	92%	52	52%	48%	92	67%	81%
13	107%	77%	53	52%	49%	93	68%	81%
14	75%	63%	54	51%	49%	94	68%	80%
15	57%	54%	55	51%	49%	95	69%	79%
16	47%	48%	56	50%	49%	96	69%	79%
17	41%	46%	57	49%	49%	97	70%	78%
18	38%	47%	58	48%	48%	98	70%	78%
19	37%	50%	59	47%	48%	99	72%	78%
20	36%	53%	60	46%	48%	100	73%	78%
21	35%	55%	61	45%	48%	101	75%	78%
22	35%	58%	62	45%	48%	102	77%	79%
23	36%	60%	63	44%	49%	103	79%	81%
24	37%	61%	64	44%	50%	104	82%	83%
25	39%	62%	65	44%	50%	105	85%	85%
26	40%	63%	66	45%	51%	106	89%	88%
27	41%	63%	67	46%	51%	107	93%	92%
28	40%	62%	68	47%	51%	108	98%	96%
29	39%	61%	69	49%	52%	109	103%	101%
30	37%	59%	70	50%	53%	110	108%	106%
31	35%	57%	71	52%	53%	111	114%	111%
32	34%	56%	72	53%	55%	112	120%	117%
33	32%	54%	73	54%	56%	113	127%	125%

[12] In a broader sense, the selection table can pertain to any kind of selection, for example, the differentiation between smokers and non-smokers as well!

Age	Male	Female	Age	Male	Female	Age	Male	Female
34	31%	53%	74	54%	58%	114	134%	132%
35	30%	51%	75	55%	60%	115	141%	141%
36	29%	50%	76	55%	62%	116	134%	134%
37	29%	49%	77	56%	63%	117	128%	128%
38	29%	48%	78	57%	65%	118	121%	121%
39	31%	48%	79	58%	67%	119	116%	116%
40	33%	48%	80	59%	69%			

1.4. THE ANALYSIS OF THE LIFE TABLE

In figure 1.8. we can see an older and a current Hungarian, and Swedish life table.

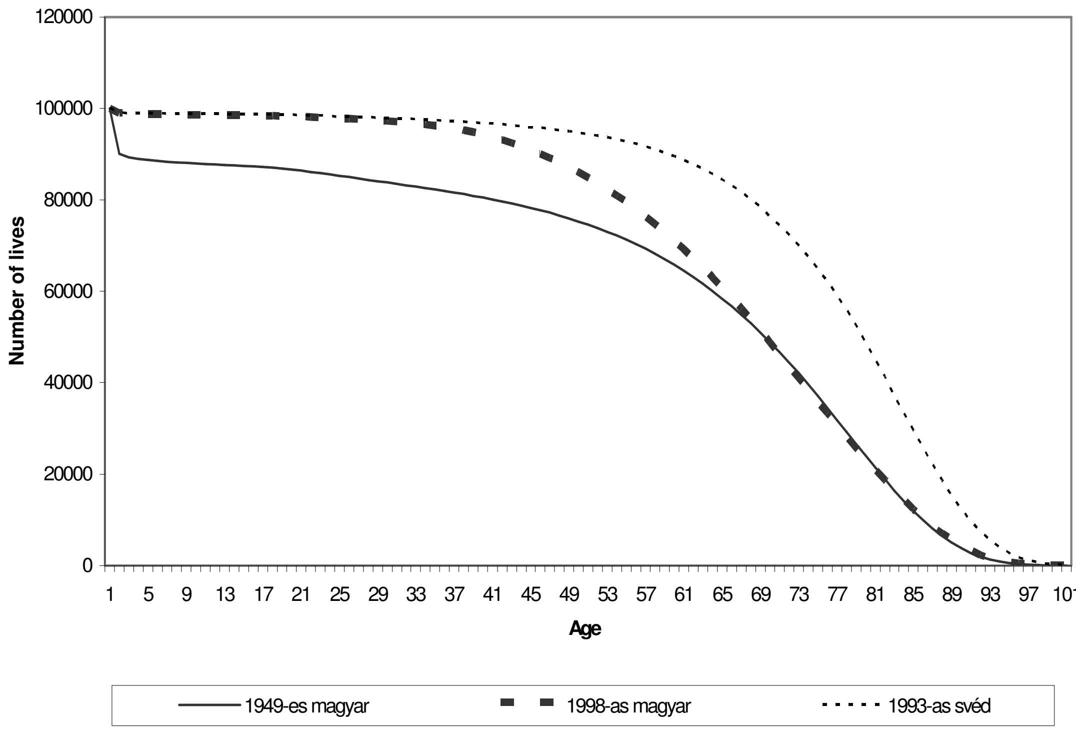

Figure 1.8.: Hungarian (1949 and 1998) and Swedish (1993) male life tables

The differences are obvious. The already mentioned infant mortality has improved a lot in Hungary in 50 years. This is shown by the fact that the curve starts a steep fall immediately in 1949, while in the '90s this decrease is very small in the first year. Above the age of 70, however, the Hungarian statistics are not any better than 50 years earlier, the reason for which is that for the middle-aged, the '98 curve falls more steeply than in '49 (so the mortality of middle-aged males is higher today than in '49). Beginning from the twenties, the Hungarian curve is significantly different from the Swedish. It seems there is barely any mortality until age 60 in the case of the Swedes.

It can be stated that:

- The later the historical era, or the more developed the country whose statistics are shown in the figure of the life table, the "fuller" it is, or the bigger the area under the curve is
- The more developed the country, the smaller the initial fall of the curve

- The more developed the country, the flatter the curve is for the young and middle ages, and the later it begins to fall.

The probabilities of mortality and survival, and the life expectancy can be calculated from the life tables in the following way:

$$q_x = \frac{l_x - l_{x+1}}{l_x} \text{ , or } p_x = \frac{l_{x+1}}{l_x} \text{ and}$$

$$e_x = 0{,}5 + \frac{\sum_{k=1}^{\omega-x} l_{x+k}}{l_x}$$

It can easily be seen that in figure 1.8., the area under the curve of the number of lives from age x represents the number of years to be lived by the x year olds (l_x people), thus the life expectancy at age x is simply the quotient of this area and the number of people who are x years old (l_x).

* * *

Due to its simplicity and ease of use, the mortality table is one of the most important statistical *inputs in insurance.*

1.5. STATUSES: MARITAL, HEALTH, ECONOMIC

Of course, insurers are often interested in the population statistics in a more detailed breakdown. Such dimensions:

- the different phases of health
- marital status
- the number and composition of households
- the distribution of the economically active and passive
- social, economic situation,
- etc.

From these numerous dimensions, we will only deal with two here, social situation (and its effect on infant mortality) and health, due to reasons mentioned earlier pertaining to the mortality table. There is a sort of "race" going on among the countries regarding health status and infant mortality, and those countries are viewed as healthier (or more exactly one with a better healthcare system) where the life expectancy at birth is higher, and the infant mortality rate is lower. (Hungary – among the more developed countries – is quite behind in this respect.)

With regards to infant mortality, a survey taken by the Argentinean Health Ministry[13] is quite interesting (their observations can probably be applied to other countries as well!). There are highly significant differences based on the mothers' education levels (which are probably positively correlated with their economic and social status and situation), which can be seen in figure 1.9.:

[13] Quoted by:: The Economist, 2000. May 6. issue 18. „Argentina Survey" p. 15. table 7.

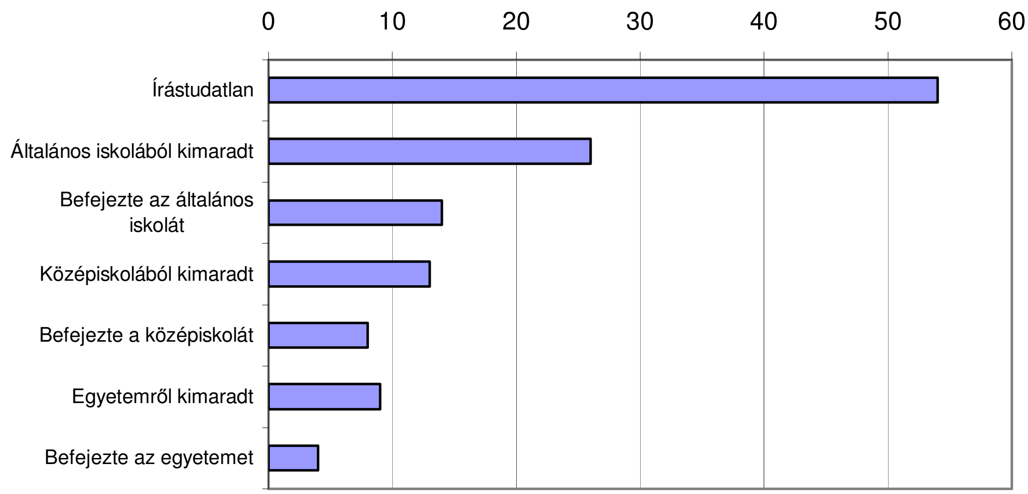

Figure 1.9.: Infant mortality per 1000 infants born as a function of mother's education – Argentina, 1998.

Nowadays, more and more people are pointing out that the life expectancy at birth is not the correct indicator when comparing countries, since health status is not qualified based on the life expectancy at birth, but rather the number of years of life spent in good health. For this reason the UN institution that deals with these issues, the WHO has introduced a new indicator, the "disability adjusted life expectancy", and this changes the picture according to the table that can be found in the appendix.

TEST[14]

1) Which of the following is true?
   a) A country's population will definitely decrease if the number of children per fertile woman falls below 2, and the balance of immigration and emigration is zero.
   b) The probability of mortality is simply the percentage of people from the total number born who die at age x.
   c) If we graph the male and female life table, the area under the female curve is typically greater than that of the males.
   d) The age structure figure of developed countries is typically pyramid shaped.

2) The life table of a country's entire population
   a) Is made by following the mortality of those born in the same year throughout their lives.
   b) Is always monotonically decreasing.
   c) Shows clearly the different infant mortality rates of the social classes.
   d) Can be used easily to calculate the disability adjusted life expectancy at birth.

[14] The test questions here and beyond are merely informative in nature, they are meant to give a taste of the nature of a written exam. Every test question has one correct and 3 incorrect answers.

2. THE INDIVIDUAL LIFE CYCLE

KEY WORDS

Incoming cash flow
Insurance
Cash flow
Structure of cash flow
Marital status
GDP per capita
Foresight
Life insurance
Life cycle
Cash flow of life cycle
Phases of life cycle
Financial planning of life cycle
Consumption
Wealth
Active phase of life
Inactive phase of life
Credit
Income
Computability
Outgoing cash flow
Risk transfer
Private insurance
Wage income
Self-insurance
Standardized life cycle
Life, accident and health insurance
Social security
Reserve
Capital income
Risk
Risk community

2.1. FORESIGHT

How long ahead do people plan their future? If we search our own thoughts, or examine the actions of people we know, that allow for such deductions, we may get very different answers. There are some people who know in the autumn exactly when and where they are going on vacation next year, and some who do not know even at the beginning of their vacation. There are some who are continually increasing their wealth, and some whose wages never last them through the month, etc. In general: the longer ahead a people plan their future, and the more of a strategy they follow, the better off they are in life. Since almost every action a person undertakes requires some form of financial-type resources,[15] the basis of long-run future planning of any sort is long-run financial foresight (or long-run financial planning).

Of course, how far one can see ahead depends on many factors. The most important are:

Subjective factors: there are simply some who have better foresight, and are careful planners, and some who are more careless or less organized, and this has a great effect on the nature of planning for the future. [16]

Age: a child can objectively foresee a much shorter time span than an adult. A child is taken care of, so the trouble of foresight is lifted from his shoulders.[17] As he grows older, he must take care of himself and others more and more, which requires him to foresee his situation for a longer period of time.

[15] Even if not in direct form, but as an „opportunity cost”, for example, if someone does not make any money while he is occupied with certain activities (a hobby, studying, having children, taking care of social relations, etc.).

[16] There are probably differences to be found between genders as well, for example, women are typically more careful, than men, who, in turn, are usually – even in a financial sense – are more driven to succeed, and more likely to think along the lines of a long term strategy in terms of their career.

[17] In many ways, and in certain countries, the relationship between the state and its citizens resembles that of a parent and child. The state „takes care” of the citizen's long-term safety (pension, healthcare) very similarly to a parent taking care of their child's, and the citizen expects this from the state (paternalism). In such cases – to which the Hungarian practice is very similar – the citizen remains a „child” in some sense regarding his own state of affairs.

Education (intelligence) level: The effect of intelligence can also be seen very clearly, which can be measured most closely by level of education, as they show a very strong correlation.[18] A more intelligent person is better at finding his way in the world, can see the connections between things more easily, and can distinguish between the important and less important factors, which all aid in better foresight. It can be observed quite frequently that social classes with low income but high education levels (as are doctors and teachers in Hungary, unfortunately) are able to do more in the long term (for example, educate their children, save for their old age rather than get caught up in the fashionable consumption trends) with the same income as classes with similar means but lower education levels.

Marital status: a single adult can much more readily live from day to day, and let their life "go with the flow" of things, than a parent with children, who cannot allow for great ups and downs in their financial situation, and must strive for stability – which can be achieved mostly using foresight.

Economic situation: it is also true as a tendency (so it is not true in every concrete example) that wealthier people have more foresight than poorer people. Of course, wealth can be exchanged or compensated for by intelligence, or frequently wealth does not compensate for lack of intelligence. We could also say that wealth allows a person to be free of the burden of worrying about day-to-day survival, and this makes it possible to foresee a longer time period. This also makes it possible for someone faced with a choice between an alternative that pays well in the short-run but has long-run drawbacks, and one that pays off in the long-run only, to choose the latter if he is wealthier, while a poor person will be forced to choose the first (for example, certain occupations that are harmful to their health, but pay well are more likely to be filled by poorer people rather than the rich, etc.).[19]

Historical situation: It is very important, and a strong determinant of the possibility for foresight whether a person is living in a consolidated society, or one that is changing rapidly or perhaps at war? The basis of all foresight is a civilization that is predictable and consolidated, the presence of rule of law, etc. During the siege of Budapest, in the bomb shelters, the time horizon of foresight was probably only a few hours (can I eat today? Will I have a place to sleep? Will I be alive at all tonight? etc.), not a few years. In a lesser degree, but it is also equally important whether an economy is in an economic boom or a recession, or perhaps undergoing basic institutional reform (as was Hungary in the first half of the '90s)?

Degree of civilization: Despite the fact that within every country there are careful and careless, single or married, richer and poorer people etc., the inhabitants of specific countries are much more similar to each other than to inhabitants of certain other countries. The reason for this is that as in the other areas of social life, people follow the examples they see, so they do things similarly as others, as they learned from their parents, at school, etc. These examples reflect the collected knowledge of the given country, which we could also call civilization. Civilization will appear in this book, in terms of foresight, as a summarizing category, or as a category that, along with foresight, mutually determine each other. In other words: the degree of civilization is higher if the time horizon of foresight of its members is longer, and vice versa: people with better foresight are at a higher degree of civilization. A very simple example: in the United States, saving for retirement is a very common and

[18] At the same time, the relationship between intelligence and education level is complicated, we cannot simply say that a person with higher education always has better foresight than someone with lower education, or that a more educated person is more intelligent than a lesser educated one, but in terms of general tendencies (so with many exceptions) this is the case.

[19] A profane example: in Hungary it is typical that the maintenance of buildings is not thought of as something that must be done continually, but rather people live in them until they almost collapse, then they are renovated. This can be seen in the public buildings or at restaurants. It is worth visiting these places for one or two years after their renovation, because the bathrooms are still clean, the paint is not peeling off the wall yet, but later one should look for a new hangout! This – in my opinion – is closely related to the fact that Hungary is not yet a rich enough country!

obvious activity, as is life insurance, etc. On a societal level, the USA has the ability to think about a defence for such far-away, unlikely risks as being hit by an asteroid. As a contrast, in Ethiopia, the provision of every day sustenance for the widest social classes presents the longest time horizon that they can foresee.

*   *   *

In the following, we will examine more closely some of the above factors (for example material wealth, age, marital status) involved in the planning of the life cycle. But first we want to find out why it is that we are dealing with the issue of the planning of the life cycle at this particular time in our history.[20]

2.2. The Human Life Cycle Throughout History

If we take a look at the changes of life expectancy in the western world in the last few decades, the gradual, and, in the 20th century, highly accelerated growth of life expectancy is very apparent. This tendency is shown in the following table as well.[21]

Country	1750-1759	1800-1809	1850-1859	1880	1900	1930	1950	1987
England	36,9	37,3	40,0	43,3	48,2	60,8	69,2	74,5
France	27,9	33,9	39,8	42,1	47,4	56,7	66,5	76,1
Sweden	37,3	36,5	43,3	48,5	54,0	63,3	72,3	77,2
Germany	-	-	-	37,9	44,4	61,3	66,6	74,8
Italy	-	-	-	35,4	42,8	54,9	65,5	75,9
Netherlands	-	32,2	36,8	41,7	49,9	64,6	71,8	76,8
Soviet Union	-	-	-	27,7	32,4	42,9	64,0	69,4
USA (white population)	-	-	41,7	47,2	50,8	61,7	69,4	74,8
Australia	-	-	-	49,0	55,0	65,3	69,6	76,0
Japan	-	-	-	35,1	37,7	45,9	59,1	78,5

Table 2.1.: Life expectancy in various western countries (1750-1987)

Among some other things, one of the main reasons behind this increase in life expectancy is the decline in the previously high rates of infant and child mortality. The historian Imhof says the following about this issue based on a graph of XVIII-XIX. century statistics (also outlined in his book):

„The computer graph[22] shows those who died each year in three dimensions, based on their absolute age. The first thing that jumps out at the observer is the large black wall in the background, which stands for the huge infant and child mortality of the time. Out of the 39251 people who died, no less than 12193, or almost a third were infants below the age of one. If we add to this the number of deaths of children below the age of eight, we've already reached half the total number of deaths (50,6%). The remainder is then pretty much evenly distributed among the 9 to 90 year olds.

[20] Todat, certain insurance companies are involved in the financial planning of the life cycle (see: M. Rimay Andrea: Pénzügyi tervezés a nyugdíjas évekre – Megelőzető az életszínvonal csökkenése – Népszabadság, 1999. december 15.). Some support their sales of life insurance with complete life-planning programs, or with acquisitional materials of such a nature (see. The satellite group of MÉBIT, or the acquisitional „Ariadné fonala" (Ariadne's yarn)!), others with demonstrative figures based on life cycles (see, for example, the figure „Emberi élethelyzetek" (Human life situations)on the website Hungária Biztosító Rt.!).
[21] Massimo Livi-Bacci: A világ népességének rövid története, Osiris Kiadó, Budapest, 1999. – p. 138.
[22] Unfortunately this cannot be shown here.

At no other point in time in the later life was there a grouping of deaths even nearly as large. One person lived to twenty years of age, another to forty, the third to sixty, eighty, or even ninety or older."[23]

The observable increase in life expectancy beginning in the middle of the 18th century fits into a more general trend, according to Livi-Bacci: "from chaos towards order". From this time on "the order of mortality dictated by age became more stable. As opposed to the chaos of earlier times, which were characterized by random and unforeseeable death, the life processes started to become more orderly."[24]

What do we mean by "orderly life processes"? First of all: children generally die later, than their parents – as opposed to the tendencies experienced up until the beginning of the 19th century. Secondly: death has its own place, it does not occur randomly during a lifetime due to war, epidemics, starvation, etc., but rather at the end of a "normal", we could say "standardized" life cycle. By standardized life cycle we mean that a person experiences within their own lifetime all the main stages of a life cycle (child, adult, and elderly stages).

The formation of the standardized life cycle was made possible by the increase in the lifespan. The life expectancy at birth became sufficiently long during the 20th century such that it became highly probable (in Hungary the probability that a newborn will live to age sixty was 70% for men and 87% for women in 1998) that an infant born would reach elderly age, or 60 years.

The evolution of a standardized life cycle had a major role in the starting of the planning of individual life cycles. It is impossible to plan an individual life cycle without some certainty about the basic frame of the life cycle. As long as death was uncertain, the "planning" of the survival of the community (the greater family, earlier the village community, even earlier the clan) was more characteristic than individual planning.

Accordingly, the planning of the life cycle as a separate financial service only appeared in the last few decades in the western countries, and in Hungary it is just beginning to gain ground nowadays.

2.3. LIFE PLANNING AND WEALTH

From the previous points, it is obvious that the ability for foresight is dependent, among other things, on the degree of wealth. Experience shows two conclusions which can be seen in large numbers:

1. The wealthier a country, the better foresight its citizens have
2. Within a country – regardless of its economic situation -, the higher the status (and thus the wealthier) the social class we look at, the more signs of foresight (and future planning) we can observe.

According to one survey[25] there is a strong correlation between the amount spent on insurance and the level of the GDP.

[23] Arthur E. Imhof: Elveszített világok – Hogyan gyűrték le eleink a mindennapokat – és miért boldogulunk mi ezzel oly nehezen…, Akadémiai Kiadó, Budapest 1992. p.214-216. The quoted graph: The distribution of deaths by age in the area of Berlin-Dorotheenstadt between 1715-1875.

[24] Livi-Bacci: p. 134.

[25] Dr. Kovács Erzsébet: A biztosítás fejlettsége Magyarországon, Biztosítási Szemle, 1999. Issue 11-12.,p.36-45.

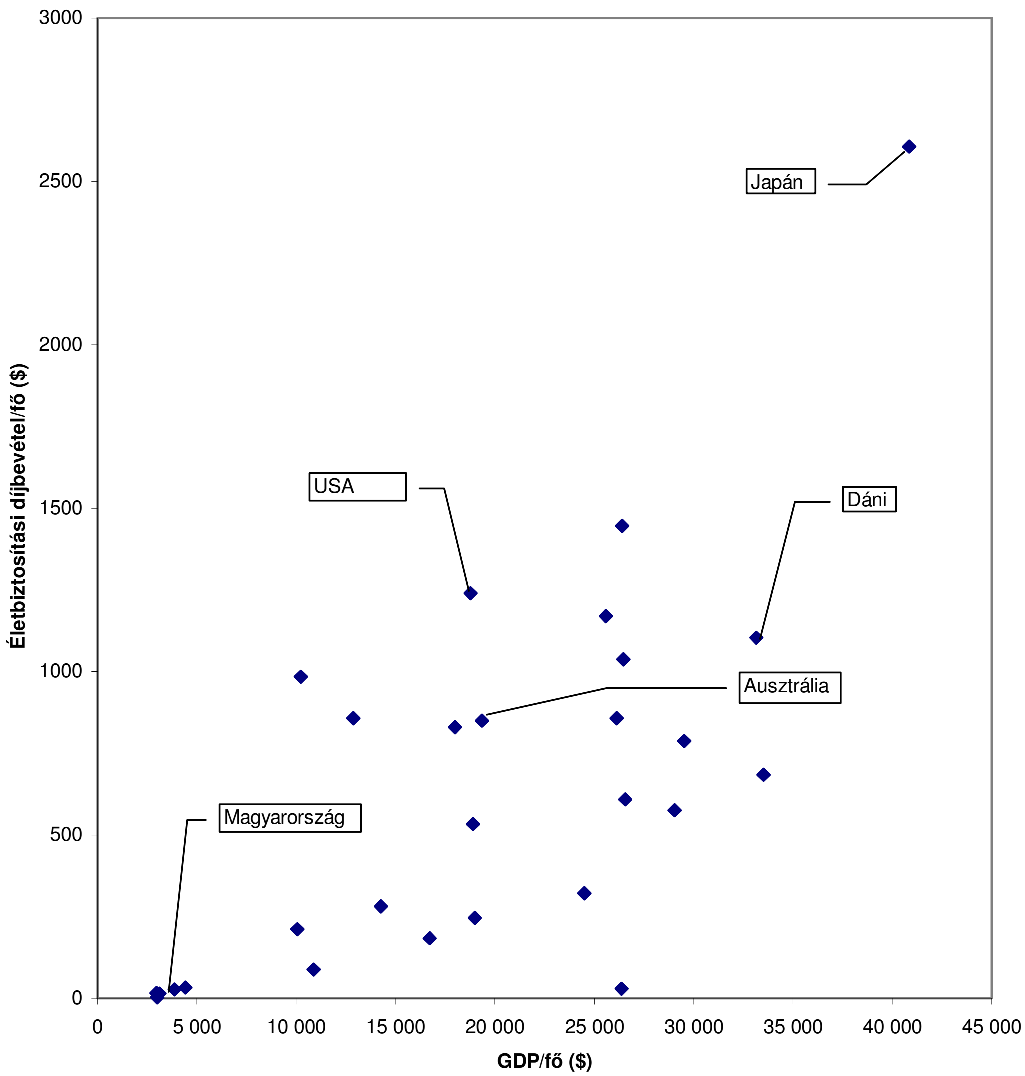

Figure 2.1.: The relationship between GDP per capita and the amount of life insurance per person in the OECD countries (with the exception of Luxembourg) in 1995

In figure 2.1 we can clearly see that:

- The wealthier a country (so the higher its GDP per capita), the greater the expenditure on life insurance per capita
- The higher the GDP, the higher the fraction of it spent on life insurance[26].

Some further examples of the ratio of life insurance/GDP: in 1995 the percentage of the GDP spent on life insurance was 0.1% in Turkey, where the GDP per capita was 2747 USD, in Hungary this was 0.72% with a GDP per capita of 4273 USD, and in Switzerland, where the GDP per capita was 43 000 USD, this number was 6.66%.

We can regard the amount of life insurance as something that is positively correlated with the degree of foresight.

[26] This cannot be seen directly, but it can be calculated quite easily!

In addressing the low ratio of insurance in the countries with low GDP, we can say that while a country is relatively poor, the pressure to fulfil the primary needs (sustenance, clothing, shelter) is so strong, that there are no resources left for higher-level needs, and we can regard the security of older age as such a higher need. So the opportunity for foresight/planning decreases – at an increasing rate – as wealth decreases. In the poorest countries and social classes we can simply see people living from one day to another, without any kind of foresight.

2.4. VARIATIONS OF THE LIFE CYCLE

Based on what was stated above: the basis for the planning of the life cycle is the evolution of a standardized life cycle that is expected to be rather long, comprised of all the possible life stages, as well as a population with a significant ratio of sufficiently wealthy people – or the development of a sizeable middle class.

In the planning of a concrete life cycle we must consider other important factors beyond the lifespan and sufficient wealth, namely the family situation, which is related to age as well. The following figure[27]shows the main variations schematically:

[27] Source: Bauer-Berács: Marketing p. 54. (based on Murphy-Saples [1979], p.12-22.!)

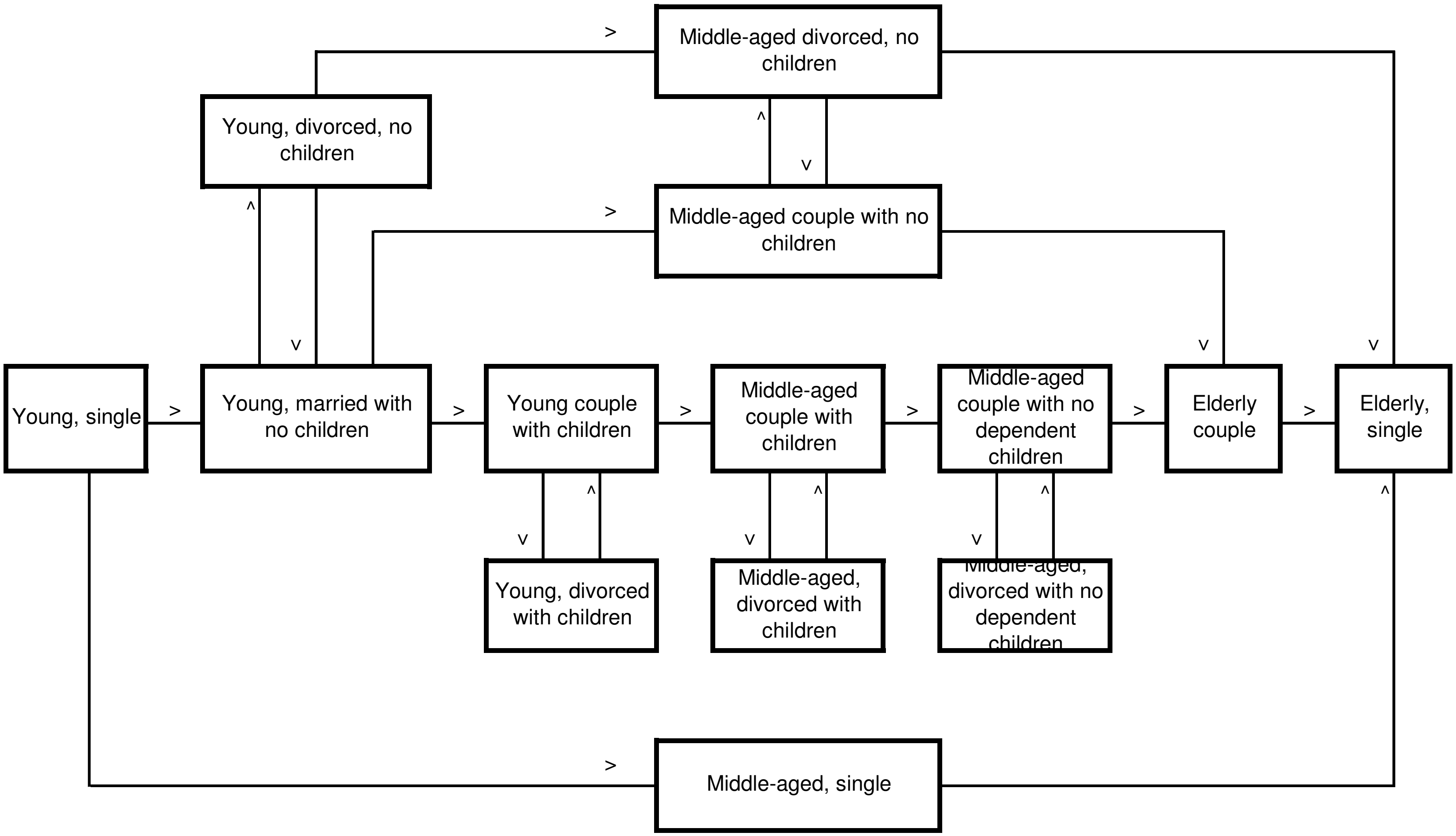

Figure 2.2.: Variations of the life cycle

A typical life cycle can be regarded as the life cycle line on the horizontal axis in the figure:

Young, single → young couple with no children → young couple with children → middle-aged couple with children → middle-aged couple without dependent children → elderly couple→ elderly, single (probably widowed)

If we are speaking of a life cycle in general, this is usually what we imagine, and this will be the basic concept in the rest of the book as well. At the same time, it is important to note the exceptions, which appear in the graph as deviations from the main axis. It is possible to get a divorce while one is still young, with or without children. It is possible to get married more than once. Perhaps a couple – by choice or not – will not have any children, or someone may not get married. Furthermore, the figure does not include some increasingly popular (or at least more common) cases, such as a lifelong partnership without marriage, or that of having children without marriage or a lifelong partner, or the more rare cases of long-lasting relationships between two people of the same gender, or communes[28].

In terms of financial planning, it is very important to be aware of these possible variations, and in concrete cases one must seriously consider them. As we will see, in the financing of the life cycle it matters how many children one raises, and whether one does so alone or with a husband or wife (or lifelong partner), or whether the children come from a single or multiple marriages? From a social perspective: if a married couple raises two children, then they pretty much pay back to society what had been spent on raising them themselves. More than two children[29] can be regarded as a net contribution to the welfare of society. At the same time, single people or married couples without children have a lesser burden, they can achieve a higher standard of living, but they do this by failing to fulfil their obligation to society.[30]

2.5. The Cash Flow of the Life Cycle

The life cycle can be divided into the following economic stages:

In the active phase of the life cycle, a person makes enough income for current needs, and usually more than that. In the economically inactive phase, there is no current income from work, so either he depends on financial support from others (parents, relatives, goodwill organizations, the state) or lives from his own (inherited or saved) assets and its interest. Since we are also the parents, relatives, and tax payers who support the state, a country does well overall if its citizens individually make enough income not just for their current, but their lifetime consumption, or somewhat more than that. If some people make less than that, then there are reasons and consequences:

- Living off of other people (or "being a parasite") or
- Living up the inherited assets, or
- An inherited or received physical or mental disability (being handicapped, or mentally retarded) in the case of which society accepts the consumption of the

[28] After all, the figure was made in 1979, and it reflects the beliefs of the time period.

[29] An important constraint! Only if they did not just give birth to the children, but also raised them in a satisfactory way, and they received proper socialization so that they can become useful members of society. Someone who has a child or children without being able to fulfil these criteria is actually not paying back, but rather increasing their debt to society!

[30] A possible solution to this could be a childlessness tax, which is really a way of getting back the cost of raising and schooling from those who are trying to avoid paying back their debt. The amount would be equivalent to the cost of raising a child.

product of others' work without contribution (or with a lesser contribution) to the creation of goods out of solidarity[31], or

- Early death, which keeps one from beginning the active phase or from completely covering one's consumption up to that point.

If it becomes the norm in a society for active people to generate less income than what they consume during their lives, then that society:

- Uses up the assets compiled by earlier generations,[32] or
- Systematically robs other countries,[33] or
- Has compiled assets[34] that it can live off of without losing the assets (and without currently making an effort).

None of the above cases are typical, so the original concept can be upheld – with some clarification:

A society is all right financially if its members who are capable of work and have average lifespan make as much income as they consume during their entire life cycle, and beyond that enough so they can contribute to society (in the amount needed) to support the members who are unable to work.

Further clarification: it is implicitly assumed in the above that the reserves that are freed up when the people who produced them died at the end of their active phase before they could consume them compensates for the shortage caused by those who died earlier. If this is not the case, so if more people die too early, this shortage will increase the solidarity burden weighing on those who had average-length life spans. If, on the other hand, there are more reserves freed up by those who died at the end of their active phase than what is needed for the above compensation, then the next generation of the society will start out better off than the previous one.[35]

The following figures represent the above relations:

[31] We will not list here among the cases of solidarity those who are theoretically able to work, but unable to support themselves due to social (lack of training) economic (regional differences between industry and work force, an unsatisfactory structure of existing and needed education, general economic recession). In these cases the need for solidarity is temporary, while those able to work but inactive can re-train themselves, or move to the appropriate location. Then our general observation will apply to them as well. If such a person still stays jobless permanently and society is forced into solidarity, then there is something wrong with the socio-economic apparatus – but we will refrain from dissecting this issue further here.

[32] Perhaps Spain could be brought up as an example here at the beginning of the New Age. Although it is true that those compiled assets came from robbery (of America).

[33] Behaviour that is typical of the colonial times. See, for example, the one-sided stream of income from India to England between the middle of the 18th and 20th centuries.

[34] Certain „oil countries" came close to being in this situation. For example, Kuwait (especially before the Iraqi invasion) but even Norway accumulated certain reserves. At the same time, members of a society that is prone to being passive and living off the interest of assets will sooner or later be left out of the „main flow of events" and be left behind, their assets losing their value.

[35] It is advisable then to avoid compensating the excess resources freed up in this way by consumption without work, perhaps due to a mistaken state solidarity policy (for example, long-term unemployment benefits, without necessary re-training programs). – Important! We are not talking about the Hungarian situation here, or valuing it, but general relations!

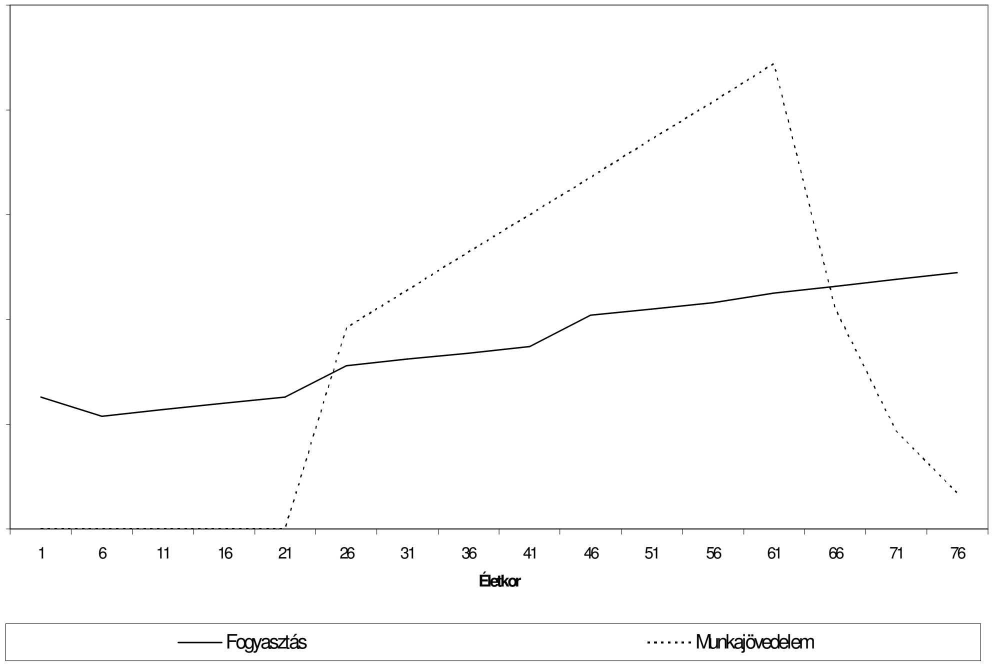

Figure 2.3.: The relationship between income and consumption

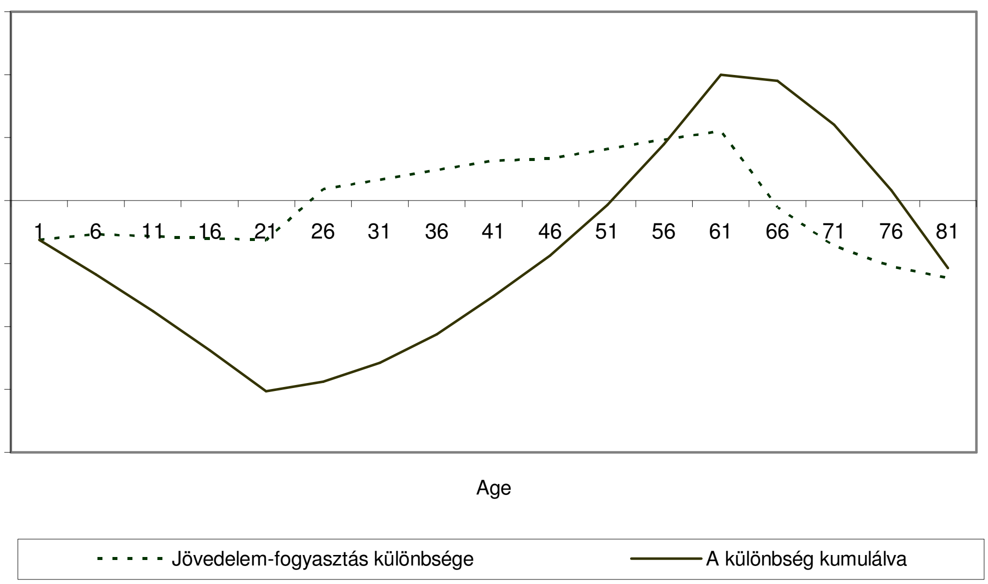

Figure 2.4.: The cash flow of the life cycle

Figure 2.3. shows the general relationship between income and consumption during an average-length lifespan. The above-mentioned relations can be seen clearly, namely that in the active phase the income is greater than consumption,[36] and in the inactive age either there is no income from work, or it is well below the consumption.

[36] Not counting the indebtedness due to the long-term „investments" (mainly buying a home)!

Figure 2.4 shows the cumulated amounts and the long-term equilibrium of income and consumption.

According to the figure, during our childhood (our first inactive phase) we surmount a huge and ever-increasing debt to “society”,[37] which we begin to pay back in our active phase, and which disappears around the end of our middle age, and turns into an ever-increasing surplus, which in turn is gradually used up in our second inactive phase. (In the figure showing the relationship of income and consumption, we still assume a significant, though strongly decreasing income during the retired years, which goes along with the tendency that today’s retired people are healthier, and thus more “active”[38].)

The usual form of form of paying back the debt accumulated during our childhood[39] is the having and raising of children. The “debt towards society” can be paid off by the raising of two children by each couple. If there are more than two children, we are paying off the debt in place of others as well (if the average number of children per couple does not surpass 2) or we are a part of society’s “investment in the future” (if the average is greater than 2 children per couple), or both (if the number of children we have is above the average).

The figure suggests that the incurred debt is paid off around the time when our children reach their active phase, and then the net accumulation begins. It is important to call this accumulation a “net” accumulation, because the above figure does not reflect the structure of debt and surplus. So while the children are raised, the creation of the surplus already begins, but due to the existing large debt at the time the accumulated outcome seen on the figure is still negative, the paying back of the debt is still going on (so this is during the young adult stages of our children’s lives – for example, during their university years, and afterwards) when overall the surplus has surpassed the debt.

According to the figure, our “lifetime wealth” starts from 0 at the beginning of our lives, and goes back to 0 at the end. This – and the assumption of a three-phased life cycle[40] itself – is a simplification, behind which there stand a few assumptions, or from which there are some deviations.

Assumptions:

- Since the length of the life cycle cannot be foreseen for each individual, only as an average for a group, we assume a mechanism that equalizes over the society, which redistributes the money required for an average-length lifespan according to the needs (this book deals with such mechanisms regarding the life insurance and social security)
- People are born without any assets and die without leaving behind any assets
- The life cycle has three phases and those phases are of normal length, so – most importantly – no one dies during their active phase, or loses their ability to work (so the active phase is not too short).

Possible deviations:

[37] We could also say that, contrary to our parents, we do not pay them back, or only in a small part, and also by raising us they were paying back their own debts.

[38] This does break down somewhat the theoretical basis for the separation of active-inactive phases, but that’s life: we can only more-or-less speak of active and inactive phases, not with clarity!

[39] „Usual”, because we can also imagine an institutionalized form of this as well. This could mean that everyone pays the costs of their upbringing to a common fund in the form of taxes or fees, and this is then used to pay for the upbringing of every child born (as a kind of child support). This solution – which is, after all, being implemented by some Scandinavian countries – has the advantage that every child receives the financial resources needed for their upbringing, to some degree independently from their parents’ financial situation. Its drawback is almost the same thing, in that it assumes that there are no income differences within the given society. In the case of large income inequality it is difficult to determine a normative support that would not be too little for the wealthier classes, or tempt the poorer classes to use the money received from the common funds for some other purpose. In such cases – as in Hungary – the „usual” solution remains, which the existing child support cannot change much.

[40] So the inactive-active-inactive phases follow each other!

- There are some people who do not start out with zero, but rather a significant amount of inherited wealth, and during their life cycle they either increase this further, or use it up, so they leave more or less to their children[41] (or: they may even leave a debt behind)
- It is possible – in fact, highly probable – that consumption fluctuates much more during the life cycle than suggested by figure 2.3. The timing of children may be at very different stages of our lives as well, or due to the differences in the ages of children, the "paying back" of the debt could be prolonged for a long time.
- The life cycle consists only of an incomplete inactive, or a full young inactive and an incomplete active phase.

Of the possible deviations, the last one is most significant from the point of view of this book. The next chapter deals with this deviation and its consequences, as do the sections dealing with different personal insurance forms in detail. For now let's just say that during the active phase of the life cycle, not only do we need to produce enough income for our consumption over life, but beyond this enough to cover the consequences of any unexpected occurrences that may keep a person from paying back their debt (or raising their children), from producing the goods needed to support themselves, and from performing the necessary accumulation.

2.6. The Structure of Cash Flow During the Life Cycle

The above aggregated cash flow must be examined in its composition, in particular what the sources of our revenues are, and exactly what our expenditures are for. This structure depends on two factors:

- age and
- socio-economic situation.

The **structure of outgoing cash flow** depends strongly on the current phase of the individual's life cycle (which can be best represented by age)[42], their social situation, and also on which version of the possible life cycles the individual is living. First we will concentrate on the differing structure of expenditures during the different phases of the life cycle assuming a typical middle-class *life cycle.*

Life cycle phase (age)	Description of phase	Typical expenditures	Financial decision maker
1-6	Small child age	Basic necessary goods (food, clothing, shelter) (=necessary)	Parent (guardian)
7-18	Elementary and high school	Necessary + educational + recreation	parent + independently regarding pocket money
19-23	University	Necessary + educational + recreation + travel	parent, state, independently
24-27	Young entry-level worker, single	Necessary + educational + recreation + travel (with different ratios!)	Independently, state
28-30	Young married without children, beginning of	Necessary + educational + recreation + home buying (also different ratios!)	same

[41] It should be noted that leaving an inheritance is not necessarily a voluntary decision. If, for example, the state taxes its citizens significantly, and from that builds roads etc., then someone can leave a significant amount to the next generation even if he dies seemingly without a penny to his name.

[42] But is should be known that at the same age – especially in the adult ages – different people are at different stages of their life cycles. Some are already parents at age 18, some only after 30. Some reach the pinnacle of their career by age 25, some move forward gradually, etc.

Life cycle phase (age)	Description of phase	Typical expenditures	Financial decision maker
	career		
31-40	Young married, with small child,	Necessary + educational + recreation + travel + child-raising + child's education + home(payment, renovation, trade) car + precautionary saving for self and children	same
41-50	Middle-aged married with bigger children, pinnacle of career	Necessary + self-training + recreation + travel + child raising + child's education + home(payment, renovation, trade) car + precautionary saving for self and children (different ratios!)	same
51-65	Middle-aged married, no dependent children, stable, high income	Necessary + self-training + recreation + travel + occasional support of child + home(renovation, trade) car trade + precautionary saving for self (different ratios!)	Increasing financial independence
66-75	Elderly married, active retired	Necessary + self-training + recreation + travel + home renovation + car trade	Financially independent
76-85	Elderly widowed, retired	Necessary + recreation + travel + healthcare	same
86-	Elderly widowed, in need of care	Necessary + healthcare + personal services/care	Decreasing independence

Table 2.2.: The structure of outgoing cash flow

Figure 2.5. summarizes schematically the most important expenditures:

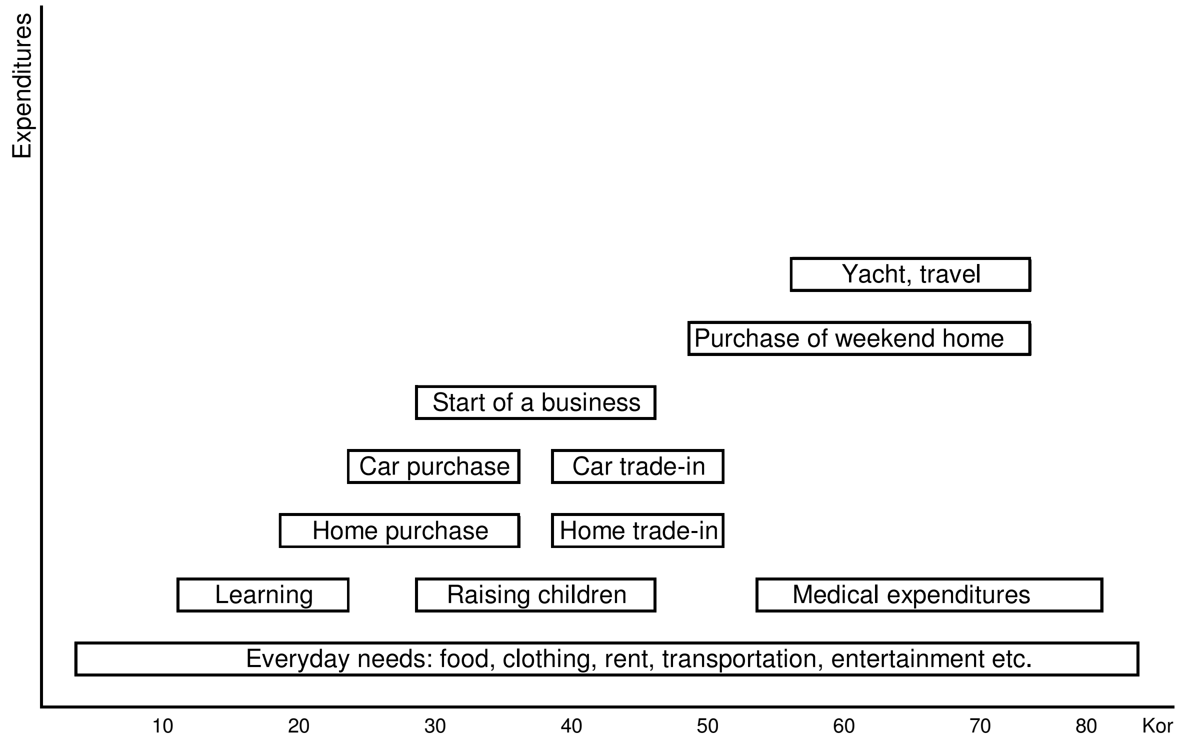

Figure 2.5.: The structure of expenditures as a function of age

The **incoming cash flow** can also be considered in a break-down similar to table 2.2:

Phase of life cycle (age)	Description of phase	Typical incoming cash flow
1-6	Small child	none (accumulation of debt)
7-18	Elementary and high school	Basically none, or some pocket money, later some part-time jobs (debt accumulation continues)
19-23	University	Regular part-time work, pocket money and further accumulation of debt
24-27	Young entry-level worker, single	Regular wage income
28-30	Young married without child, beginning of career	Regular wage income
31-40	Young married, with small child	Regular wage income (paying back of debt)
41-50	Middle-aged married with older children, pinnacle of career	Regular wage income, beginning of income from capital (paying back of debt)
51-65	Middle-aged married without dependent children, stable, high income	Regular wage income, significant income from capital (debt is paid!)
66-75	Elderly married, active retired	Basically income from capital, perhaps pension from pay as you go social security system, irregular wage income

Phase of life cycle (age)	Description of phase	Typical incoming cash flow
76-85	Elderly widowed, retired	Basically income from capital, perhaps pension from pay as you go social security system
86-	Elderly widowed, in need of care	Basically income from capital, perhaps pension from pay as you go social security system, perhaps income from the activated LTC[43] private insurance

Table 2.3.: The structure of incoming cash flow

The main tendency of incoming cash flow as age increases:

credit → wage income → capital income (+pension) → (perhaps) income from insurance

Naturally this varies depending on social situation. In social classes with higher income, the capital income is dominant, possibly even the only form of income during the life cycle.

Figure 2.6 contains a summary of the most important cash flows and reserves. The horizontal axis shows the phases of the life cycle, on the vertical axes (since we put two graph on top of each other!) are the Forint amounts.

[43] Long Term Care: see in the Explanation of terms appendix!

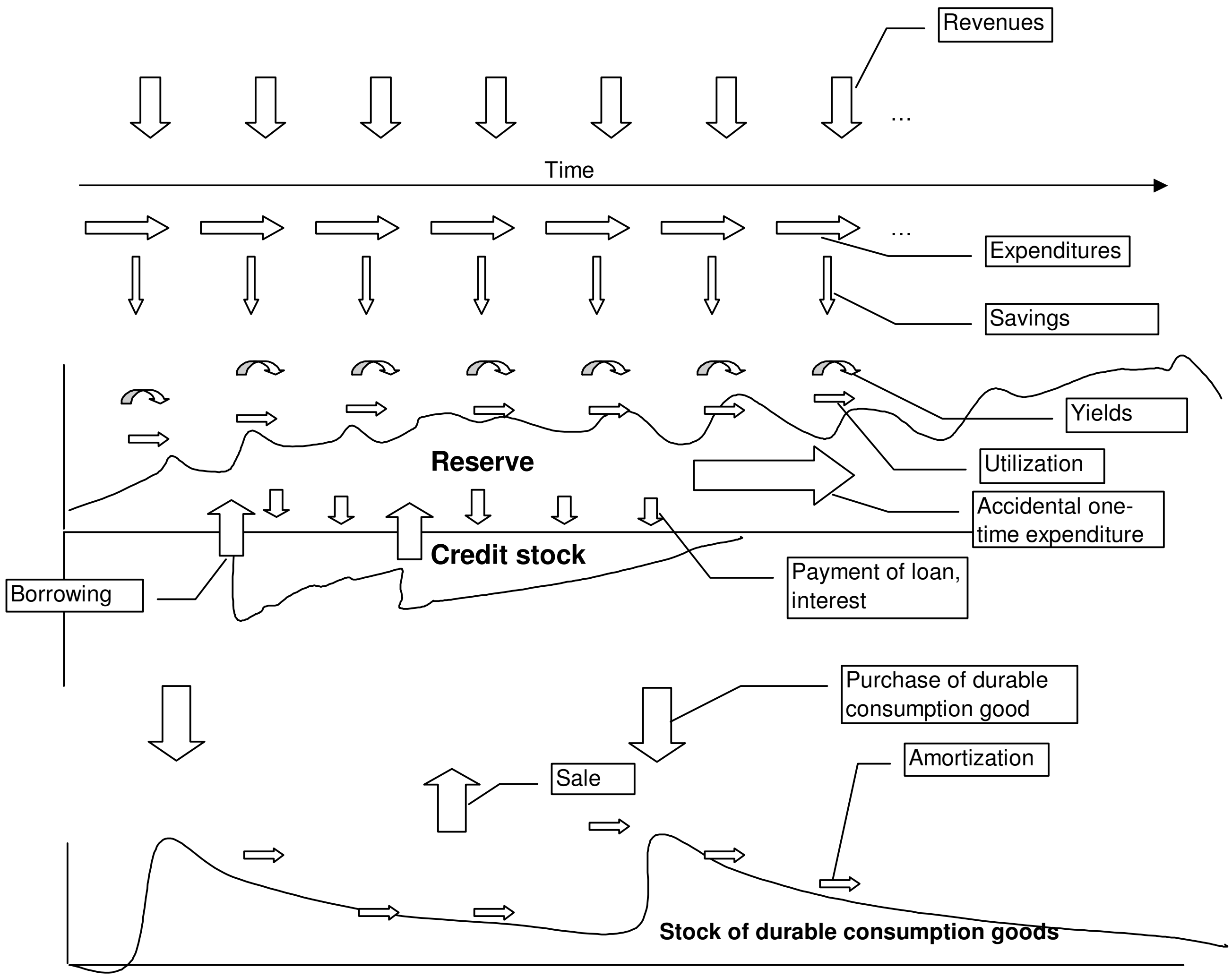

Figure 2.6.: The structure of cash flow during the life cycle

The “debt to society” that is accumulated during childhood is not marked on the figure, it concentrates on the stocks formally showing up as debt. The stock of debt can be regarded as negative surplus accumulation, so it is shown along with the stock of reserves, as its mirror image. The net reserves are the difference of the reserves and the stock of credit.

2.7. RISKS THREATENING THE CASH FLOW AND THE METHODS OF DEFENCE

The following requirements must be met by the cash flow of the life cycle:

- Liquidity should be assured at every moment, so the needs of the individual must be financed
- There should be sufficient funds for the achievement of goals reaching beyond the individual (care, leaving a legacy, other obligations to society)
- Great fluctuations of the standard of living should be avoided if possible (especially large drops in it[44])

The standard of living is also expected to grow continually[45]

Let us systematically examine what risks threaten the financial life program reflected in the figure of the last chapter, or the attainment of a three-phased life cycle, and how these can be defended against – using financial-type instruments?

The most important threats:

- Death
- Becoming unable to work
- Inability to produce income
- The devaluation of reserves, the falling apart of the system of institutions that serve as the frame of our life cycle

Of the above risks we will only deal with the first two here, and to a lesser extent the third. The description of the last threat is not the subject of this material, but it is important to be aware of the fact that objectively thinking there exists the possibility of a war, a large scale economic crisis, regional or world-wide catastrophe, which the country, or the suitable institutions must prepare for, and which must be avoided along with its consequences.

Death is unavoidable, so it is not its occurrence in itself that causes problems (at least not from the point of view of long-run financial planning), but rather when death doesn’t occur at the “appropriate time”, or not at the social average. In this respect, two kinds of deviations are possible:

- Earlier than average, or
- Later than average death.

Later than average death causes a problem if we want to end our lives with 0 wealth (or if we don’t want to leave anything behind). In this case there is a need for a risk community, which redistributes our accumulated wealth among those still living. The solution to this (as we will see!) is the annuity, which is dealt with by life insurance.

If someone wants to leave some wealth behind, then he can live off of its yield for any length of time.

The picture becomes more differentiated in the case of earlier than average death. The problem is different if death occurs within the following intervals:

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[44] Large increases are also better avoided, because they may cause a fallback later on. When someone’s actual income rises, the temptation to immediately increase their standard of consumption is strong. Before anyone does so, he should ask the question: „Will my increase in income be permanent, will it make this high consumption level sustainable in the long-run?” It is also wise to include: „Did I raise the level of my savings by enough to ensure that I will not have to lower my consumption after I retire?”

[45] Historically, this is a relatively new expectation, but nowadays it has completely assimilated into the expectations of western man, and seems only natural. More precisely: our expectation is that our standard of living should not grow more slowly then that of the groups relevant to us (for example „neighbours”). Overall, it is important that a person should feel that they are „constantly moving forward”.

- 0 – about 30 years old (from birth to the birth of the first child). We do not deal with this case here, we assume that at this point the person has not seriously considered the planning of the life cycle. From a social point of view, we can say that we should save up for such cases as well, so couples should raise more than two children on average for this reason as well.
- about 30 years old – about 50 years old (from the birth of the first child to the end of raising the last). In this case, death occurs when the individual has not yet finished paying back his debt to society,[46] but has started to do so. For this purpose, a risk community with this specific aim should be created (see below!). (We'll discuss this later under term insurance when introducing the types of life insurance! Of course this problem does not come up in the case of sufficiently large inherited or later – with small probability – attained wealth.
- about 50 – about 60 (after raising the last child, before retirement). In this case the debt to society has been repaid, and the individual just started accumulating for retirement. Death is a problem here if there is still an obligation to support someone (for example a non-working spouse). In this case as well, the term insurance is the solution. If there is no such obligation, then a risk community based on pure endowment insurance is ideal, since then the money accumulated for ourselves will go to those who may still need it (and, of course, this is worth doing because we I do not know if I will be the one who receives the money).
- about 60 – about 75 (after retirement, before the average lifespan). This case is the opposite of the case of a long lifespan discussed earlier, so its solution is also the annuity.

People may become temporarily unable to work (and thus make a living) during some time of their active life due to illness or accident. During an average life cycle we can defend against this with the forming of a risk community (social security, accident, medical or disability insurance). The chapters on accident and medical insurance[47] deal with this in detail. The costs of forming a risk community are added to the costs of a standardized life cycle beyond what we have discussed (since anyone can have an accident ...!)

It is very important to consider what expenses these risk communities should provide coverage for at these times:

- for the current treatment of the consequences of the illness or accident
- to compensate for the living costs of the income earner (as long as the inability to work lasts)
- for the current consumption of dependents (the foregone payment of the "debt")
- for the foregone savings for retirement age

There may be other reasons for inability to work beyond accidents or illness – basically in the case of unemployment. This is sometimes related to the previous (so someone cannot perform his earlier job duties due to illness or accident, but is not in general unable to work), but mostly it is due to socio-economic reasons, and so its discussion is beyond the scope of this book. What we should note regarding the planning of the life cycle is that the individual can do the following to avoid or defend against the effect of threats:

- does not rely solely on one occupation, but trains himself to perform multiple jobs, and educates himself continually
- „keeps his eyes open" for new opportunities and for signs of problems, and tries to take advantage of these or defend against these ahead of time
- tries to become independent of employers, and start his own business
- always has a suitable size of reserves for transitional situations

[46] This is similar to the first case as well, but does not cause personalized financial problems for others.
[47] These do not yet appear in this book, but we hope they will in the later versions!

The most important tool in our defence against the financial consequences of various threats is the accumulation of **reserves**. This is such an important instrument, that it can be substituted for all others[48] – which is why, in the above, we always treat the suitable size of wealth[49] as an exception.

The reserves can be temporary ("precautionary reserves") or for the longer term. Here we must primarily deal with long-term reserves, but the various temporary reserves are also important, which smooth out the occasional fluctuations of the cash flow. The various financial-type fields deal specifically with the issue of reserves.

The form of reserves can be of many different types, and this also depends on the aim of the reserves. The temporary reserves must be liquid (cash, demand deposits), or easily turned into liquid form. The form of long-run reserves is more likely higher interest bearing, less liquid pension funds, life insurance, investment fund, treasury securities, bonds, stocks, property and possibly (only in special cases) durable consumption goods (automobile, furniture, etc.). The following figure summarizes the major forms of saving, according to three important points of view: liquidity, interest, and risk.

Form of savings	Liquidity	Interest (and its possible fluctuations)	Investment risk
Cash	immediately usable, the most liquid	none	none
Bank deposit	quickly mobilized	little	negligible
Savings deposit	can be mobilized quickly with a loss	medium	negligible
Treasury bill	mobilized in a few months	medium	none
Long term government securities	sold in the market – mobile after the sales procedure	medium-high	none
Bond	same	medium-high	low - variable, depending on issue date
Stock	same	high	high
Voluntary pension fund	can only be mobilized after ten years	medium-high	medium
Private pension fund	can only be mobilized after retirement, only as a payment	same	same
Traditional life insurance	before maturity with a large loss, mobilizing is a several day long procedure	medium - high	low
Unit Linked type life insurance	as the duration moves forward with decreasing loss, but mobilizing is a several day long procedure	low - very high	depends on us: we can choose from low or very high risk
Property	the length of the sales process is unpredictable	negative – very high	very high
Own business	same	depends on us	mostly depends

[48] As a reminder: it can not be substituted for threats to the reserves themselves (catastrophe, war, social decay etc) but this is not the subject of this book.

[49] Naturally what this "suitable size" is requires further investigation, and depends on many factors, and also varies by country and social class. In any case, even relatively large wealth can only partially substitute for other solutions, so the spectrum of transitional solutions between wealth and others is very broad.

Form of savings	Liquidity	Interest (and its possible fluctuations)	Investment risk
			on us

Table 2.4.: Major forms of saving

The form of reserves also varies by social class. Starting from the lower-middle class, the ratio of long run reserves starts to shift from pension funds first to life insurance, then to securities (investment fund, government securities, bonds, stocks in this order) and property. The reason for this is that the fluctuations of the value of these differs (it increases in the order listed), and that the defence against these (portfolio-generation) is only possible with relatively high reserves on one's own, with smaller amounts it must be left to professionals (pension fund, life insurance, investment fund).

The entitlement to pension in the pay-as-you-go pension system can also be seen as a specialized form of reserves, which is guaranteed by the stability of the state's institution system (and its payment depends on this stability). The risk community discussed below can also not function without reserves.

The **risk communities** are a substitute for traditional communities, and, along with the larger communities (local governments, the state) can even take over their roles. The traditional communities (clan, extended family, village community, multi-generational family) have basically disappeared nowadays (in the developed countries),[50] and their earlier roles of protection-defence were taken over by the society's specialized system of institutions. The main elements of this system of institutions:

- risk communities,
- wealth generation,
- charity organizations (for example churches, foundations),
- local governments – state.

We have already mentioned the creation of wealth (reserves). Perhaps it is surprising to call this an "institution", but thinking about it: could a symbol on the computer function as wealth in Laos or Ethiopia? For this to be possible it is necessary to have a system of institutions that guarantees that everyone should view that symbol as a stock expressing ownership in MOL, etc.

The goodwill institutions and the state cannot be regarded as an instrument of self-protection – as we have already mentioned earlier – and thus will not be discussed here.

The risk communities have in their name the fact that they have taken the place of earlier "natural" communities. It is a crucial difference that compared to those:

- the risk community only fulfils a single, specialized task,
- the members of the risk community do not usually form a community otherwise, and don't even know each other,
- the risk community has a formalized set of rules of operation.

In its simplest form, a risk community is a community of solidarity (they jointly help those in trouble – this is more or less built on a natural community, for example that of a village), in its more evolved form (which is what we are speaking of here) it is the creation of a common reserve and its operation. The basic principle of the common reserve is that the unexpected events do not hit everyone at the same time, so it is not necessary for everyone to generate and maintain the total amount of funds individually, it is enough to do so in common. Thus

[50] With the exception of the grouping called the single "family", but what happens here can be thought of as a game of "who can define it smaller?". Does a single man with an adopted child constitute a family? etc.

the amount of necessary reserves can be lowered. This is especially important when the financial situation of the members of the risk community would not make it possible to generate the total amount of reserves otherwise.[51]

The formation of a risk community also has the following advantages:

- The entire amount of the reserves is always available, even if the members only contributed a small amount to its creation (so if the member had tried to create the reserves on his own, it might not even have been available to a sufficient degree at the time of the risk's occurrence).
- The reserves don't run out even after the repeated occurrence of the risk (so even if someone has enough reserves for a negative event, luck may have it that it occurs twice in a row, before the sufficient reserves for a second occurrence has accumulated, etc.).
- It makes planning the expenditures on random events possible (see below!).

The accumulation of reserves by the individual and by the risk community are substitutes to some degree. It can be clearly seen from the previous that the larger someone's wealth, the fewer risk communities he will have interest in, and vice versa. It is important to note, however, that with the increase of wealth the character of the risks and the risk communities that concern the individual change. The risk communities formed for the purpose of protecting physical wealth, for example, change with the type of wealth. In the case of the middle-class, a risk community against the burning down of their home is important, but the risk of their yacht sinking is probably not relevant. And a sufficiently expensive car is probably protected by its owner using more unique methods (for example, hiring a chauffeur), rather then via a risk community.

The risk community as a virtual community is nowadays mostly organized as an economic (so self-sufficient, profit-oriented) entrepreneurship, or perhaps as the specialized institution (social security) of an even larger community (local government, state).

The relationship of the risk community and its member – not forgetting that the risk community means the management of common reserves – can be thought of as a **transfer of risk** (from the member to the risk community, or the economic agent representing it). The main goal of the transfer: in exchange for a fee, the economic agent accepts a risk from the individual that he could not handle on his own (so its handling surpasses his own reserves). This fee is composed of three parts:

- contribution to the creation of common reserves (from which compensation takes place in the event of a risk's occurrence),
- the part of the fee needed to handle the administrative tasks of the risk community,
- a risk premium paid to the economic agent who runs the risk community in exchange for taking over the risk.

The transfer of risk simultaneously represents trade in several directions (realized in several projections):

1. **Risk transfer as trade in the market**: in this projection we can say that the transfer of risk is realized via trade in the market, so: the risk is sold by one who cannot undertake it to someone who can. In this case, someone whose potential loss surpasses his financial resources shifts its responsibility to some agent in the market (private individual, more likely a business) for whom it is not so large relative to their own assets. The acceptor of the risk is motivated by the risk premium he receives in exchange.

[51]This, of course, is not a contradiction only if it can be assumed that the event the reserves were created for does not occur in the majority of cases!

2. **Risk transfer as an evening out in "time" and "space**: the individual does not necessarily avoid the financing of the consequences of the risk that occurred because of the transfer of risk, in fact, in the majority of cases he has to pay more, than without the risk transfer. Still, it is a very useful and important thing, because it results in the evening out of the cash flow, so via the risk transfer he trades the uncertain, large loss (including the catastrophe – the collapse of the cash flow!) for a series of certain, small losses (payment of the fee).

This idea is equivalent to regarding the risk transfer as a way of evening out in "space", so something which allows a one-time large loss to be distributed "in space" among the current members of the risk community, since the large risks are unpredictable with respect to each individual, but with respect to the risk community as a whole it is a rather regular occurrence.

The usability of the risk transfer depends on the frequency of each given risk. Three different cases should be distinguished:

- occur frequently during a person's life (for example, a cold)
- occur rarely, but probably a few times during a person's life (for example, illness that lasts a few weeks)
- occurs only once or never during a person's life (catastrophic) large loss (for example, complete loss of ability to work due to car accident).

In each case a different strategy should be followed, and there are different expectations of what the risk transfer should provide.

In the **first case**, it is not appropriate to use any kind of risk transfer, this can be solved individually (with the use of short-run reserves).

In the **second case**, it all depends on the extent of the risk. If the loss that occurs infrequently is significant, it is worth using the risk transfer, if not, then the solution via individual means is best here as well. At the same time, if risk transfer occurs, another requirement appears regarding the evening out in time, namely: for each person the amount paid during the life cycle (subtracting the risk premium and the administrative costs of the risk community) should be in balance with the amount used.[52] Then the main purpose of the risk transfer is the evening out of the cash flow.

In the **third case**, risk transfer is pretty much mandatory, and, naturally, the addendum above regarding the evening out in time does not apply.

Insurance is the most important private method of forming a risk community, and gives the best examples of risk transfer. We can also say that the

Insurance = risk transfer realised via a virtual (risk)community

The remaining chapters will deal with insurance in greater detail.

One of the most important functions of both the risk community (or of the risk transfer and insurance) and the accumulation of reserves is that they allow the cash flow to be evened out in the case of unexpected events, so the person does not have to diverge significantly from his original life plan. So they make it possible to plan our life cycle ahead of time, and to stick with this plan. In a financial sense they make **computability** possible.

There remains one important question, which has to be made clear, namely: **who has interests in the individual's life**, and its suitable financial planning, how do these appear, and how can these be asserted? In the following we will summarize the most important

[52] Some authors, mostly dealing with policy issues, have a tendency to only look for state solutions to risks of this nature, and they cover up the question of the balance between contribution and usage with an ungrounded and not well-defined notion of "solidarity". As it will be seen later on, I try to define the operational space of solidarity in a much smaller space and more precisely.

observations in table form, specifically noting the most important private solution from the point of view of this material, personal insurance.

Who has an interest regarding the life of the individual?	what are the interests?	How are these realized?	The role of personal insurance
The individual himself	Maintaining the living standard attained in the active phase, ensuring its safety; protection against the inability to work; ensuring the re-establishment of the ability to work	Precautionary saving; annuity; accidental and disability insurance, medical insurance; leaning on the state; leaning on acquaintances, relatives, altruist institutions	saving-type products, annuities, accidental and medical insurance on a voluntary basis
The state	- social stability - management of obligation to provide for citizens - maintenance of international competitiveness via the improvement of the state of the population	- redistribution, mandatory precautionary saving and planning These can be achieved in a separate state system or via market agents Left to goodwill organizations	Can do everything except for redistribution – if allowed to
Dependents of the individual (children, spouse)	Financial security in the event of a fallout in earned income	The individual takes care himself leaves it to the state, acquaintances, relatives, altruist institutions	Either in its entirety, or as a complement to the state
Those effected by the individual's life (creditor, employer)	So they get their money even in the individual's absence, or to substitute for him quickly	Via insurance on the individual's life, health etc.	Almost entirely on a private insurance basis – often with group insurance
Voluntary social organizations	Lowering of social inequality	Voluntary redistribution	Not much – can also support these
Business partners	Taking over business ownership, organization of other business advantage via the provision of personal insurance (for example, gift of accidental insurance built into a card)	Private insurance	Entirely his own business

Table 2.5.: The interests related to the life of the individual and their methods of realization

It can be seen in the above that the individual does not always assert his interests himself (although this would be ideal, and that is the message of this material, that everyone should realize these consciously and as independently as possible), and that in some cases, the individual is not the only one with an interest in his life. If someone else acts in the place of the individual, in an optimal case it is for the following reasons:

- They see the interests of the individual better than he himself (for example the state recognizes earlier the need for saving for retirement and makes it mandatory).
- If the individual does not directly have interest in his life (health, etc.), or if his interest in this respect is less, than others' (for example he is the key worker of his employer, or the creditor's interest).
- The individual is in a dependent position (e.g. child, dependent, disabled).
- The individual is sometimes (or in some cases always!) irresponsible.

We also assumed the ideal of the self-reliant middle-class person. Self-reliance of other classes (as we have seen earlier) may differ from this. For example:

- Upper classes: have no need for the risk transfer realized via risk communities
- Lowest classes: cannot support themselves. Some of their care is taken over by the middle classes (through goodwill organizations and individual donation), since it is also in their interest to assure they do not spread epidemics, and that they do not worsen the public situation.

TEST

1) Which of the following statements about the cash flow of the life cycle are true?
   a) in main lines it contains the revenues (incomes) and expenditures of the individual, and shows their accumulation and balance as well.
   b) it starts from 0 when an individual is born always, under any circumstances.
   c) its structure does not change in its basic principles during the life cycle, though it differs by social class.
   d) defence against the risks threatening the cash flow is mainly the task of the state.

2) The risk transfer
   a) allows the individual – in exchange for the risk premium – to sell those risks, which he faces during his life cycle, but which surpass his ability to bear risk.
   b) need for the risk transfer does not depend on the individual's financial situation, but can be regarded as a universal human need.
   c) is typically a good solution for defence against frequent, relatively small losses.
   d) allows people to pay less overall (in expected value) for the negative effects of risks than would be the case without a transfer of risk.

3. THE LARGE SOCIAL SECURITY SYSTEMS

KEY WORDS

Transparency	Institutional care
Equivalence	Income transfer
Individual account	Self caring
Insurance consciousness	Open system
Pay-as-you-go system	Solidarity
Transferability	Defined contribution principle
Institution	Closed system
Institution system	

Institutions that enable people to plan their lives have been discussed throughout the previous chapters only in a very general way. References have been made to institutions like individual insurance, social security and accumulation of wealth – and to principles such as ''self-care'' and solidarity. Readers have been informed that institutions and methods connected to certain institutions will be discussed in later chapters. Here comes a very general summary of what have been said about the institutions, and we will compare their main features. In a developed society, the different methods and institutions of care/"self-care" usually form an interconnected system, the elements of which are in interaction with one another, and mutually define each another.

Above, all through discussing the planning of cash flow, the need for "self-care" has been emphasised. So now, in connection with the institutions, first of all, we should examine the question whether "self-care" is opposed to other institutions of providing care? The answer beforehand is: no, and it is highly important to note that, contrary to a number of "popular" opinions of these days, self-care and solidarity are not opposite of each other.

3.1. SOCIAL INSTITUTION SYSTEM AND SELF-CARE

We have to notice that self-care and relying on social institutions are not conflicting ideas, thus the presence of one of them does not exclude the other. In a way, self-care is a kind of social institution to "rely on". So to say, contrasting self-care and other ways of providing care is rather a question of consciousness and methodology. But what is all this about?

Let's suppose that somebody completely cares for himself, meaning that he does not belong to any risk community (he does not take out an insurance, does not resort to the services of social security), and does not accept any services gratis, not even if offered by his family. He makes his living by the incomes of his labour and capital throughout his life cycle. If somebody is able to do this, we can be sure that the society he is living in is quite a developed one, and that it has built up a system of social institutions (or ''caring'' institutions), which makes this possible. In such a society we certainly can find:

- a system of capital investments,
- institutions that protect property,
- a healthcare system that provides services on a wide range (including home care services),
- a comprehensive financial accounting network,
- a wide range of rules and conventions, both formal and informal, which are known and accepted by members of the society,
- appropriate mental attitude of people, which enables them to accept such rules, and to operate the institutions.

Someone who was born into a society like this takes the operations of these institutions for granted and does not realise that it must have taken a long, historical period for this system to develop, and that there are societies even nowadays, where such institutions have evolved only partially (see most of the Sub-Saharan countries, Laos, Papua New Guinea, etc.); or where it is just being born on the ruins of a past regime (e.g. Russia), where due to the lack of institutions, the stronger and more aggressive members of society consider the others as ''resources" and form their own society in smaller groups (i.e. they are operating a mafia).

After all, such institutions aim to provide for the needs of individuals in a very general sense (in a way that, they provide everyone access to the common goods based on equal principles, and breaching of these principles is sanctioned in order to maintain and operate the system).

Of course, ''self-care" does not only mean the situation in which someone relies only on his own resources. Relying on voluntarily founded common resources (risk communities) is also considered to be "self-care", since it is based on voluntary and conscious sharing of risk. In such a case we can speak of the "self-care'' of a group.

If a country doesn't deprive another country systematically of its resources with nothing in return (exploitation), or vice versa[53], then the population of that country, as a group, can be considered to be ''self-caring", for they can rely only on themselves[54].

Such national level ''self-care" can be organised in several forms, but it is certain that its source is the work of the individuals living in that country and the return on their capital. When organizing ''self-care'' on country-level, it is worth respecting the following principles as fully as possible:

Principle of solidarity: life situations, where people get into through no fault of their own (e.g. congenital diseases, belonging to disadvantageous social groups), or get into through their own fault, but cannot get out on their own (e.g. different types of addictions or having a criminal record), are defined as precisely as possible, and people in such situations are supported by the others free of charge[55]

Principle of equivalence: above their parts of the solidarity contribution and the amount needed to maintain the social ''self-care'' institutions, every individual should receive – in expected value – as much material goods and services from the institutions as he pays in – in expected value.

Principle of transparency: it must be clear to individuals what costs what in the institution system, and what is happening with their money.

Principle of insurance consciousness: individuals should pay their contributions consciously to the system, after considering their own long term needs.

A great number of countries (including first of all Hungary) operate a social "self-care'" institution system that respects the above mentioned principles only partially. In Hungary, for instance, only the first principle is realised – more or less![56] Due to the partial operation of these principles, the role of the individual is lost in the institutional system, thus it is usually not labelled as ''self-care''. Moreover, ''self-care'' appears as an alternative to this institution system, contrasted to it.

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[53] And nowadays, lack of exploitation is one of the most important requirements in international relations, so we may assume that it does not exist. It is not our task here to discuss the problems of international exploitation.

[54] Except, of course, in case of huge disasters, e.g. earthquakes, when international actions of solidarity take place, and the activities of certain international charity organizations, by which goods flow systematically, and free of charge, from one country to another. However, these actions do not represent a significant volume, at least in case of the ''developed'' countries that we examine, that we consider more or less the OECD countries (so Hungary is considered here to be a ''developed'' country!)

[55] Solidarity is a very interesting and important topic, and it will soon be discussed in more detail. But it is important to note here, that from the point of view of a society, solidarity only makes sense if its goal is to enable those who are supported to leave the situation in which they need to be supported (considering the fact that – as far as we know at present – in some cases this is impossible).

[56] It is important to note, that defenders of Hungary's outdated social care system refer to the principle of solidarity in a scope much wider than it would be justifiable, they are glamorising it, and most of all, they contrast it with the other principles.

In our view, ''self-care'' is when an individual is consciously planning his future needs, and then, taking the number of alternative social institutions he chooses the ones that mostly adhere to the principles above.

3.2. Institutions of Care and Self-care

The question is what kinds of institutions the individual can choose from (if he can choose at all)?

3.2.1. Institutional Care Throughout History

The first question arising is what can be considered as institution? The notion ''institution'' can be defined in a way that any kind of more or less standardized method, which is used not only in a limited sphere of life (e.g. a joint family looks after the elderly) is considered to be an institution. Here in the following, ''institutional'' will be used in opposition to ''personal'', so it refers to impersonally organized, standardized social functions[57].

Throughout history, as standardized personal methods of caring turned more and more indirect and impersonal, institutional forms defined as above have gradually grown out of them. The main (standardized) personal forms of care (and their social frameworks) are:

- tribe,
- joint family,
- family,
- village community,
- professional group.

The extent of the care provided is determined by the resources available for that particular group of people. In rough circumstances communities used to ''get rid of'' the ones who were, in the long run, incapable of living.[58] But normally these individuals were taken care of by their relatives, and since due to high mortality rates the ''web of relatives'' might have been rather incomplete and it was not rare that members fell out of it, people kept count of relatives in a wide circle. Orphans were brought up by their uncles, the elderly were looked after by their children, and if they were already dead, then the closest living relatives provided care for the ones in need. As society was developing, an increasing number of impersonal modes of care (''institutions'') came into existence, and relieved people, who started to draw the boundaries of relations in a narrower circle. It is understandable that people preferred institutional solutions, because in spite of the fact, that personal relationships ensured safety for the ones in need, but on the one hand they meant a certain threat to those who had to provide care, since they might have had to bear a disproportionately huge burden and responsibility. All these are shared within a wider range in case of institutional forms of care giving; however this can be arranged only in a society that operates within rather stable conditions. (As soon as the frameworks of a society start to loosen, due to a war or depression, personal relationships are again highlighted, and people rediscover kinship, and get in touch with relatives they never knew before.)

The caring function of personal and family relations are quite apparent in such reciprocal activities as for instance “kaláka” (a team that gathers to build up the new house of one of its members), or sending “kóstoló” (titbits sent to relatives, friends or neighbours from the meat made at the pig-killing), which are still common practice in some regions of Hungary.

The tribe – joint family – family line can be interpreted also in a way that keeping count of relations started to reduce on this line parallel to the consolidation of social conditions and more institutional solutions were created (e.g. charity institutions of the Churches). Village communities and professional groups (e.g. guilds that looked after the family of a deceased

[57] E.g. a home of elderly, where people get into through impersonal mechanisms – like paying the fee – instead of personal relations.

[58] See Taigetos in Sparta, where crippled children were taken, or the Mount Narayama in Japan, where those who got older than 70 years were taken to die.

member) represent transitional modes of care on the way from the personal towards the institutional forms.

The role of the family/joint family can still be observed in less developed[59] countries and in poorer layers of societies, that fall out of the institutional system of care. In such cases, having a number of children can be interpreted as "saving up for the pensioner years", since if someone has many children, there is a greater chance that they will be able to look after him in the later years. (See population data in African countries, or the number of children – highly above the Hungarian average – in Hungary's roma population, that lives much below the Hungarian average living standard.)

Nowadays we are even witnesses of the breaking up of the primary human group, the family, and we also see the trend that functions of the family, one after the other, are transferred to different social institutions[60].

3.2.2. Main Institutions and Institution Systems

Caring and self-caring solutions form a system in every more or less stable (but not necessarily well functioning!) society. In less developed countries (as well as in the past of developed countries) standardized personal (i.e. not institutional) methods form such a system. In developed countries, however, these systems are institution systems.

It can be noticed, that every country in general has its dominant institution system (or non-institutional solutions form a system), but at the same time, institutions and solutions of other systems are also present. It is very rare, that two fully developed institution systems are competing within a society, since that would create theoretical problems. Coexistence of two institution systems is possible only if they operate within two separate layers of society, and if there are strictly defined rules of transfer between them.

Generally speaking, there are two main types of institution systems, with several different variations and transitional forms:

1. organized by the state (or ''compulsory'') and
2. institutional system of self-care.

A kind of transition between these two versions is where the state makes self-care compulsory.

Institutions, which are vital for the proper operation of care giving though do not strictly belong to it (e.g. institutions of legal security), have been referred to previously and will not be listed in the followings.

3.2.2.1. Elements of the (''Compulsory'') Institution System Organized by the State

Very general principles will be discussed in this chapter and we do not intend to categorize strictly the big social security systems. Thus – contrary to the practice of relating literature – the author does not differentiate between systems operated by the state (financed by taxes) or the social care system (financed by contributions). Here both types are referred to as organized by the state.

Main elements of the institutional system organized by the state:

- pay-as-you-go system and/or national pension system
- state-owned healthcare system financed by public contributions
- disability pension, allowance for orphans and the bereaved, financed by public contributions
- unemployment support system

Since in national systems entitlement and its extent are usually not bound to contributions, or to the extent of contribution paid to maintain the system[61], participation is compulsory in a

[59] Countries which are euphemistically called ''developing'' countries.

[60] The systematic support given to bringing up children can be reckoned among such institutions, since the whole society provides the basis for it.

[61] Or this ''bond'' is not a strict one.

national system (at least for an individual who is in a well defined social position). In this sense, choice, and thus competition, between a national system and a voluntary system is either impossible or very limited.

3.2.2.2. Elements of the Institutions of Self-care

Several references have already been made to these institutions. Here we give a list of them:

- institutions for savings and capital investments (insurance companies, banks, investment funds, stock exchange, real estate market)
- institutions of market-based risk transfer (insurance companies, options exchange)
- private or foundation-owned health care scheme
- charity institutions

3.2.2.3. Transitions Between the Institution Systems

As it was mentioned previously, different institutional systems (or non-institutional solutions) may coexist at the same time, but usually there is one comprehensive system, the institutions of which are dominant, and the other institutions are complementary in nature. In Hungary, for instance, national solutions are dominant, while self-care and non-institutional methods have a complementary role[62]. In the United States the situation is just the opposite: the dominant system is formed by the institutions of self-care, which is complemented by the national solutions (mostly applying to disadvantageous groups of people – the poor or the old, etc.). In Germany national and self-care institutions exist side by side: joining to the national system is not compulsory above a certain level of income where the state allows self-care, and as a consequence, an appropriate, comprehensive institution system has evolved.

Looking back in history we can see that at first certain institutions (mostly those of self-care, e.g. charity institutions) grew out of the system of non-institutional solutions, which turned into a kind of institution system in the upper classes. The state gradually created a comprehensive institutional system, mostly for the middle- and lower classes, where anticipation needed for self-care was not available at the necessary extent. From this point of view it may be said that state solutions are necessary transitions for the middle and lower classes from the non-institutional systems towards self-care, that can be realised at the time when these groups become wealthy and provident enough.

3.2.3. Expectations Related to Institutions and Institution Systems

Some principles have already been mentioned previously:

1. Principle of solidarity
2. Principle of equivalence
3. principle of transparency
4. principle of insurance consciousness

These principles also formulate certain expectations in connection to the institutional systems and their operations. But different interested parties may have different expectations. The most important expectations from the point of view of the **provider/operator** of the system:

- Sustainable management, incomes should cover expenditures in the long run. This partly means that current incomes, i.e. payments made by the active population should cover all current expenditures, i.e. the services that the inactive population needs, and partly that possible deficits should be managed through planned utilization of accumulated reserves and their interests.

[62] Nevertheless, there are certain groups of people who practically do not draw on the benefits of the state system – either because they are wealthy enough to take full care of themselves or because they have not integrated properly into the society, so they rely on non-institutional solutions.

- Participants should be satisfied with the system.

From the point of view of the **participants**:

- There should be a long term equilibrium between payments of participants and the services/benefits they receive from the system.
- The system should (transitionally or for a lifetime) take care of those who, momentarily or permanently, are not able to contribute to the system.
- There should be a timing equalising mechanism between contributions made to the system and payments received from it, so that contributions should be made when the contributor is able to pay in, and payments should be received when he needs them.

The followings are often only latent expectations of participants:

- transparency, i.e. they know what is happening within the system, and what costs what, and
- individual account, i.e. keeping account of every contribution made, and every payment received by the participants.[63]

Finally, as mobility plays an increasing role in people's lives, which means that it becomes more and more usual to live and work in several different countries during their lifetime, and as countries tend to join supranational organizations (e.g. the European Union), there is an expectation in connection with these systems that becomes apparent now:

- Portability: entitlements gained in one country should be easily portable to another.

The main problem with portability is that contributions to the system and benefits received from it do not occur at the same time. It very often happens that at the beginning of their life cycle members of the system only pay in, while later they only take out of the system. Thus, from the point of view of a country, it matters whether people arriving or leaving want to pay in or receive its benefits.

In connection with portability, two kinds of systems can be differentiated:

1. Closed and
2. open systems.

Portability does not cause any problems in between open systems, which nearly automatically get connected to each other. However, in case of closed systems, a ''sluice gate'' mechanism is needed, which is not always a simple process.

A pension system based on the defined contribution principle is basically the element of an open system, especially if no minimum pension is guaranteed (the amount received depends on the management of the institution). In this case, the cumulative pension fund is ''portable'' between countries, though in fact this is already unnecessary, because money can be invested anywhere, and benefits can sent to any place. But the ''pay-as-you-go'' systems are fundamentally closed systems, since here it is not the real money that is accumulated, but entitlements. So, for a country which intends to join the European Union, it is worth working out a strategy to introduce a portable pension system instead of the pay-as-you-go system.

Differentiating between open and closed systems has also another aspect, which is in connection with the need for sustainable management. Namely, this is the question, whether the system relies only on the current output of the country, or not. In a closed system, only the current output of the given country appears as current resource, thus a significant decrease in the number of active people in comparison with that of the inactive means a real threat. No such problem arises in case of an open system, since if there are enough reserves backing up the obligation of providing benefits, then the current output of other countries can be the source of the goods allotted to the inactive of the given country, which will be covered by the accumulated reserves. Thus, ''demographic problem'' threatening pension systems is not general at all; it is only a problem of the fundamentally closed, pay-as-you-go systems.

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[63] In a sense, this is a sub-case of transparency, but at the same time it is wider concept.

3.2.4. Functioning Principles of Different Institutions

In my view, the principles discussed before and quoted in the previous chapter are to be applied to any kinds of institution system, either national or self-care. However it is a very general attitude, that the first two principles are highlighted, and national systems are described as having solidarity as their functioning principle, while self-care systems are referred to as having equivalence (proportionality in risk) as their functioning principle; and thus the two types of system are contrasted with each other.

I say, on the contrary, that the principle of solidarity applies to a well defined (narrow) social group (who are permanently unable to take care of themselves mainly due to their state of health), with whom the majority of society shows solidarity. To the others, however, the principle of equivalency should apply, irrespectively of the type of the system.

Thus, it is a basic requirement for any system to define precisely the boundaries and the extent of solidarity. Notwithstanding, experience shows that national systems define things in a rather loose way, which they explain by the ideology that this shows solidarity with everybody in general.

It is extremely important to note, that the principle of solidarity is highly abused in Europe, and especially in Hungary. Mystic significance is ascribed to it (it is ''the force that keeps the web of society together'' etc.), it is morally elevated[64], and then the self-care institutional system, accused of seeking after profit only, is pounded down.

Principle of solidarity, in an economic sense, is a claim for redistribution of incomes. This means that income has to be transferred from the more fortunate, wealthier members of a society to the less fortunate, poorer people. All this should be done by using the authority of the state. Thus the state would execute an income transfer from the rich to the poor.

If we want to analyse the directions of such income transfers, we surely find two main directions in present-day Hungary:

- income transfer between the sexes
- income transfer between generations

Income transfer between the sexes: men pay more into the ''pay-as-you-go'' pension system than women do (characteristically higher wages, longer time of employment, and until now: higher retirement age), but women receive more (lower retirement age, higher life expectancy, pension determined by the latter years, not by the wages of one's whole career). The explanation for this is: solidarity. But, if society systematically admitted the household work of women (e.g. the husband would have to pay after it formally, from which the wife could pay contributions for her later pension), or the time spent with bringing up children, as a service provided for the society[65], then the system could be organized in a clear, transparent way, on the bases of the principle of equivalence.

Income transfer between generations: in the most simple form of the ''pay-as-you-go'' system, and in its counterpart in the health care system, the currently active people finance the utilization of the currently inactive. If a significant difference occurs in the number of people belonging to certain generations, it may happen that somebody, back in his active period, financed a group of relatively few inactive people (which means that his expenses were low), but now he is taken care of by an active group small in number, whose expenses are much higher than the amount he paid in as contribution to the system. All in all, the present generation pays more than the previous did, and it will not get it back in the future. Such income transfers could be eliminated by reserves and foresight, and its absence –

[64] The French point of view, expressed by Prime Minister Lionel Jospin, is characteristic: ''Pay-as-you-go is the symbol of the chain of solidarity which links the generations. This is one of the most important expressions in the National Social Contract" – Quoted from: The Economist, 25 March 2000., pp. 35-36., ''France – Pensions funk''

[65] Of course, it also should be taken into consideration that women owe much of this to the society! But they redeem this obligation also instead of their husbands, which should be admitted!

contrary to the opinion of certain representatives of social policy[66] – would probably not cause any kind of collapse of the society.

In a national system, solidarity can be realised quite evidently, together with, and within the same institution system as the provision of the ones who do not need solidarity. In a self-care institution system, however, it is important that solidarity should be realised in separate institutions, because a voluntary and a commercial organization will not be able to deal with the systematic income redistribution. Such separate institutions can take the form of charity institutions, which are usually operated by Churches, with the help of voluntary offers. It is frequent, however, that in a coherent system of self-care institutions solidarity function is fulfilled by a national institution. Between these two there is a transitional solution, namely that the state gives planned ''instructions'' to voluntarily operated charity institutions so that they can form a comprehensive (sub)system.

Primary addressees of solidarity[67] are those who are, through no fault of their own (they were born cripple or handicapped, etc.), in a situation, in which they cannot take care of themselves. Nevertheless, it is also worth widening the range of addressees and to include those who are in difficulty through their own fault (the alcoholic, e.g.), without losing sight of the primary aim of doing away with the cause of their tough situation[68]. If no solidarity is shown with these people, they can mean a threat to the rest of the society (criminality), and their lifestyle has a negative effect on the quality of lives of others (e.g. unwashed homeless people using public transportation).[69]

3.3. SOCIAL POSITION AND SELF-CARE

As we have already analyzed, positive correlation can be found between a country's economy and the extent of foresight (thus self-care). This is also true for the different layers of society (between people of different financial status).

Unfortunately we have no data about the correlation between individual income and foresight, and within that, life insurance, but practise shows that the rate of the income spent on life insurance increases with income, up to a certain high level, and then it starts to decrease. The reason for the decrease is that concerning his risks occurring throughout his life cycle, after a period of time, a rich man does not need the risk equalization provided by a

[66] Although misinterpretation and mystifications of solidarity are the mistakes of professionals, it would not be correct on my part if I forgot to mention that, looking at the present-day Hungarian society, all this is not without any objective reason. One of the most important reasons is that the work of several groups of society is not paid according to value. Here I refer mainly to the employees of the educational and the health care sector. (It is not surprising, that these are not market based fields! At the time of their retirement, people working in these sectors, in a sense, receive compensation for their arrears of salaries of their active period, since then they worked altruistically (showing solidarity), so they are entitled to receive something in return. But at the same time, this system operates like ''compensating a bad practise with an even worse practise''. It is also justifiable that the people in question are afraid of the declaration of the claim for self-care, because they have experienced that rights are cancelled sooner than duties, so there might be a chance for them to lose their pensions and health care services without appropriate adjustment of their wages.

[67] I try to discuss solidarity on the basis of economic rationality, and not in the usual emotional way. The reason for this is that discourse on solidarity is dominated by the representatives of humanities, who – due to the absence of economists and their use of concepts – made their own system of concepts dominant. This possibly makes the false impression that solidarity cannot be discussed any other way than poetically, and, which is even worse, that if somebody tries to apply down-to-earth economic principles, they are threatening solidarity itself.

[68] Lajos Bokros [Hungarian ex-minister of finance] analyzes requirements of solidarity from the perspective of the system of economy (in *Népszabadság*, 3. March 2001). In his view, the most important things to do are creating equal opportunities, and creating the (market-based) system which ensures them. As he puts it: „One must handle separately the means used to improve equal opportunities and the means that serve keeping the dignity of the hopelessly miserable. …No-one is entitled by social solidarity to morally differentiate between the poor on the bases of the origins of their situations. Solidarity, however, allows us to offer priority in receiving support to those who are willing and able to make efforts to ascend from poverty."

[69] According to an English proverb (perhaps said by Keynes), if I owe somebody 100 pounds, it is my problem, but if 1 million pounds, it is theirs! Paraphrasing this proverb, we may say that if someone is undemanding towards himself, it is his problem, but if this goes too far, it becomes the problem of all the others.

risk community, since his fortune covers all (self-insurance!) From this we may formulate the hypothesis that there is a level of GDP per capita, which is so high that from that point the amount of money spent on life insurance (in the ratio of the GDP) is already falling.[70] (Even if such a high level of GDP is not experienced yet.) To put it in another way, life insurance is the typical form of precaution for the middle class (that of the upper classes is their big fortune; from several point of view, life insurance and fortune can be replaced by one-another, and many types of transitions can be observed between them, when somebody relies partly on his fortune, partly on his life insurance during the planning of his life cycle.) In the developed countries, as well as in Hungary, most of the society belongs to this middle class, thus theorems relevant to this class are formulated here, and the statements concerning the other layers of society are made in relation to this class.

One **main rule** can be derived from the previous train of thought: Social/financial position fundamentally affects our self-care strategies, so when planning our life cycle, this is what we must first face, and our strategy has to be adapted this.

A model of social layers is shown below, with the most important self-care (insurance) solutions and institutions.

It is important to know that the institution system of accident-, sickness- and life insurance is strongly connected to the layers of society determined by the financial position. Which institution system and which solution become dominant in a country depends on which social layer determines the operations of the social institutions[71]. Let's have a look at the layers of society and the solutions and (insurance / care / self-care) institutions belonging to them![72]

[70] Carrying on with this train of thought, it can be added that insurance itself is a product of a historical period, when people were already wealthy enough to have a foresight for a relatively long period of their life cycle, but they are still not wealthy enough to take all the risks on their own (self insurance). So there is a chance that insurance (or at least its currently known forms) may disappear in centuries. But this is not relevant in the case of insurances for risks of disasters which will remain necessary, and may gain even more importance, furthermore, due to the growing complexity of our society, new forms of disasters may appear (consider electronic networks that did not exist previously, or the growing level of addiction of people to these networks, and their necessarily (in a mathematical sense) chaotic way of working!

[71] This does not necessarily mean the largest in number! Before the civil war, in the American South slaves were more in number than oligarchs, though the latter determined such operations!

[72] Here I will not deal with property insurance. I deal only with life-, accident- and health insurances that are in close connection with planning of the life cycle.

Layer	Characteristics of social position	Safety interest	Safety solution	Who takes care	Institution
Upper	inherited fortune, high standard of living; often occupation of high prestige /of artistic or scientific nature e.g./, or managing of own business, resulting in high current income	To maintain and augment wealth	Nothing, or investment-like life insurance, for a great amount of money, with single premium (but they fundamentally prefer separate investments)	themselves	Stock-exchange, investment consultant company, their own companies, elite private insurance company
Upper middle	Significant inherited or gained fortune, but fundamentally earn their own living in the first part of life cycle, while live on the yield of their assets – leave great fortune	To maintain and augment assets + in the accumulative phase of the life cycle to ensure bequeathing and ability to work	Financial planning of the life cycle as a separate service; term life, accident- and disability- insurance with high sum assured before retirement + (around the middle of life the cycle) single premium investment-like life insurance with high sum assured	themselves	Stock-exchange, investment consultant company, life cycle planning consultant, their own companies, investment funds, elite private insurance company
Middle	Some inherited and gained fortune, fundamentally earn their own living with a little income from capital, they leave significant fortune	Almost all possible insurance interest: insuring life and ability to work, providing for those left behind, for pension and leaving a legacy	Individual planning of life cycle + all classic solutions: term life, savings type, accident and health insurance, annuity and whole life insurance	themselves + state	Life insurance company, life cycle planning consultant company, investment fund, social security, pension and health insurance, complementary pension and health funds
Lower	Usually no inheritance,	Taking care of dependants,	Standardized planning of life cycle	state +	Social security,

Layer	Characteristics of social position	Safety interest	Safety solution	Who takes care	Institution
middle	gained fortune runs out by the end of life, earn their own living, at the end of life living on their own accumulations, leaving no legacy	keeping ability to work, safe pension	included in the product, selling through agents, Mass life insurance, combined life-, accident- and health insurance, combined term and savings type (endowment), almost exclusively regular premium annuity	themselves + charity institutions	pension and health insurance, mass life-, accident- and health insurances, pension fund, charity institutions
Low	No inheritance, earn their own living in active phase of life, later on problems, no accumulation	Taking little care of those left behind and themselves – due to demonstration effect and their ignorance, not able to economize income of life cycle	Simple, cheap life-, accident- and health insurances (e.g. *CSÉB* – group life and accident insurance), compulsory solutions	state + charity institutions + themselves	Social security, compulsory part of social security given to private insurance, charity institutions
Outcasts	No inheritance, no accumulation, do not earn a living, paradoxically primarily rely on themselves all the same, – by losing ability to survive from day to day they lose their life, too	-	none	themselves + charity institutions, private individuals	Charity institutions

3.1. Table: A summary of social position, safety interest, safety solutions, the care-taker, and the institutions

TEST

3) Solidarity
   a) Its principle is the main ideology lying behind the institutional system of self-care.
   b) It means that we try to minimize the income redistribution between different layers of society.
   c) It is an expression of the workers' movement, it is out of date since 1989.
   d) The notion is often used too widely, without any reason, and it is often given a kind of mystical overtone.

4) Institution system of care/self-care
   a) From the point of view of portability, it can be nationally organized or private.
   b) It is open, if people can easily move between the institutions of different countries.
   c) It is either national or private, there are no mixed systems.
   d) From the point of view of its provider, one of the most important expectations is that contributions made by, and payments received by the participants should be in equilibrium in the long term for every individual.

II. THE STUDY OF LIFE INSURANCE PRODUCTS

4. THE ROLE, CONCEPT AND MAIN TYPES OF LIFE INSURANCE

KEY WORDS

Transfer between funds	Asset Fund
Unit Linked Life Insurance	Accumulated Units
Accident Insurance	Conditional Annuity
Accidental Death	Term Insurance
Accidental Disability	Annuity
Sickness Insurance	Beneficiary
Maturity	Initial Sum Insured
Insurance Term	Initial Units
Insured Event	Term Insurance
Sum Insured	Hospitalisation Daily Allowance
Insurance Benefit	Critical Illness
Insured	Term Fix Insurance
Age of Insured	Surgery Benefit
Table of injuries	Pension Insurance
Pure Endowment Insurance with Premium Refund at Death	Sum Insurance
Whole Life Insurance	Critical Illness, Dread Disease
Health insurance	Disability Waiver of Premium
Unit	Disability Annuity
Offer Price	Policyholder
Life Insurance Policy	Endowment Insurance
Top-up Payment	Bid Price

In the following sections of the book we will take the life insurance from the earlier mentioned solutions that facilitate the planning of the life cycle, and discuss it in more detail. First we'll try to find the concrete situations of life and the concrete types of life insurance that can be used to achieve individual (in some rarer cases organisational) goals. We have to mention right at the beginning that nowadays life insurance, and so the concept of life insurance is changing, and the boundaries between different financial institutions are loosening up, and we currently are in the state of redefining these boundaries. This way the formerly unambiguous situation, when life insurance meant a group of products and also a well defined institution is starting to disappear. The following discussion focuses primarily on the product, briefly mentioning the most important institutional specialities, and at the same time referring to the changes that both are currently undergoing.

4.1. FINANCIAL NEEDS

4.1.1. The Logic of Changes in Financial Needs

The change in demand for financial – and within this life insurance – products has its own logic, that the insurance companies have to follow both on the sales and on the product development side. The most important elements of this logic, as they appear at the beginning of the 21st century are:

- As people are becoming wealthier, new financial needs arise, and these needs are differentiating – parallel to the differentiating society.
- Parallel to the enrichment of people, their financial literacy is developing, and consequently:

- They also understand more the functioning of life insurance, and they have a growing need for more comprehensible products (so in a certain sense demand is gradually shifting from traditional to modern life insurance).
- Needs are becoming more differentiated, which requires more and more individually fit products[73].

- Because of enrichment and competition the consumers' comfort and demand level is increasing, and this way individuals are less satisfied by only the product itself, they expect a complex solution to problems. They respect much less the argument that "my competence as an insurer ends here, seek the advice of other kinds of institutions with your further problems".
- With the integration of financial areas it will be much more difficult to define what exactly life insurance is. Due to integration, the competition of other financial institutions (banks, brokers, mutual funds) is becoming more definite, but at the same time new opportunities open up for insurance companies, they won't be locked up as much into a relatively narrow field of action.

4.1.2. Financial Needs in the Life Cycle

Behind the change in consumer demands – e.g. the change in demand for financial products – we can discover a kind of order. We see the same order in the specialisation of institutions on different financial areas. Residential banks are specialised in handling mostly daily, short term financial affairs, that is in handling cash flows, deposit collection (also short term), that is logically connected to these and consumer loans (again short term). Insurance companies are specialised in creating the financial cover necessary for the long term, "strategic" goals of the individuals, and security handling companies in the investment of "excess" capital of greater volume. One of the fundaments of the integration of financial areas of our days is that compared to the above, in-between needs have emerged that are close enough to all, so it is not evident which existing institution should satisfy them. Whichever way the current definition of life insurance should change, it seems a fix characteristic that it supports the realisation of "strategic" goals of the life cycle requiring a greater volume of money, and it neutralises the dangers threatening the realisation of these goals. So the areas accessible by life insurance can be defined by two dimensions, the financial need and its term.

The order behind the demand for financial products (especially those that are long term and require the consumption of greater volume of money) is: First the most pressing need is satisfied, after that the most pressing among the remaining, etc… If we want to order – by main points – the "strategic" needs by "how pressing they are", then we get the following:

1. "Some kind of" housing (sublease; a room in the parental house; a house gradually built and modernized in a lifetime's work) – its special feature is that any "excess" of money is immediately "built in".
2. Precautionary reserves for "general use"– the smaller, the more liquid (so the order is: under the pillow, sight deposit, short term saving account, time deposit, life insurance). Because the reserve is not satisfying, the goals are not really differentiated: It is spent on whichever comes first, and they hope that there won't be too many needs "coming in" at once.
3. Complementing the general reserve: A low premium, low sum combined insurance covering a number of life and related risks (Basically the CSÉB, Group Life and Accident Insurance).
4. From the "general goals" the one that first becomes independent, but still remains undifferentiated is "providing for children" – that is, saving up money dedicatedly for them.
5. Providing for children in a differentiated way: term insurance to the benefit of the child for covering the expenses of upbringing. Term fix insurance for "starting life"[74].

[73] Which has the sales and network-organisational effect that they have a growing need for quality counselling.

6. Buying a car, or a gradual change in quality.
7. Immediate solution of quality housing – on the debit of discounted future excess income –, and the periodic change of housing satisfying higher and higher level needs
8. Saving for the case of sickness
9. Pension complement, or saving up the necessary capital for an adequate level of pension.
10. Saving up the pension capital for the spouse
11. Changing the pension capital to annuity (in case it is of middle size; if it is very large, the annuitization – and the insurance company – can be left out)
12. Leaving wealth to the dependants
13. Advance provision of our helpless elderly selves, or organising the concrete care – long term care (LTC), home for elderly
14. "Liquidating" the housing[75], i.e. changing its value to annuity[76]

4.2. The Relation of Life Insurance with Other Insurances, the Nature of Life Insurance Risk

Let's look at what life insurance is, and what distinguishes it from other types of insurances.

We gain the definition of life insurance[77] from the general definition of insurance. We get its specialities if we delimit it from other insurance types. The delimitation can be of several aspects. In the following we will analyse the relation and differences of life insurance and other insurances in the following aspects:

1. the insured event,
2. the character of claim and reimbursement,
3. the specialities of reserving.

4.2.1. Insured Event

The term "life"-insurance itself is partly correct, but partly euphemistic, since primarily those insurances are called life insurance, where the insured event is related to the death of the insured. This – given by the nature of the matter – can be exactly of two kinds:

1. the incurrence of death,
2. the non-incurrence of death.

More precisely the possible life insurance events can be phrased as

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[74] Because of the „free" state education this is based on „feelings", but it is becoming more obvious that the parents have to bear the expense of education, and this may become a significant sum. Moreover the parents don't necessarily have to provide for the children. As the goal of educational expenses is presently missing, a small starting apartment takes over the goal of the „starting life" support.

[75] That is: changing the "frozen" value of the apartment to a cash flow, or "liquid" money, and at the same time losing the proprietary rights.

[76] Currently (in 2003) the Hungarian insurance market is somewhere around satisfying goals 2-4. So despite the fact that insurers are marketing "pension insurance", in reality they are selling reserve building insurance for "general use". The same way: they are practically not yet selling adequate coverage for the event of a parent's death. We see that buying a car precedes quality housing, moreover it comes before pension, which is not particularly rational, but – knowing the preferences of Hungarian consumers – it can be handled as a fact. Actually, if a consumer is rational, immediately after providing for children will come providing for his own retirement (before the car and housing), but we can take this order as a fact in the Hungarian practice. "Liquidating" the apartment is a current need, but typical of a later part of the life cycle, so actually it stands out a little bit from the logic of the above ordering (where we primarily had a middle-aged consumer in mind).

The insurance market is on "stage" 4., but consumers are somewhere around stage 7. Buying a car is not really the area of life insurers, but housing mortgages classically are.

At the time of writing this book the Hungarian insurance market is in the middle of a strategic crisis. Sales cannot be significantly intensified any more through the former types of methods, but the new products and/or methods have not yet been found.

[77] We have to pin down right at the beginning that here and further on, under life insurance we primarily mean the insurance of profit-oriented joint stock companies, or private life insurances. On the other hand most of this discussion holds for the plans of mutual insurers, co-operative insurance societies.

1. death as an insured event, if the death of the insured happens during a pre-determined term,
2. living through a term as an insured event means that death doesn't happen during a certain pre-determined time-period.

Consequently we get the two elemental insurances that are most important in many respects:

1. term insurance (for death) and
2. pure endowment insurance (for living through).

In the following sections this book is mostly persistent in regarding as life insurance only the insurances that can be characterised by these two insured events, but we have to add that this conceptual clarity is difficult to hold due to two reasons. These are:

1. **Tradition**, which has several layers. First of all, life insurances traditionally tend to contain accidental and health elements, too. Conceptually these cannot be regarded as life insurance, but practically they are accounted under life insurance[78]. On the other hand there are two risks that do not really fit into the definition of insurable risk, but from time to time they appear in life insurance policies all the same. These are marriage and childbirth. Even European Union guidelines recognise this tradition and allow the cultivation of life insurance policies containing these risks under the life insurance branch. Anyhow, we do not handle these two risks here.
2. **The development of life insurance products** takes us in the direction that the former complementary function of life insurance, long term saving is becoming the main function in many products, and in these products we do not find an element that can be pointed out as insured "event". The book will handle these products as life insurance with full rights, and doesn't require a life insurance to contain one of the above insured events.

4.2.2. Claim and the Character of Reimbursement

Life insurance principally cannot be a reimbursing insurance, which would mean that if the insured event occurs, the level of reimbursement paid by the insurer is determined based on assessment of damage, depending on the size of actual damage occurred, as for example in property-casualty insurances. Even the usage of the term "claim" itself can be questioned in relation with a life insurance insured event, since this term has been developed mainly to describe material, or materialised losses. Of course the parties of a life insurance contract also have losses if the insured event occurs, but on the one hand this is not really tangible (if the insured event is living through the insurance term), and on the other hand it appears through more transpositions (if the insured event is death during the insurance term) than in case of the typical reimbursing insurance.

If possible, we do not use de term "claim" in life insurance, or only in an analogous way with other types of insurance, to refer to the occurred insured event.

Because of the above, insurance can only be so called "**sum insurance**", which means that the benefit paid by the insurer is not determined by the size of damage, but a level, a "sum" defined in advance in the policy.

4.2.3. Life Insurance and Reserving

Regarding reserving, life insurances (at least 99% of them) differ totally from other insurance types, from all non-life insurance (at least from 99% of these). The cause of this lies in the typical length of insurance terms, and in whether there is a characteristically

[78] There is a kind of "dominance" rule in the Hungarian life insurance practice, according to which if a product covering several types of risks contains also life insurance risk, then the whole product is regarded as life insurance, regardless of whether the premium part of the life insurance risk is the major part of the premium or not. Later we will see that within a plan life insurance risks can only appear together with accident and health risks, and no other types of risks.

different, definite change in the probability of claim during the term. So let's look at the difference between the two mentioned insurance categories in these respects.

A typical life insurance policy has a term of several years, or rather several decades. The mortality rate continuously increases throughout the term, and the actual death rate is usually – with a little fluctuation – around the theoretically expected value concerning the whole portfolio. From this, and from the fact that – contrary to e.g. property-casualty insurance – in case of life insurance maximum one claim can occur during the term, follows that the insurer uses the premium received during the whole term to pay claims, so most part of the premiums have to be put aside, reserved. This has the effect that a life insurance company has long term, stabile reserves.

We can take for example property-casualty insurance as typical non-life insurance. A property-casualty insurance policy – contrary to a life insurance policy – is typically signed for a year (even if it is generally automatically prolonged in the next year). In the next year the risk is generally – disregarding some special factors – the same as in the previous year. So there is no need to gradually save up from received premiums in a reserve (at least not to the degree as in case of life insurance). In case of property-casualty insurance the claims of a given year are generally covered by the premium of the same year. On the other hand the fluctuation of claims is – contrary to life insurance – very erratic. This way property-casualty insurance is profitable in one year, and results losses in the other. This is caused by – among other factors – that we can question property-casualty claims in three dimensions, unlike the 1 dimension of life insurance. In case of life insurance the only question that can be asked regarding a claim is "when?". In case of property-casualty insurance beside this we can also ask "how many times?" and "how big?".

Because of the above differences we distinguish two branches when categorising insurances (according to EU guidelines): life- and non-life branch. The Hungarian Act on Insurers and Insurance Activities treats the subject the same way.

The life – non-life categorisation has an organisational consequence: in many countries insurance companies cannot operate if they are dealing with both life- and non-life insurance. This is caused exactly by the hectic claims experience of property-casualty insurance, which in years of greater loss might tempt the management of the company to cover the losses from the stabile life insurance reserves. In order to prevent this from happening, measures have been taken concerning the separation, that the Hungarian insurance act has also introduced to newly formed insurance companies starting from January 1st, 1996.[79]

4.2.4. The Characters of a Life Insurance Contract

Let's start the introduction of a life insurance policy and its characters by defining one of the most well known life insurances, the term insurance:

> **In a *term insurance* contract the *insurer* undertakes the liability – against the premium payment of the contractor (*policyholder*) – of paying a sum specified in advance (*sum insured*) to the person specified in advance (*beneficiary*) if the person specified in advance dies (*insured*) during a certain period of time (*insurance term*). If the insured lives at the end of the insurance term, the insurance policy is terminated without benefit payment.**

[79] In reality the reserving characteristic is tied not so much to the life- and non-life type, but to the long term and short term type of policies. But in the practice it would be difficult to separate different types of insurances according to this aspect. The life- and non-life distinction, on the other hand covers the long term and short term categorisation quite well, and it can also be easily realised in practice (although here we have to refer to the uncertainties related to accident- and health insurances that we have already mentioned). In the practice we see examples of long term non-life policies mostly among health insurances. Legislators often solve problems arising from this by accounting these health insurances under life insurance. Nevertheless the term "long term insurance" is often used for life insurance, which is not totally correct, but covers the essence well enough.

It is worthwhile to analyse this definition a little bit, because it contains a number of key words that we can use later on. First of all the characters (or subjects) of a life insurance policy. As we can see from the definition, there are four of these:

1. the insurer
2. the policyholder
3. the insured and
4. the beneficiary.

Taking the insurer as obvious, we have to know the following about the others:

The ***policyholder*** is who signs the insurance policy and pays premiums. He is the “owner” of the insurance policy (he “holds the policy”), he can make legal statements relating to the policy, he names the beneficiary, and he has the right to terminate the policy (surrender). The policyholder can be a natural person or a legal entity. As a natural person, he is usually – but not necessarily – at the same time the insured, too, and can naturally be also the beneficiary. Nevertheless it is worth distinguishing whether the same person is mentioned in the role of the policyholder, the insured or the beneficiary. When this role is not important, or the same statement can be made in the case of several roles, then the place in the policy is only important compared to the insurer, and we can use the term **client** (as we will in this book). From a legal aspect the main character of the life insurance contract is the policyholder (naturally beside the insurer).

The insurance policy is about an event related to the ***insured*** person’s life. In the insurance technical sense he is the main character, and can only be a natural person. If the policyholder and the insured is different, then the policy requires the written consent of the insured to become effective. But from the legal aspect this is almost the only right he has. Apart from this, if the policyholder wants to surrender the policy, he has the right to take his place as the policyholder. We have to point out, that the fact that from the insurance technical and from the legal aspect the main character of the insurance policy is different, sometimes leads to problems, mainly when interpreting the subject of tax allowances, if the state incites life insurance through tax allowance.

The ***beneficiary***, who receives the benefits paid by the insurer if the insured event occurs. In many respects he is in the most favourable position, since he has no obligations, only rights, but on the other hand his position is the least stabile. The policyholder can change the beneficiary any time, even without the former beneficiary having knowledge of this. Naturally the beneficiary doesn’t have to be only one person or a natural person.

If we further analyse the above definition, the next keyword is **term**. Life insurance policies most often have a definite term, but it might also have an indefinite term. Policies of definite term have usually a length of whole years. These whole years usually do not overlap calendar years, but start with the signature of the policy, and the so-called insurance anniversary is every year on this same day. The time period between two insurance anniversaries is an insurance year[80]. The minimum possible term of definite term insurances is generally 5 years, and the maximum is – varying from company to company – between 25 and 40 years. The most typical term in Hungary is 15 years, in more developed countries it is generally longer.[81] A maximum relative to the age of the insured is also generally used, e.g. the insured cannot be older than 75 years at the maturity of the policy.

Life insurance policies with indefinite term usually end with death or surrender. In certain types the expected ending event is rather death (e.g. in whole life insurance policies), in other types it is rather surrender (basically in unit linked policies in which the term is not marked).

The last important term is the **sum insured**. Since life insurance is sum insurance (it principally cannot be anything else), this way declaring it in the policy is essential. A life

[80] The generalisation of the insurance year is an insurance period, which is usually defined by the time-period between two premium payments, in other words the period covered by the premium. The most common periods are insurance month, quarter, half-year and year.

[81] Since from 1995 a life insurance with a term of at least 10 years receive tax allowance, this is a strong incentive for policyholders to choose a term of at least this length.

insurance policy can theoretically have more than one marked sum insured depending on how many insured events are allowed. According to this, we can speak of term, pure endowment (and accidental death, accidental disability, etc…) sums insured. These might be the same, but might also be different.

4.3. Introducing the Most Important Life Insurances

In the following sections – without any kind of logical order – we introduce the most common life insurance products (plans) and how they are used (function).

4.3.1. Term Insurance

We have already seen the definition of term insurance above. Based on this it seems that the functioning of the insurance means that the money collected by the risk community will go to only a few, to those (or persons connected to those) who die during the insurance term. Because of this it is possible to receive relatively high levels of benefits with low levels of contribution, but this has the price that in case of living through the term nothing is received (since the collected premium has been distributed to the beneficiaries related to the deceased).

When using a term insurance, the most important thing to consider is that a death in the life of the family – if it is the death of a wage-earner – causes great difficulties, or it may even lead to total bankruptcy. The more the life of the family depended on the wage-earner, the greater the bankruptcy.

> **Example**: A 30 year old woman raises two children on her own, and at the same time is building a house. She thinks that if everything goes well, the building operations will be finished in 5 years. If, on the other hand something happened to her, a term insurance sum would save her children from bankruptcy. This way she takes out a 5 year term insurance policy.

Every family can be regarded as an enterprise. A distant example is: a small factory works with two high-capacity machines on 100% utilization, with constant over-ordering and constantly renewed loans necessary for further development. If one of the machines suddenly has a break-down, this small enterprise can be auctioned. (Not to mention the stress that this constantly threatening possibility causes.) This way in all such enterprises the machines are insured against such "outage". In the family the wage-earners are such "high-capacity machines", and the term insurance corresponds to the outage insurance.

It logically follows that from this analogy we immediately move on to the area of enterprises. Here the entrepreneur himself is a "machine" of even higher capacity, so losing him would cause an even greater problem to the family.

During the normal operation of the enterprise death (due to physical wear and tear) or accident can happen to anyone. Enterprises are responsible for their employees, which also means that the enterprise has to provide for the family of its employees, this way – although in Hungary not many of the managers think this way yet – it is almost a duty of the enterprise to arrange for the negative financial consequences of such an event. The responsibility is general, and every manager has to know this. This way the government often allows enterprises to account the premium of the term insurance policy he pays in favour of the employee as expense (this is the situation also in Hungary).

According to the above thoughts we recite a few concrete situations when it is useful to take out a term insurance:

- Term insurance is the "cheapest" insurance in the sense that the benefit received is greatest compared to the premium paid. This way this type of insurance can especially be recommended to people who at present are not in a financial situation to have savings of greater volume. These can be for example young householders, who are currently trying to build the bases of their living (building a house, starting a business, etc…). They don't have much money that could be saved up, but are afraid that their family could be deprived of a promising possibility of financial prosperity due to their sudden death.

- In relation to the above example we can also mention using term insurance as a credit life or loan insurance. If the guarantee of repaying a loan on an enterprise or simply on building a house is the entrepreneur or the householder himself, his family is in a difficult situation if he dies. This situation should be parried by a loan cover, or credit life insurance.

Some insurers provide the option to the client of taking out a risk insurance rider or riders with shorter term or terms beside a term insurance main policy. This option might be useful in both of the above two cases. This way the policyholder can achieve a higher sum insured in the first part of the term. In the first example this could be useful, because if he dies earlier, raising his children requires more time and also more money than if he dies later. In the second case its reason would be that the capital of loan to be repaid decreases in time, and this way later on a smaller sum insured provides sufficient cover.

A further – more general – area of use of this plan is to level out earnings inequalities within the family. In the consuming structure of a family husband and wife both spend about half of their total joint income. So if the income of the wife is higher, she consumes less than her actual income, and her husband consumes more than his actual income. In case of such asymmetric earnings the loss of the partner is particularly threatening for the party with lower income. Here the life insurance on the partner with higher income serves to evade the financial consequences of his death.

4.3.2. Pure Endowment Insurance

In case of the **pure endowment insurance** the insurer undertakes the liability – against the premium payment of the policyholder – of paying a sum specified in advance (*sum insured*) to the insured (or to a beneficiary specified in advance) if the insured is alive at the end of a certain period (*insurance term*). If the insured dies before the end of the term, the insurance policy ends without benefit payment.

If we think about it, this design works in a way, that the sum gradually accumulated over a long period of time by the risk community is distributed exclusively among the survivors. In term insurance the risk is early death (and consequently leaving dependents who need to be cared for), against which the insurance gave protection. Here, on the other hand, it is life that means a "threat", that someone lives through the period in which he can live on his own earnings. This is not a threat to those, who don't live beyond the end of this period.

The pure endowment insurance is an important theoretical design, and is a building stone of many other insurances, but it practically doesn't exist on its own (although after thorough research undoubtedly many would find concrete examples of it throughout the world). We can roughly imagine why this is so, even based on the above logic of the pure endowment insurance. Anyway, one of the most important reasons is that – as we will see later on – pure endowment insurance cannot be surrendered, that is, if someone cannot go on with premium payment any more, he will lose a relatively large amount of money.

This defect is corrected by the design of the so called pure endowment insurance with premium return guarantee (in other words this has a surrender value), which has a death benefit also beside the living benefit, which is the value of the premiums paid so far, calculated in a certain way. Because of its name it is often regarded as a pure endowment insurance, although it would be more correct to account it as an endowment insurance with a special death sum insured.

In case of the pure endowment insurance with premium return guarantee we are talking about the calculated value of premiums paid, because when determining the death benefit, the premium additives or loadings applied upon premium payment are not taken into account, but the effect of inflation handling is calculated.

4.3.3. Endowment Insurance

Instead of the pure endowment insurance, in practice insurers recommend to clients the combination of a term insurance and a pure endowment insurance, the so called **endowment insurance**. Technically the endowment insurance is simply the sum of these two plans. Since in this design the insurer pays in all events, it is accepted by clients a lot easier than a pure endowment insurance.

Some people by life insurance because they are afraid of death, others because they are afraid of living. The two goals naturally require a different kind of insurance. The endowment insurance unites in itself the two life insurance types that satisfy these goals, i.e. the term and the pure endowment insurance. The risk part of the endowment insurance serves the same purposes as a term insurance, basically providing for those who are left behind. The pure endowment part on the other hand aims mostly at the insured providing for himself.

Naturally the pure endowment part can also serve as providing for others. E.g. the insured wants to create the starting basis of the living of a child through an endowment insurance. This way the term insurance part provides for a child in case of death, and the pure endowment part has he same purpose in case the parent is alive at the end of the term. In these cases it is thoughtful to choose a term such that the maturity date falls together with a special age of the child (18 years, when he graduates from high-school, or 23, when he receives a diploma, etc…).

We have to add, that the endowment insurance is not as good a design as it appears at first sight. The term of the two main goals (providing for others and providing for myself) is generally not the same, this way the exact purpose of the insurance in a particular case can hardly be defined. Probably in Hungary, where endowment insurance is the most popular life insurance this popularity rather shows the under-developed state if the insurance culture, since it is not so much the insurance benefit, i.e. a conscious foresight that sells this type of policy, but it is rather used as a kind of general reserve.

The primary form of an endowment insurance is the design, when the death sum insured is equal to the maturity sum insured. In a more general sense we can accept as endowment insurance every life insurance that has both a death sum insured and a maturity sum insured, even if these two are not the same. E.g. in the beginning of the 90s in Hungary several companies experimented with plans where the death sum was half or double of the maturity sum insured. (They never really became popular.) Some companies have built more than one maturity into the design (these are so-called “stepped” designs) – e.g. the insurance with a term of 20 years has a partial maturity and a corresponding partial maturity sum insured at 10 and at 15 years. As we have already mentioned, the pure endowment insurance with premium return guarantee can also be regarded as an endowment insurance rather than a pure endowment insurance, and from the aspect of surrender it also works like an endowment insurance. Finally the term fix life insurance that will be discussed later on can also be regarded as a special endowment insurance, similarly to a pure endowment insurance with premium return guarantee, with a varying death sum insured.

From the side of product design we have to add that the (primary) endowment insurance can not only be handled as a term + pure endowment insurance, but also as a simple “savings account” complemented by a term insurance with varying death sum insured. This varying death sum is in every moment such, that it could supplement the current account value to a certain sum defined in advance (the sum insured).

4.3.4. Whole Life Insurance

Since the possible age of humans is not infinite, if we start to stretch the insurance term of the above definitions, the term insurance and the endowment insurance “meet”. We can view this as a new type of insurance, the so-called whole life insurance. Whole life insurance differs from others in insurance term, which is so long, that the remaining life of the insured fits into it, this way in any case it will end with the insured’s death, and consequently with benefit payment. But generally the premium term is limited e.g. until the insured reaches age 85, and after this the policy is in effect without premium payment.

Since the term of such an insurance is very long, the question, whether it will surely be longer than a certain period, e.g. 10 years is meaningless. Yes, it is longer, since the term is not the same as the period in which the insured is alive. If it were, a policy of 10 years would have to be subsequently redefined to 5 years if the insured would die at the end of the 5th year. But we never do this, so the term of the whole life insurance cannot be identified with the period until the insured dies, only with a longer, but not necessarily precisely declared period.

The features of a whole life insurance are also between a term and an endowment insurance, e.g. it has a surrender value (see later on), but not at the level of the endowment insurance.

The whole life insurance has special purposes.

1. Coverage of funeral expenses (ceremony, shrine, etc…). If it is important to the insured that his funeral should be of appropriately high level, then he can collect the money this requires through such an insurance in small fractions.
2. To pay for legacy duty. If the testator doesn't want the inheritors to sell his property in order to be able to pay the legacy duty, then it is useful to take out a whole life policy of a significant sum insured, that will pay exactly when the legacy duty has to be paid.

Relating to this it is important to mention a very favourable feature (in the Hungarian law and order) of life insurance, namely that it is not part of the legacy, this way the beneficiary receives it before the – sometimes very long – legacy procedure[82]. Moreover the life insurance benefit (as most of the insurance benefits) is free of duty.

4.3.5. Unit Linked Insurance

Unit linked insurance that has been introduced in the United Kingdom in the 50-s at first has been nothing else but the combination of a traditional term insurance and a few investment funds.[83] The client regularly paid a premium to the insurer, which had two components of fixed size, that the client could also see:

- the premium of the term insurance
- the premium part filling the investment funds

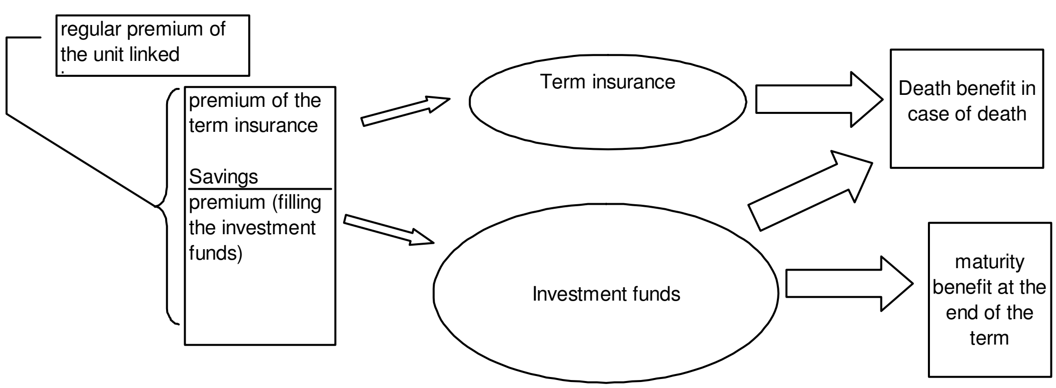

Figure 4.1.: The premium and benefit structure of the first Unit Linked Insurances

The benefit paid in the event of the insured's death had two components:

[82] Of course only if a beneficiary was named in the policy, and it is not stated as "the inheritor of the insured" – in which case naturally the benefit can only be paid after the legacy procedure has been ended, so that the insured knows whom to pay to.

[83] The British Unit Linked insurance is practically the same as the American Variable Universal Life Insurance, that has been developed as the generalisation of a whole life insurance. In the first step they made the death sum assured variable during the term (Universal Life), and after that they made the investment options also variable during the term (Variable Life).

1. the death sum assured
2. the current value of investment funds

At maturity the term insurance – the usual way – ended without benefit payment, and the beneficiary received the current value of investment funds as insurance benefit.

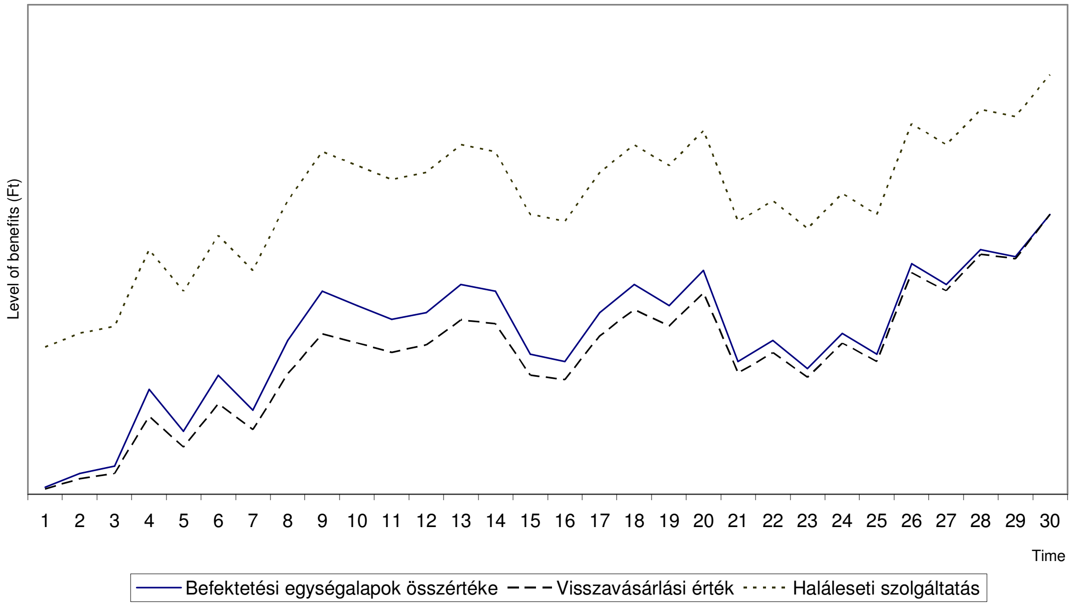

Figure 4.2.: The benefits of the first Unit Linked Insurances

Similar product can also be found nowadays, but the basic construction of unit linked insurance has changed significantly. First of all, regulation separates investment funds and the funds of unit linked insurance. The former is rather referred to as “unit fund” or “asset fund”. Secondly: the total premium currently goes to the asset funds – with the following restriction:

1. a certain part of the premium is subtracted right at payment, to cover expenses of the insurer,
2. certain other types of premiums are also immediately taken – for other types of expenses.

If we disregard these second types of expenses, then the premium and benefit structure of modern unit linked insurance is the following:

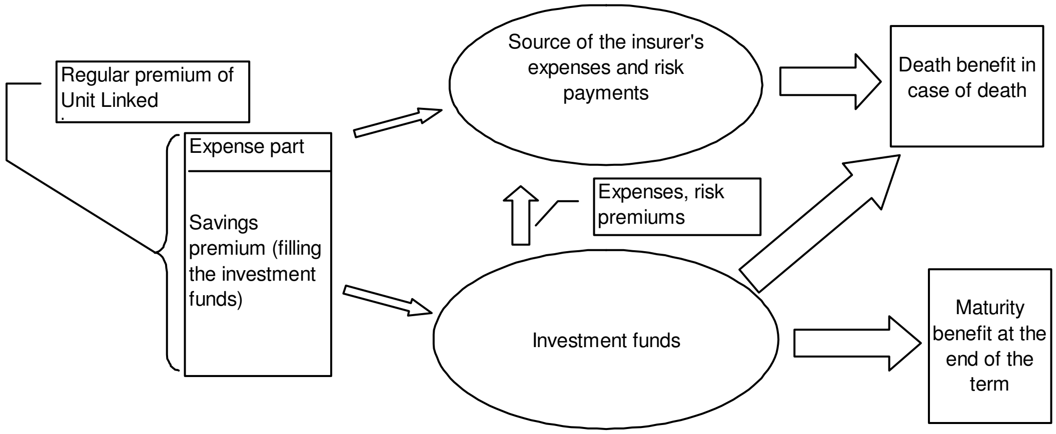

Figure 4.3.: Premium and benefit structure of modern Unit Linked insurance

In the event of death, the benefit received from the insurer is a fixed sum declared in advance, or the current value of unit funds, if it exceeds the death sum insured. At maturity the benefit paid is the current value of investment unit funds once again, as shown by the following figure:

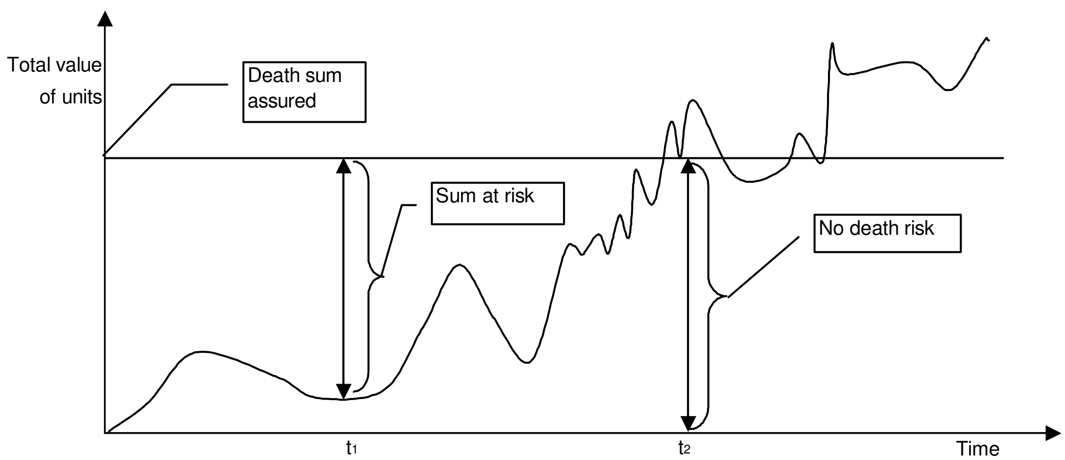

Figure 4.4.: The value of unit funds in a Unit Linked Insurance

In the name of the insurance "unit" means that the clients money is accounted in the asset funds – the same way as in investment funds – in units. Investments are usually evaluated daily, this way the value of units can change daily, which is brought to the inquiring client's knowledge through the internet, or an automatic telephone line (as it is required by the regulation in force). Units are accounted on the client's "account". The units of the different offered asset funds are accounted within the client's account on separate sub-accounts. We get the current value of all money in an account by multiplying the number of units with their current price.

The insurance company evaluates units on two prices:

- the offer price
- and the bid price.

Buying and selling are viewed from the insurer's side, so we can look at it the way that when the policyholder pays the premium, the insurer sells (offers) him units, so this is made on the offer price, and when the insurer pays the benefit, he buys units from the client, so he uses the bid price in this case. The same happens when he subtracts units from the client's account during the term upon different grounds.

Naturally the offer price is higher than the bid price, usually by 5-6%. The difference is immediately taken at premium payment by the insurer to cover expenses. On the sub-accounts units are practically accounted on bid price, since after changing the premium to units all accounting is performed only on this price.

The expenses and profits of the insurer have 4 sources in the unit linked insurance:

1. the above mentioned bid-offer spread
2. subtraction of certain types of units
3. regular subtraction of units from the fund
4. fund management fee

Two types of units are distinguished:

1. accumulation („ordinary") units
2. initial units

What has been said so far regarding units concerns mostly the accumulation units. The insurer uses the initial units technique to cover initial (mainly) acquisition expenses. The essence of this is that part of the premiums of the first (or the first two) years (e.g. the part of the first year's premium not exceeding 100,000 Forints) are marked, they are not changed to accumulation units, but to initial units. Formally these initial units work the same way as the accumulation units, but with one significant difference: A certain percentage (generally 5%) is subtracted during a certain period (usually 10 years) at the beginning of each year. After a definite period the remaining initial units are converted to accumulation units.

The continuous subtraction of initial units is only a way of dressing, since the initial units to be subtracted during the whole term are in reality subtracted at the payment of the premium, so they are changed to units only formally. We can easily calculate how much needs to be subtracted (supposing 10 years and 5% yearly): $1\text{-}(1\text{-}0{,}05)^{10} = 0{,}4013$, i.e. 40,13% of the initial units. The surrender value of the insurance is determined in such a way that it only contains the initial units remaining until the end of the term.

The above 4 expense sources cover characteristically different expenses:

1. primarily renewal commission,
2. primarily acquisition commission,
3. continuous administration expenses,
4. fund management and the insurer's profit.

The insurer usually subtracts for covering administration expenses a monthly fixed portion, determined in a Ft sum from the policyholder's funds, so that he mobilises as many units as are necessary on the current bid price to cover administration expenses. The insurer generally increases this fixed expense part yearly by the inflation rate.

The premium of the death risk and the possible rider risks is usually collected similarly to the administration expenses, by subtracting the adequate number of units monthly. The premium of insurance riders is typically fixed, but the premium of the death risk varies depending on the total value of all units in the asset funds. If this exceeds the death sum insured, then the insurer doesn't have any death risk in that particular month, this way he doesn't collect a separate death premium. If, on the other hand the death sum is greater, then the difference, the sum at risk is the risk of the insurer, and a premium is collected to cover it. This premium part is determined simply by multiplying the sum at risk with a multiple corresponding to the age and gender of the insured (which is basically derived from q_x), and subtracts the number of units from the client's account having the value of this Ft sum.

An important feature of unit linked insurance is flexibility. Both the premium, the death sum and the sum insured of riders can be relatively freely modified during the term. But if this is not the consequence of a regular, inflation-handling indexation on policy anniversaries, then – in case of a larger sum insured – it requires a new underwriting procedure.

Flexibility can be detected in other forms, too. Unlike traditional insurance, unit linked can handle top-up premium payments above the regular premium payment of the client. Since the insurer changes premiums to units when those arrive, this way the insurer can easily handle the situation if the client is not paying premiums accurately when due, or maybe leaves a few premiums out. The exact declaration of maturity is also not so important, it can be smudged together with the surrender of the policy.

A paid up policy practically doesn't require any special procedure, and surrender can have many directions. It might be partial surrender (simply withdrawing some money from the investment funds) or a regular, annuity-like withdrawal of money.

The insurer offers several kinds of asset funds to clients, and they can freely choose which ones and in what ratio the insurer should place their money in. The most frequent funds in Hungary currently are:

- Hungarian bond fund (primarily government bonds)
- Hungarian stock fund
- International bond fund
- International stock fund

The client decides whether to place regular premiums only in one fund, or to divide them in a certain ratio between all or some of these. The dividing ratio can be changed at any time concerning future premiums, and can apply a separate, one-time dividing ratio to the occasional top-up payments.

He can also regroup existing units into other funds – but the insurer usually charges a separate changing fee for this.

By choosing funds the client gets – compared to traditional insurance – a greater degree of freedom, but consequently he also has to take over the investment risk from the insurer. But if he follows an aggressive investment strategy (if he chooses stock funds with higher risk), then in the long term he can achieve higher returns than on a traditional insurance.

An important feature of unit linked compared with traditional insurance is that the expense structure of the former is transparent to clients (with the exception of the expenses hidden in initial units). The goal of unit linked is in the long run the same as that of the traditional insurance, only here the savings motif is especially emphasized. Almost all traditional insurances can be "simulated" with unit linked insurance. For Example:

- Unit linked itself can be viewed as a generalised endowment, or whole life insurance.
- If the death sum is chosen high enough, so that at the end of the term the client's account is practically emptied, then we get a traditional term insurance.
- If we complement the unit linked insurance with a conditional annuity, that in the event of the insured's death pays an annuity equal to the regular premium to the client's account until the end of the term, we practically get a term fix insurance.

4.3.6. Term Fix Insurance (à terme fix)

Term fix insurance (in Hungary also referred to as *à terme fix* after the French name) is in Hungary the most popular type of life insurance after the endowment and the unit linked insurance. In this case the insurer undertakes the liability – against the premium payment of the policyholder – of paying a sum specified in advance at the end of the premium term under all circumstances (and not conditionally, as in a pure endowment insurance) to the beneficiary (or if the beneficiary dies during the term, the insurer pays back the premium reserve or the premiums paid so-far). The period of premium payment is until the end of the term, or the death of the insured, if this happens earlier (from this point on the insurance is paid up).

A typical area of using this insurance is saving in advance for a child, e.g. for a starting life support (starting a business, contribution to housing), or the initial expenses of founding a family (as a trousseau insurance). The insurance is more than a savings account in the way that the child receives the required sum even if the parent – because of an earlier death – cannot save up the total initially planned sum.

If we think about it, this is really an endowment insurance with a varying death sum, since when the insured dies, the claim has happened from the insurer's point of view, even if he doesn't have to pay at once. But from this point on he doesn't receive premiums to fill the reserve, so the insurer has to fill it up at once to the level that will reach the maturity sum at the end of the term, compounded by the guaranteed (technical) interest rate. This way we can look at the term fix insurance as an endowment insurance with a death sum insured always equal to the value of the maturity benefit discounted to the time of death. To be more precise, we have to add that the term fix insurance would only be an endowment insurance with varying sum insured if in the event of death the benefit would immediately be paid. Since it is not, we can suppose that this endowment insurance with varying death sum implicitly contains another policy, according to which the policyholder can change the death benefit to a single premium term fix insurance, that has the same maturity date as the original policy (and its sum assured is the original maturity benefit).

This single premium term fix insurance is rather a theoretical design, since the insured event is missing. But as a theoretical design – similar to the pure endowment insurance – it has an important role in other insurances, e.g. in the case of investment-profit sharing of the term fix insurance itself.

The term fix insurance can not only be regarded as an endowment insurance with varying death sum, but also as a savings "account", with a conditional annuity rider. The condition that starts the annuity payment is the death of the insured, and the annuity benefit is that it performs further payments of the savings part of the premium to the "account".

Which one of the above theoretical constructions we imagine has significance in how the insurer handles the problem of reserving and investment profit sharing after the insured's death.

In case of the term fix insurance we have to mention that it is usually signed by parents as insured persons for children as beneficiaries, this way here it has particular significance who the insured parent is. If it is mostly one parent who makes a living for the family, then he has to be named as insured, because it is his death that causes economic difficulties to those left behind. A typical sales mistake is, when instead of the householder of the family it is the parent who is at home who is named as insured, because it is easier this way, or because the insurance is cheaper this way.

4.3.7. Annuities

Annuities have many forms, and often they are also combined with other insurances. Not every form of annuity can be regarded as life insurance, only those that have a beginning, an end and a term depending on the insured's death. There are single premium and regular premium, immediate and deferred, paid in advance or in arrears, temporary or lifetime annuities.

4.3.7.1. Single Life Annuity – Single Premium, Immediate Lifetime Annuity

Lets look at, first of all, the *single premium, immediate, lifetime annuity paid in advance*. Here the insurer undertakes the liability against the single payment of a greater sum, of paying at the beginning of every (insurance) year (or as this happens in the practice, in monthly instalments), starting from that moment, a certain sum to the insured while he is alive. When the insured dies, the annuity ends.

It can be clearly seen that this insurance in reality is a series of pure endowment insurances, that have the same sum insured, but different terms, the successive ones have a term one month longer than the preceding one.

This type of insurance serves us as a good starting point, since we can easily place the other types in relation to it.

Whether an annuity is paid *in advance, or in arrears* is simply a question of technique. The difference is whether the insurer pays the annuity at the beginning, or at the end of each year. Of course there only is a difference in the first year. In case of an annuity paid in advance the insured receives the first instalment right at premium payment, while in case of payment in arrears only a year later.

The difference between *immediate and deferred annuities* is that in case of the immediate annuity benefit payment starts in the first year (in advance or arrears), while in case of deferred annuities only a few years later (which is naturally defined at contracting). During those few years the sum of the singe premium works like an interest deposit, an investment, or a single premium pure endowment (or maybe endowment) insurance.

This way we can also say that the deferred annuity is the combination of an interest deposit or a pure endowment (or maybe endowment) insurance and an immediate annuity.

The difference between *single premium and regular premium* annuities is when the insurer receives the capital providing the base of the annuity. If the client pays in a single sum, all at once, then we are dealing with a single premium annuity, if it is collected gradually, in instalments, then it is a regular premium annuity. But the capital accumulation phase can also be regarded not as an annuity, but as an interest deposit or a regular premium insurance, as in the previous case. This way the regular premium annuity can be regarded as the combination of an interest deposit (or a regular premium pure endowment insurance) and a single premium immediate annuity.

Deferred and regular premium annuities are connected, since premium payment has to end before the annuity starts, this way regular premium payment is only possible during the deferred phase. A special case can be an exception of this, when the policyholder and the insured is not the same person, and the insurer starts to pay the annuity benefit to the insured, while the policyholder gets the allowance of paying the premium in instalments.

The difference between a *lifetime annuity and a temporary annuity* is that the lifetime annuity is paid until the death of the insured, while the temporary annuity is paid either until a fixed date (e.g. until the insured reaches age 18), or during a certain term (e.g. it is paid for 5 years). If this is not the *certain annuity* mentioned in the chapter covering financial mathematics, then its beginning generally depends on someone’s death (e.g. parent, spouse, etc…).

From time to time certain concrete annuity plans appear in life insurance practice that are named “Pension insurance”, “Widow/widower’s annuity”, “Orphan’s annuity”. These names can cover several kinds of constructions, or combinations.

A pension insurance plan is usually a lifetime annuity or a deferred lifetime annuity with or without a guarantee period (see later on). The temporary annuity is decidedly unsuitable for pension insurance, since this would mean that the insured receives the annuity until alive, but only until reaching a certain age, e.g. 80 years. A temporary annuity is naturally cheaper than a lifetime annuity, since it generally provides less benefits. This way it can be attractive to those, who think that they won’t reach age 80. If this happens all the same, the annuity benefit will end just when this might be the main income of the insured, when he is too old to make a living from his own work.

4.3.7.2. Joint Life Annuities

An orphan’s annuity on the other hand should only be non-temporary in very special cases (e.g. physically or mentally disabled child). Here mostly the different types of temporary annuities can have a role, since usually the orphan child only needs the orphan’s annuity until reaching the working age, or until his/her possible death. Of course many orphan’s annuity constructions can be imagined. A possible design is, e.g. a two- or three-person temporary annuity, where one insured is the child, who is also the beneficiary of the potential annuity, and the other (two) insured(s) is (are) the parent(s). The annuity can be single or regular premium. The term of the annuity is the period until the child reaches a certain age (say, 18 years). The payment of the annuity only starts after the death of the parent (or in the three-person annuity after the death of one of the parents), and lasts up to the end of the term, but only until the earlier death of the beneficiary (the child). (Since the probability of death during infancy is very low, the annuity after the parent’s death can also be a certain annuity, so the child isn’t necessarily insured.) In the regular premium version premiums are paid until the end of the premium term, or the earlier death of the parent. But this regular premium version has problems connected to the premium reserve, that we will mention in relation to the discussion of the premium reserve.

The widow/widower's annuity can also have several types. The annuities with guarantee period, that will be discussed later on, can be regarded also as widow/widower's annuity, but the widow/widower's annuity is most often a two-person joint life annuity.

These joint life widow/widower's annuities can be divided into two large groups. Using my own terms we can say that there are “symmetric” and “asymmetric” annuities. In case of the symmetric annuities either one of the insured persons can be regarded as widow/widower, so we do not declare beforehand who the widow/widower will be, but simply say that it is the survivor. So here there will always be a survivor, with the low-probability exception of the two deaths occurring at the same time.

In case of the asymmetric annuities it is not certain, that there will be a survivor, because we declare beforehand that the widow/widower can only be the insured that we declare in advance, and only if this insured lives out the other insured (which is of course evident).

Let's look at a few possible widow/widower's annuity designs! Within the symmetric annuity type a version is possible in which case after the death of only one insured a yearly annuity of 1 Ft is paid to the other insured as a beneficiary, until his/her death.

An example of an asymmetric annuity is:

Let there be a primary insured (whose death makes the other insured a widow/widower) and a secondary insured (the potential widow/widower). The construction is single premium. After the death of the primary insured the secondary insured receives an annuity of 1 Ft until his/her death, if he/she is alive when the primary insured dies. If the secondary insured dies before the primary insured, the insurance ends without benefit payment.

Naturally a number of other pension-, widow/widower's- and orphan's constructions can be imagined beside the above.

It is worth mentioning the concept of “conditional” annuity. We have already given several examples of it in the above sections. The essence is: the beginning of the conditional annuity is subject to an insured event, so the benefit payment might not begin at all.

4.3.8. Pension Insurance

Pension insurance is usually mentioned separately, although logically it belongs to life insurance. The name “pension insurance” typically refers to the goal of the life insurance policy rather than a special type of product design. This goal can be achieved through many kinds of plans. Basically two types of plans are used as pension insurance, and their function is different:

1. savings type (capital accumulating) life insurance
2. annuity insurance

According to the logic of pension insurance everyone saves during the active part of the life cycle the required level of capital, that he changes to annuity at retirement, and uses up by the end of his life. Naturally these two insurance types can be combined into one plan. Then the term of the combined insurance can be divided into two parts: a capital accumulating phase and an annuity payment phase.

During the capital accumulating phase the savings type insurance might be an endowment insurance, a pure endowment insurance with premium return guarantee, a unit linked insurance, or a simple “deposit account” type saving. Naturally capital accumulation can be performed outside of the insurance sector, and the capital saved up this way can also be changed to an annuity.

During the annuity payment phase a single premium annuity starts to function. This phase usually begins with retirement.

Pension insurance generally receives outstanding government attention because of its outstanding social significance. This outstanding attention can have different forms in different countries. Tax allowance is generally used, the definition of a special pension product (“account”, or investment fund) is also frequent, or even the creation of a separate institution system (e.g. pension fund). Where government regulation has created a separate institution system for pension insurance, it seems that this is a separate sector, but in reality pension is an organic part life insurance (although not necessarily private life insurance).

4.3.9. Complementary Risks – Insurance Riders

Life insurance is very often sold with coverage provided also for the following risks:

- accidental death,
- accidental disability,
- certain "critical illnesses" or "dread diseases",
- disability,
- surgery,
- hospitalisation.

The death of the householder always causes financial difficulties. These difficulties can appear in a cumulative way if the death was unexpected, due to an accident. This way in these cases the insurer offers a supplemental cover beside the "normal" death sum insured, if the cause of death was accident. **Accidental death** usually means a sudden, outside effect[84] independent of the insured's intention, that causes the insured to die within a year. The accidental death sum insured is unusually 1-2-3-times the "normal" death sum insured, with the restriction that the insured sets an upper limit on this sum.

Accidental disability insurance is usually sold independently and also as a rider to life insurance. In insurance companies offering independent accidental death coverage, the accidental death risk is not included in accidental disability, while the insurer that doesn't sell accidental death as independent coverage, includes this risk in the accidental disability coverage. If accidental death and disability are separate insurances, the insurer has to pay attention to the following rules:

- Accidental disability cannot be offered without an accidental death coverage (but it can be offered the other way around).[85]
- The accidental death sum insured cannot be lower than the benefit payable in case of 100% accidental disability (it would not be too easy to sew the insured for part of the sum paid for 100% disability if he dies within a year after the accident).
- The covered term of accidental death cannot be shorter than the term of the accidental disability cover, and the accidental death cover cannot be ended sooner than the accidental disability cover.

The definition of accidental disability is quite similar to that of the accidental death: accidental disability means that the insured suffers severe and permanent deterioration of health caused by a sudden, outside effect independent from the insured's intention, within one year. (sunstroke, etc. is usually excluded from the definition here also.)

The level of permanent health deterioration is determined by the insurer's doctor. The "table of injuries" or – after the common German terminology – "gliedertaxe", that is part of the insurance terms and conditions helps him in this. This enumerates the most common losses of body parts and functions. The individual insurance companies might use different percentages, but they usually work with very similar tables. The table of injuries measures a kind of general health deterioration. It doesn't take into account that the body-parts, capabilities (e.g. because of his occupation) of the given insured might be more important than generally for most of the people (e.g. the fingers for a surgeon, a piano player). If someone requires coverage fro such special risks, he has to take out an individual accident insurance.

An example of the gliedertaxe[86]:

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[84] Usually not including death caused by overstrain from lifting, sprain, freezing, sunstroke or heat-stroke.

[85] It is interesting that when I teach this rule to people actively practicing in the insurance business, there are always a few who argue on this rule, saying that the accidental disability benefit is payable while the insured is alive, and has the function of facilitating the insured's further life, while the accidental death benefit has a principally different function, as it goes to the dependents. It can easily be imagined that someone doesn't have dependants to provide for, so it is unnecessary for him to purchase the accidental death coverage. This is an acceptable train of thought, but it doesn't count with the fact that after an accident the formation of the final level of disability or death is a gradual process. According to this there might be several benefit payments (because of the disability becoming more severe), and it is not too purposeful to sue for repayment. The problem doesn't have a clear solution, but it is wiser to take the points of the insurer into account here.

[86] This was used by the former ABN-AMRO Insurance Company.

Injuries of body-parts	Degree of disability
Total loss or loss of function of one of the upper limbs from the shoulder joint	70%
Total loss or loss of function of one of the upper limbs above the elbow joint	65%
Total loss or loss of function of one of the upper limbs below the elbow joint or the total loss or loss of function of a hand	60%
Total loss or loss of function of a thumb	20%
Total loss or loss of function of an index finger	10%
Total loss or loss of function of any other finger	5%
Total loss or loss of function of one of the lower limbs above the middle of the thigh	70%
Total loss or loss of function of one of the lower limbs up to the middle of the thigh	70%
Total loss or loss of function of one of the upper limbs below the elbow joint or the total loss or loss of function of a hand	50%
Total loss or loss of function of a foot at the level of the ankle	30%
Total loss or loss of function of a big toe	5%
Total loss or loss of function of any other toe	2%
Loss of the sight of both eyes	100%
Loss of the sight of one eye	35%
Loss of the sight of one eye if the insured has lost the sight of the other eye previously to the insured event	65%
Total loss of ability to talk	60%
Total loss of ability to smell	10%
Total loss of ability to taste	5%

Table 4.1.: Table of accidental injuries

Under the term health insurance we usually mean accident or sickness insurance. It primarily has insurance technical causes that the uniform health insurance is divided into two categories. Accident insurance – as we have seen – is under all circumstance an insurable risk. This cannot be stated of most of sicknesses, the auto-selection and moral risk is too strong. But this is not true for some sicknesses, that – we can say – behave in an accident-like way. Insurers should offer cover against the sicknesses that satisfy all of the following criteria:

- they appear in relatively rare instances,
- people know them well and are afraid of them,
- they would do anything to avoid them,
- if they happen, they cause significant financial consequences.

These illnesses are called by an overall term "**Dread Diseases**". (Instead of this name the term "**critical illness**" is becoming more frequent nowadays.)

Dread diseases are defined differently from insurer to insurer, but the following usually belong to the insured circle everywhere:

- heart attack,
- stroke,
- cancer,
- the need for artery-bypass operation,
- kidney failure.

With such a cover, if one of the sicknesses in the policy terms and conditions is diagnosed in the insured, then the insurer pays a sum (which serves the purpose that the insured can finance the treatment of the given illness) independent from the other benefits, or the benefit payment (or a part of it) of the life insurance – that was the main policy of the dread disease rider – is brought forward.

Severe disability is a case, when the insured (if he is at the same time the policyholder) cannot necessarily continue the premium payment of the life insurance, while he still needs the coverage it provides. In case of traditional insurances the **disability waiver of premium** cover or rider solves this problem. According to this insurance if the insured becomes severely disabled during the (premium) term of the main policy, then the risk community takes over further premium payments (so the main policy becomes paid up for him). The paid up term is usually the remaining term of the main policy (or, naturally the period until the earlier possible death of the insured), but sometimes the insurer declares that if the state of the insured should get better, the premium payment might be restored.

Disability is usually considered severe by the policy terms and conditions if it is 67% caused by accident, or 100% caused by illness. In Hungary there is no coverage offered for disability of lower degree.[87] The social security distinguishes two types of 100% disabilities caused by illness, and this practice is taken over by the private insurers, moreover they tie the benefit payment to the declaration by the social security system of category I or category II disability. The difference between these two categories is that while the category II disabled is able to take basic care of himself, the category I disabled needs care in the everyday life.

Premium payment in modern insurances (primarily the unit linked insurance) is not as well defined as in case of traditional insurances, this way the benefit of the disability waiver of premium also cannot be well defined. This problem can be solved if we realize that the disability waiver of premium is implicitly an annuity insurance, namely a conditional annuity that begins with the disability and has an annuity payment equal to the premium of the life insurance. This implicit annuity can be made explicit and then we get a **disability annuity rider** that is practically the same as the disability waiver of premium rider. This can serve primarily in case of unit linked the same function (naturally it can be taken out as a rider of traditional insurance, also) as the disability waiver of premium rider by traditional insurances, but it can also be made an independent benefit. But insurers don't really like this, because they are still afraid of the disability risk "tamed" this way.

Beside the disability annuity insurance the most common sickness insurance ride in Hungary is:

- surgery benefit and
- hospitalisation daily allowance rider.

In case of the surgery benefit the insurer groups possible surgeries into categories of "severity" (generally 5 categories), and if a surgery is performed on the insured, then a part of the sum insured of the surgery benefit rider is paid as a lump sum payment that corresponds to the category of severity of the surgery performed (e.g. 20% of the sum insured in case of category 1 and 100% of the sum insured in case of category 5). The goal of the insurance is to cover expenses arising in relation to the surgery.

If the insured is hospitalised, this also has costs, and are mainly proportional to the number of days spent in the hospital (loss of income, supplementary expenses, "gratitude money"). The hospitalisation insurance rider satisfies the needs arising due to these, which has a benefit of a certain daily allowance (e.g. 5,000 Forints a day). This is usually not paid for short (3-5 days) hospital stays (saying that the financial needs are not as demanding, and the costs of handling the claim would exceed the degree of benefit payment), only if it is longer than this lower limit – usually with an upper limit also applied (e.g. 60, 180 or 365 days).

[87] At the beginning of the '90s 67% disability caused by illness (in other words category III. disability of the social security system) was also part of the coverage, but it has been left out due to the numerous cases of frauds. 67% disability in Hungary seems to be too subjective and can be manipulated too much.

5. CATEGORISATION OF LIFE INSURANCE

KEY WORDS

Main policy
Group insurance
Individual insurance
Single premium
Elemental life insurance
Main policy
Traditional life insurance
Long term care insurance
Insurance rider
Modern life insurance
Regular premium payment
Reduced premium payment
Multiple life insurance

5.1. USUAL CLASSIFICATION OF LIFE INSURANCES

The study of life insurance is an applied science. Its subject, life insurance has been created based on practical considerations centuries ago (or thousands of years ago – according to other opinions) and it has developed by practical challenges, not theoretical discoveries. Because of this, the theory also tried to follow and reflect these challenges and did not aim at the axiomatic structure of the "classical" sciences (primarily mathematics and physics) that serve as examples to other sciences. This way – although it would seem as a necessary first step and basic requirement – there is no widely accepted common categorisation of life insurance, but it changes from author to author, is strongly inferior to the later matter, and usually gives an ad-hoc impression. Now let's look at a few examples from the Hungarian and the English literature.

5.1.1. Categorisation of Life Insurance in the Hungarian Literature

In the Hungarian market, considering questions of insurance theory it was Dr. Dezső Csabay in the last decades, who has created the most enduring foundations, and his writings serve as a standard up to our days (although in many respects it would be useful to rethink them). Considering life insurance he[88] states the following under the title "The Classification of Life Insurance"[89]:

"There are several usual classifications of life insurance according to several aspects:

I. According to the conditional or unconditional liability of the insurer. …

II. According to conditions depending on the health status of the insured: normal or abnormal (high risk) life insurance.

III. According to the insurance benefits: capital and annuity insurance.

IV. According to the insured event: term endowment, pure endowment insurance and annuity.

V. According to business management, mostly based on the administration and transaction method of policies:

a) major life (or regular life),
b) minor life (or popular, or industrial, workman)
c) group insurance"

The individual (life) insurance products themselves can be further categorised according to several aspects[90]:

By the type of premium payment:

1. single premium
2. annual or monthly premium (recurring premium)
a) fixed (level) premium
b) variable premium

[88] dr. Csabay Dezső: Általános biztosítástan, KJK 1971 pp. 333-366.
[89] p. 352.
[90] p. 356.

As per the number of insured:
1. single life insurance
2. double o multiple life (mutual) insurance

„The most important categorisation of insurance products is categorising based on the *purpose of the insurance* – or the insurance term. Basically all products of this aspect can be derived from two basic products or their different combinations. We distinguish four main types:
1. Term insurance, ….
2. Pure endowment insurance, … annuities belong to this type, …
3. Endowment insurance, which is the combination of the two above …
4. Fix term insurance …”[91]

dr. Csabay makes similar statements 10 years later[92]: „LIFE INSURANCE is the most important type of the personal (sum)insurance branch. It is an insurance where the declared sum assured is to be paid when the insured lives to a fixed date or a fixed age (pure endowment insurance), or upon the death of the insured (term insurance), or in both cases (endowment insurance). Annuities also belong here (recurring, periodically payable pure endowment insurance).”

“The different types of life insurances are usually divided into several groups. These are the major life (or regular) insurances of greater sum assured, and the minor life (or “popular”) insurances of smaller sum assured; in the western world: industrial or workers' life insurance, short term life insurance (risk insurance) type credit insurance, group insurance, etc.

The insurance types of the individual groups are divided to insurance products (or tariffs) – according to their purpose. The three major products are: term, pure endowment and endowment insurance. All of these exist in a number of forms.”

The few remarks on the next couple of pages are also very interesting and typical, although they cannot be considered totally accurate in an actuarial sense:

“With the exception of the pure risk insurances (term and pure endowment) the net premium of the other life insurance types consists of two main elements: a risk premium and a savings premium. (Premium reserve.)

In the western insurance literature life insurance with a fixed sum assured – in order to differentiate from annuities – is sometimes also called “capital insurance”.

A lot of insurance theoreticians and lawyers since the 19th century consider the endowment (and the pure endowment with premium refund) insurance according to its economic purpose not as insurance, but as a savings deposit, because it contains only a minor risk element.”

László György Asztalos in 1995 stated – basically agreeing to the above classifications in the chapter “the categorisation of life insurances”[93], that “An individual life insurance policy can only be categorised, professionally defined by applying at the same time (using a combination of) several, at least the following 6 aspects (groups).” These are:
A. the purpose of the insurance policy,
B. the date(s) related to the insurance policy, or the term,
C. the mode of benefit payment,
D. the number of insured lives,
E. the technique of premium payment,
F. the return of yields.

Variations of the first aspect are:

„The purpose of the insurance is in every case that the insurer takes over the risk caused by the uncertainty of the duration of life. All insurance products can be listed under one of the 4 basic types.

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[91] p. 357.
[92] dr. Csabay Dezső: Biztosítási Lexikon, Állami Biztosító Terv- és Közgazdasági Főosztálya, 1980, szerk.: dr. Ébli Györgyné p.107. Életbiztosítás címszó. (Insurance encyclopaedia, “Life insurance” entry, p. 107.)
[93] Asztalos László György: Biztosítási alapismeretek, BOI, 1995. p. 330.

1. In case of the term insurance the sum assured is to be paid in the event of death of the insured, that can occur at any time (Whole-life Policy). If, on the other hand the insured doesn't die during the term of the policy, the insurance contract expires – without any benefit payments received from the insurer.

2. In case of the pure endowment insurance (Versatile Endowment Policy) the insurer pays only if the insured survives a fixed age (survival age).

3. In the combination of the above two methods, in case of endowment products the insurer pays the benefits if the insured

(a) dies before reaching a specified age, or

(b) reaches the specified age alive.

4. In case of the so called term fix (fix term, "a terme fix") insurance it is not the fact of death or being alive that is important, but (the time of) some other event that may not even happen. At this time the sum assured is paid under all circumstances,

(a) either to the living insured person, or

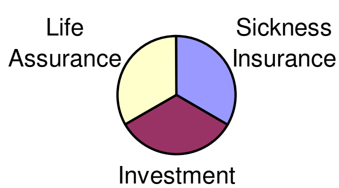
(b) the beneficiary that the insured has declared in advance (e.g. inheritor)."

Interesting in the above categorisation is that – uniquely in the literature on the subject – it identifies term and whole life insurance, and the author only mentions annuities in the subgroups of aspect C).

5.1.2. Life Insurance Classification in the English[94] Literature

The English literature handles categorisation in a much more practical way even compared to the above discussion. The volume CII (Chartered Insurance Institute), that – in a certain sense – serves as an official curriculum in England, discusses life insurance under the title "Basic types of long-term insurance policy"[95]:

"..., as a brief revision and introduction to those policies, you might find it useful to think about the vast range of policies as falling in one of the sections of the pie chart below.



Some of the policies you will encounter will fall easily into one of these sections, being entirely either life insurance, sickness insurance, or investment-based, but others may fall under two or even all three sections.

Consider the following examples, and decide where each of them could most appropriately be placed in that chart.

- term assurance
- endowment assurance
- whole of life policies
- permanent health insurance
- annuities."

The book gives the following solutions:

- term assurance = pure life insurance,
- endowment assurance = minor part life assurance, major par investment,

[94] Unfortunately I'm not proficient in the German professional literature, but I might not be too far from reality in supposing that the German categorisation has fundamentally influenced Csabay's grouping.

[95] Popplewell, K.: Life Assurance, Study Course P45, The Chartered Insurance Institute (CII), 1992. p. 3/2.

- whole life = major part life assurance, minor part investment,
- permanent health insurance = totally health insurance,
- annuity = important type of assurance, but cannot place it in the diagram.

He also remarks that: “Finally, you will probably be aware of a number of other types of contracts not mentioned above, such as “critical illness policies” (being partly sickness insurance, partly savings, and also usually partly life insurance) and “long term care” (sickness insurance).”

The volume „Life Insurance”[96] of imposing size, that has a history of a century doesn’t deal with the categorisation of life assurance in a comprehensive way. As an introductory remark it says that “The simplest form of life insurance protection is yearly renewable term (YRT) insurance.”[97] It gives the following definition of this type of insurance: “Yearly renewable term life insurance provides coverage for a period of one year only, but guarantees the policyowner the right to renew (i.e., continue) the policy even if the insured suffers poor health or otherwise becomes uninsurable. Each year’s premium pays the policy’s share of mortality costs for the year. The renewal premium rate increases each year to reflect the annual rise in death rates as age advances.” After this the author considers the discussion of premium payment types to be most important.

He only returns to the categorisation in a later chapter (“Overview of types of life insurance”)[98]:

“As suggested in Chapter 2, life insurance policies can be constructed and priced to fit a myriad of benefit and premium-payment patterns. Historically, however, life insurance benefit patterns have fit into one or a combination of three classes:

- Term Life Insurance
- Endowment Insurance
- Whole Life Insurance”

He also mentions that[99]: “Another class of insurance issued by life insurers is annuities… Most annuities are savings instruments designed to first accumulate funds and then systematically to liquidate the funds, usually during one’s retirement years.

The above life insurance classification scheme remains valid today, although it is not always possible to determine at policy issuance the exact class into which some types of policies fall. As discussed in Chapter 6, some policies permit the policyowner flexibility effectively to alter the type of insurance during the policy term, thus allowing the policy to be classified as to form only at a particular point. For presentation purposes, these flexible forms of life insurance are discussed as if they were an additional classification, even though all can properly be placed (at a given point in time) into one or a combination of the three traditional classes.”

The following discussion is also interesting. Term assurance and endowment assurance appear in the same chapter, as subchapters, but the Whole Life, the “Flexible-premium Life Insurance Policies” (the 6th chapter) and the “Annuity and Special-Purpose Policies and Benefits” received separate main chapters.

Concerning endowment assurance it declares that[100] “There are two ways of viewing endowment insurance: in terms of (1) the mathematical concept, and (2) the economic concept.

Mathematical Concept. The insurer makes two promises under endowment insurance: (1) to pay the face amount if the insured dies during the endowment period, and (2) to pay the face amount if the insured survives to the end of the endowment period. The first promise is identical with that made under a level term policy for an equivalent amount and period. The second introduces a new concept, the pure endowment. A pure endowment promises to pay

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[96] Black, Kenneth, Jr. – Skipper, Harold D., Jr.: Life Insurance, Twelfth Edition, Prentice Hall, Englewood Cliffs, 1994.
[97] p. 24.
[98] p. 82.
[99] p. 83.
[100] pp. 95-96.

the face amount only if the insured is living at the end of a specified period; nothing is paid in case of prior death. Pure endowment insurance is not sold as a separate contract in the United States. It is said that few people are willing to risk the apparent loss of all premiums paid in the event of death before the end of the endowment period. …

Economic Concept. Another analysis of endowment insurance, the economic concept, divides endowment insurance into two parts: decreasing term insurance and increasing savings. The savings part of the contract is available to the policyowner through surrender of or loan against the policy."

Chapter 6. mostly discusses the Universal and the Variable Universal Life type assurances.

It is interesting that Life Insurance, Theory and Practice, that was also published in America a few years earlier, has also lived many publications and is also a thick book – although splits assurances into similar groups – doesn't define the same categories as Black and Skipper. In the chapter "Basic Types of Life Insurance Policies"[101] it writes the following: "Life insurers issue numerous types of life insurance contracts. Many of the policies are special combinations or variations of what are often considered to be the basic forms of life insurance: term, whole life, and universal life." The authors insert here in a footnote that: "Life insurers also write annuities … Some persons like to argue semantically that annuities are the only form of true life insurance as they insure persons against outliving their income. These persons argue that what is called life insurance should be called death insurance as it insures a person against loss caused by death." The very beginning of the chapter emphasizes that two important variations must be remembered: the insurances called variable life and variable universal life.

The authors handle "other" life assurances in a separate chapter ("Product Diversification and Special Purpose Policies"). They find that many types of new insurances have been introduced in the '70s and '80s that give the customer many new options. This had the effect that „In recent years, the diversification trend has resulted in some blurring of the demarcation lines that traditionally distinguished the various types of financial institutions. Most observers predict that further breakdowns in institutional distinctions will occur as diversification continues."[102]

The first special purpose insurance it discusses is the endowment insurance. „Although once considered to be one of the basic forms of life insurance, endowment coverage has declined drastically in popularity in recent years, partly due to the development of more flexible products, such as universal life. During 1984, less than half of 1 percent of all new ordinary insurance purchased was endowment coverage."[103]

Funny, that although both books have the title "Life Assurance", both discuss health insurance and different kinds of welfare plans in full detail without considering them conceptually as life assurance.

5.1.3. General Features of the Categorisations

Based on the above, one can make the following statements concerning the usual life assurance categorisations.

1. There is no unambiguous tradition concerning the classification that could be continued or that has to be followed.
2. There is no general aspect that gives a basis to everyone's categorisation. There probably is no single aspect by which all products can be satisfyingly categorised.
3. The authors do no aim at comprehensiveness, they handle exceptions quite easily.
4. The authors tend to regard the most common products as basic products (e.g. the handling of endowment assurance by English language authors, and also the absence of fix term insurance in their writings).
5. It is uncertain and has always been so what they regard as life insurance and what not, but this also doesn't really bother the authors.

[101] p. 51.
[102] p. 93.
[103] p. 96.

5.2. PRACTICAL CLASSIFICATION OF LIFE INSURANCE BY DIFFERENT ASPECTS

In the following we will outline a multi-aspect, practical classification – mostly following Csabay's track, but also deviating from it at many of points. The aspects taken into account are:

1. the historical order of development,
2. the logic of the internal structure, and the type of benefit,
3. administrative and legal considerations,
4. the relation of the status of the policyholder and the insured,
5. the number of insured persons,
6. the term and frequency of premium payment.

5.2.1. From the Aspect of the Historical Order of Development – Traditional and Modern Life Insurances

There is a classic form of life insurance developed in the 19th century, that has changed very little during the 20th century. But the challenges (e.g. inflation, competition with other financial products) and potentials (primarily information technology – IT) of the 20th century gradually began to undermine the well established and well-thought-out traditional product design and new types of products, characteristically different from the former ones have evolved. Because of this, it is useful today to distinguish the classical type insurances from the new types, so the terms "traditional" and "modern" life insurance are widely used.

The term traditional insurance usually means the products designed by the combination of term insurance and pure endowment insurance, that is: term insurance itself, endowment insurance, fix term, pure endowment with premium refund, whole life and annuities. Their characteristics are fixed technical interest rate, and that all major parameters of different points of the insurance term can be well foreseen.

The modern life insurances differ from the conception of traditional insurances in two respects:

1. product design and
2. insured events.

It was the Unit Linked Insurance that has brought innovation in the product design by integrating the main features of investment funds into life insurance, and by practically eliminating the pure endowment part as a building stone through the uncertain development of the reserve.

From the insured event side it was the newly emerging risks at the boundary line of life insurance and health insurance that widened the field of life insurance (and at the same time smudged the boundary line between the two insurance branches). Some well defined, serious diseases (Dread Diseases or Critical Illnesses) and the need for care have been introduced into life insurance plans as insured events. The new insurances corresponding to these events appeared either as insurance riders (mostly Critical Illness), or as independent policies (typically Long Term Care Insurance).

Modern life insurances first appeared in the '50s, but their world-wide spreading can be placed in the '80s and '90s.

5.2.2. By the Logic of the Internal Structure, and the Type of Benefit

Traditional life insurances form a well constructed system, they can be built from certain building blocks – as we have mentioned earlier. They can be built from two basic building blocks, this way we can also call these atomic life insurances. The two atomic life insurances are

- the Term Insurance and
- the Pure Endowment Insurance.

A few brief examples: endowment is the combination of these two, whole life is the border-line case of term Insurance, annuities are basically series of pure endowment insurances, etc.

We can conclude that traditional life insurances are the combinations of atomic insurances. The most common combinations are usually called **basic life insurance**. It changes from country to country what the basic life insurances are. In Hungary it is primarily the:

- endowment,
- fix term,
- term and
- immediate, single premium annuity

that can be considered basic life insurance.

5.2.3. From the Administrative and Legal Aspect – Main Policies and Riders

The insurance companies form “commercial packages” from the above life insurances, and sell these packages. When the policyholder signs an insurance contract, he actually buys such a commercial package. This commercial package can be constructed in two different ways:

1. The insurance company builds into a single design, the so-called “insurance product” one or a few of the above life insurance types, and maybe ads on – also built into the plan – a few non-life (accident or health insurance) elements, too.
2. A policy is built from several optional elements. In this case one element, that has to be a life insurance is dominant, this is what we call the main policy, and the other elements – that can also be non-life elements – are additional, supplementary elements, called riders to the main policy.

On the relation of life- and non-life insurances we have to mention that according to European Union guidelines (that the Hungarian insurance legislation naturally took over) a life insurance company can only sell life insurance, with the exception of accident and sickness insurance, if the life insurance contains these as “supplementary risks”. (In this case these non-life insurances are accounted under the life branch.) Nor the guideline, nor the Hungarian insurance legislation defines precisely just when a risk can be considered supplementary, but one point is certain: a life insurance company cannot sell accident or sickness insurance separately.

In Hungary at the time of the insurance monopoly separate insurance riders didn’t exist, these were added to the set of insurance designs only with the development of market economy. On the other hand, built-in non-life risks have been part of life insurance policies even then, mostly health risks, where the insurer provided a waiver of premium benefit if the insured became disabled[104].

There are two separate types of insurance riders, providing:

1. independent benefits, or
2. options.

The **rider providing independent benefits** has two subtypes, depending on whether the benefit of the rider is:

- independent from the main policy, or
- depends on the benefit of the main policy.

The former subtype can be life, accident, or health insurance. Typical examples are: term assurance rider (life assurance), accidental death or accidental disability insurance rider (accident insurance), and hospitalisation daily allowance insurance rider (sickness insurance).

An example of the later is the waiver of premium disability insurance rider (sickness insurance).

An example of **options** is the premium increase option that allows the policyholder to increase the premium at certain periods without repeated underwriting.

The technical solutions of insurance riders (inflation handling, premium frequency etc.) usually follow that of the main policy, and expire when the main policy expires.

[104] See subchapter 4.3.9.!

5.2.4. By the Relation of the Status of the Policyholder and the Insured – Individual and Group Insurance

Insurances can also be categorised as individual and group insurances.

Individual life insurance is usually signed by individuals as policyholders with individuals as insured. In case of group insurance, the policyholder either signs the insurance contract for a total collective (e.g. members of an association, employees of a company, etc.) at once, or a prerequisite of becoming member of a certain group (e.g. customers of a bank taking out a loan) is signing the insurance contract.

Group insurance technically doesn't necessarily differ from individual insurance, but if signed as a group, the insurer may offer a premium discount. This is one of the main reasons for the marketing of group insurance. Because in this case the underwriting and possibly also the administration becomes simpler, thus resulting cost-saving, and this saving can be the base of a premium discount. On the other hand, the so-called "group calculation" is also possible, which means that the members of the group pay a level premium for a unit of benefit, independent of the differences in their own personal risk (e.g. age). In this case premium calculation is based on certain "average" characteristics of the group.

The marketing of group insurance, entering the risk community in a simplified way and offering a level premium is based on the fact that the group was not organised for signing the insurance, but for some other goal, but the members automatically have the insurance. This is the only way that anti-selection – otherwise undoubtedly appearing – can be eliminated.

Regarding group insurance we cannot go by without mentioning one of the most important group insurances of the Hungarian insurance history, the Group Life and Accident Insurance (CSÉB). This group of policies, that was started in the '60s and sold up to the beginning of the '90s was held by more than 4 million Hungarians in its days of glory (the '70s and '80s). It basically was a simple, standardised insurance providing cover (at a low level) for all significant personal insurance risks, that was only called group insurance because of the method of premium calculation and marketing (since in other respects it was an individual type insurance). The premium calculation didn't distinguish insured persons by age and gender, this way it was particularly advantageous for older males to sign the policy and particularly disadvantageous for young females. Since this tendency, if only older males had signed the CSÉB contract would have caused a problem for the insurer, a special marketing technique was used to avoid this situation. It was only sold at places of work. The socialist mammoth companies typical of that period were subdivided among agents, and an agent was allowed to move on to the next company with selling CSÉB only if he had reached a certain level of saturation – projected on the total number of employees – (which meant that he didn't only sell the insurance to older males). It is obvious that this policy was the product of the period of insurance monopoly and group insurance with this principle cannot be sold on the competitive market.

Today group insurance is usually short-term, without premium reserve, so in this respect it can be regarded as non-life insurance.

5.2.5. By the Number of Insured Persons – Single Life or Joint Life Insurance

Individual life assurance can be single life or joint life insurance, or there may be one or several insured persons declared in the policy. The default is one insured, this way we made this implicit supposition in all the above cases. On the other hand, joint life insurance policies are also relatively common. This is usually taken out by married couples, which gives an explanation to why this type of policy is so rare in Hungary. Because of the high divorce rate, a major part of these policies would be surrendered, even if the need that it satisfies is still there.

Clearly, a joint life insurance is not the same as two – otherwise similar – single life insurances. The difference lies in the definition of the insured event. In joint life insurances death usually means the first death, or (in certain kinds of annuities) we differentiate benefits according to which insured died. Practically all traditional insurances can be designed as joint life insurances. E.g. the joint life term insurance pays if either insured dies during the insurance term (and the insurance ends with this death, so the possible death of the other insured is irrelevant in regarding the insurance), and if both insured persons are alive at the end of the term the contract expires without benefit payment, etc.

The need for joint life fix term insurance is quite logical if the family has two wage-earners with similar salary levels. In this situation if either one falls out, the education and generally starting a career for the children could present a problem.

Another type of multiple life insurance – that is at present unknown in Hungary – is the so-called *co-partner insurance*. This is usually signed by the owners of small enterprises, so that if one of them dies, the legacy can be paid from the insurance benefit, and the enterprise doesn't have to be reorganised or terminated.

5.2.6. By Premium Term and Premium Frequency

Life insurances can also be categorised according to the *frequency* of premium payment. Most often the premium of life insurance is paid at *regular intervals* (monthly, quarterly, semi-annually or annually) throughout the term of the contract. But policies also exist with premium payment term shorter than the total insurance term. In these cases we are dealing with *limited premium term* insurances. One extreme case, usually mentioned separately, is when premium is paid once, at the beginning of the insurance term. This is the so-called *single premium* insurance.

The limited premium term insurance behaves mostly like a regular payment insurance during the premium term, but after that – until the end of the insurance term – behaves basically like a single premium insurance.

Modern insurances – mostly the unit linked life insurance – allow premium payment frequencies that are not specified in advance, and allow also so-called single premium top-up payments, additional premium payments supplementary to the regular payment. In case of traditional insurances such additional payments are not allowed.

6. COMPARING MODERN AND TRADITIONAL LIFE INSURANCES

In the following section from the modern group we will only analyse the unit linked insurance in detail, since the Critical Illness and Long Term Care type insurances have not yet appeared, or at least are not really popular in the Hungarian market, and on the other hand because apart from extending the concept of insured event – that is, by the way, also important – in other respects they follow traditional insurances quite well.

6.1. MODERN LIFE INSURANCE IN GENERAL

The Unit Linked life insurances was the great novelty of the Hungarian life insurance market in the '90s, that very quickly gained a significant market share from the total life insurance market, and its portion is still growing steadily at the expense of the other life insurance products that are called "traditional products" starting from their appearance. These unit linked products have significantly changed our views of life insurance and our expectations from regulation (that regulation has not always followed). Unit linked insurances have made a great step toward making life insurance products more transparent to clients, and gave them new kinds of options compared to the former ones, but in return also gave the clients more responsibility in handling their own financial affairs. Summarizing: the unit linked insurance expects a consumer who is more mature than the former one, and in a certain sense it also "raises" a more mature consumer.

6.2. LIFE INSURANCE BEFORE THE APPEARANCE OF UNIT LINKED

Traditional life insurance has a more than 100 year past of settled, refined format and product design, and a well constructed, closed, independent calculation methodology and notation system. The basic traditional life insurance policies were signed even in the 19th century practically (regarding the most important features) in the same form as nowadays. This product design reflected well the age and the environment in which it was created and formulated. The most important characteristics of this environment (from our point of view) were the following:

- the stability of money, low or no inflation, stable (usual) interest rates
- paper based, manual administration
- only a very small, selected part of the upper classes had access to the capital markets
- relatively low life expectancy and therefore large mortality risk – paired with typically high birth rates

These characteristics had a fundamental impact on the construction of long term insurance of that time (that is – with a little uncertainty – almost the same as the presently available life insurances). The most important elements of this product design are the following:

- The incidences related to the life insurance policy starting from its signature until its termination are tied to a fixed, rigid "scenario" allowing only a few variations (that means that the premium term, premium frequency are specified in advance and cannot be changed, the reserve runs on a pre-calculated path, etc).
- The most important parameters of the insurance (premium, sum assured, yield of reserve, term) are specified in advance and fixed.
- They tried to avoid all in-between changes (raising or decreasing the premium, changing the insurance term, changing the face amount, changing the dates of premium payment), or place these changes outside of the product design (e.g. they regarded late premium payment – from the reserve aspect – as if it had arrived on time, but charged a late interest to compensate for the profit loss, etc.

Regarding the product design all traditional life insurances can be built from two basic building blocks, two final elements: the term insurance and the pure endowment insurance[105]. The most frequent "ready" insurance products that dominate the market of traditional life insurance up to this day are:

- Term insurance (including also whole life assurance)
- Endowment insurance
- Term fix insurance
- Annuities (primarily single premium, immediate, lifetime annuity paying in advance without any guarantee period)

The common talk on life insurance has been such (and is still determinant at present), that when the advantages and features of life insurance were described, it covered mainly the term insurance. This was further supported by the rhetoric of life insurers, who emphasized mainly the benefits provided by term insurance. On the other hand, the weight of these particular insurance products has been (and remained) very different on the individual markets. On Anglo-Saxon markets were dominated by the term insurance types (so here the rhetoric was appropriate), but the French, German and Hungarian markets were (and still are) dominated by endowment and term fix insurances, beside a negligible portion of term insurance. (It would be interesting to investigate in detail why this is so, but we won't do this now.)

Long term savings were the most important motivation of signing life insurances even on the Anglo-Saxon markets, where this need was primarily satisfied by whole life insurance. Furthermore, life insurance has been the only option for long term savings for those not particularly wealthy. Because of low life expectancy and large families the goal of this type of insurance was evident: to provide for the dependents in case of a life too short, which meant that uniting death risk with long term savings was quite self-evident.

6.3. The Development and Circumstances of Development of Unit Linked Insurance

The development of Unit Linked insurance started in the '50s in England, but its world-wide spreading only started in the '80s and '90s. Parallel to the English development (where the unit linked insurance was at first a traditional term insurance tied together with investment funds) an American evolution, somewhat independent from the English one ran its course, where they started to make the cover of whole life insurance flexible (Universal Life), and later allowed also free choice between investments (Variable Universal Life), and this way a life insurance plan similar to the English version was created.

If we look at those factors in the '80s and '90s that we considered the most important "environmental" factors having influenced the formation of traditional life insurances, we see a completely different picture:

- The comparable stability of money and low inflation rate in the developed world has more-or-less returned by the second half of the '90s, but after a long, uncertain period, and with the perspective that prices could become instable once again very easily. And in the greater part of the world inflation hasn't become low up to our days. The stability of interest rates is history, not in the least due to the increased and diverse investment options, and naturally because of the instability of inflation and the typically large stock of state debts.
- Paper based, manual administration is completely replaced by computer administration, and this way financial services have been freed from the stocks.
- Almost anyone has access to the capital markets directly and indirectly – not in the least through unit linked insurances.
- Although it cannot yet be felt in Hungary, but the remaining life expectancy has radically increased in the whole developed world (that means that the death risk has decreased), and parallel to this, birth rates and the number of children per household have very strongly decreased (also in Hungary).

[105] To be precise we have to add that the construction of certain plans requires a certain kind of pure endowment insurance, that calculates with certain survival, that we can consider as a third element, as a "funded contract".

Another new, changed environmental condition can be fit in the above enumeration, namely that in the past 50-100 years the living standard and the individuals' life quality expectations in the developed world have increased dramatically, and not independent from this process the wealth of the upper, as well as the very wide middle classes has also increased.

The new environmental conditions have formulated new needs concerning unit linked insurance, and at the same time created new potentials:

- Computer administration makes it possible to create flexible products, with certain parameters variable even daily. The face amount, the premium, its frequency, its arrival date may be varied, the insurer can handle single premium top-ups, and these changes can be handled within the product design.
- The increasing life expectancy and the increasing living standard expectations of the individuals have upgraded the significance of self-care in life insurance. This has been enforced by the decreasing need of providing for others caused by the lower birth rates, and this way in life insurance the needs have shifted from death benefits toward living benefits.[106]
- The greater wealth increased the risk-bearing willingness of individuals, and with it their demand for investments carrying greater risk, but higher yield. This tendency increased the number of innovations not only in the insurance market, but generally on the capital markets.

The new environmental conditions have also increased the elbow-room of life insurance substitutes, and this way – primarily from the part of the newly developed investment funds – life insurance had to face new challenges. The creation of unit linked insurance is the answer to this challenge, since this type of insurance integrates many features of the investment funds into the traditional life insurance.

The change in environmental conditions meant, at the same time, that from the historical antecedents, or compared to them, the need for an independent (long-term) investment option has evolved that is independent from any specific risk, which raises certain "philosophical" questions about the nature of insurance.

6.4. Similarities with Endowment Insurance – Definition Arguments

Unit linked insurance is mostly similar to endowment insurance (and its special case, the whole life assurance). We can also say that endowment insurance is the ancestor of unit linked insurance, or that Unit Linked can be viewed as the generalisation of endowment insurance.

If we look at the premium-construction of the (regular premium) endowment insurance, it can be partitioned into more simple elements in two ways.

1. Traditional partitioning: the endowment insurance is the sum of a term insurance and a pure endowment insurance.
2. Modern and generalised partitioning: the endowment insurance consists of two elements, one is an instrument providing fixed interest on the net part of current payments (premiums) and the cumulated part of earlier payments, the second is a term insurance with variable face amount having the following properties:
    a. The face amount is a sum that can complement the current capital level of the instrument to a pre-defined level in the event of the insured's death.
    b. This pre-defined level is not arbitrary, but the sum that the capital level of the instrument reaches at the end of the term.
    c. The premium of the term insurance is net premium, and is always equal to the risk of the period, i.e. it changes from period to period.

[106] We have to mention that in Hungary this shift has happened without the earlier significant spread of term insurance. This way the overall insurance cover is significantly under the optimal level, which makes it theoretically possible for the Hungarian market – regarding insurance covers – to develop in a different way compared to the international tendencies.

d. The premium of the term insurance is deducted from the capital cumulated in the instrument at the beginning of the period.

For a very long time only the traditional partitioning was mentioned, but that cannot be generalised. The modern partitioning comprises the possibility of generalisation (if we leave the fixed nature of the interest and the strictly pre-defined sum of 2.b, and at the same time formulate 2.a more precisely, so that the face amount is zero if the pre-defined level is lower than the current level of the capital).

The first partitioning doesn't interfere with the traditional, general definition of insurance, of which I quote two phrasings: one is quoted from the Act on Insurers and Insurance Activities, the other from the current draft of the IASB[107]:

> **Section 4** Insurance activity is an obligation based on an insurance contract, legal regulations or membership during which the party engaged in the activity organises the community of parties exposed to identical or similar risks (risk community) assesses risks eligible for insurance with mathematical and statistical means, establishes and collects the price (fee) for the undertaken risks, sets aside the required amount of provisions, assumes the risks and performs services on the basis of the established legal relationship.
>
> **Draft of IASB**: An insurance contract is one under which one party (the insurer) accepts significant insurance risk from another party (the policyholder) by agreeing to compensate the policyholder or other beneficiary if a specified uncertain future event (the insured event) adversely affects the policyholder or other beneficiary.

Neither phrasing contains as possibility or as element the "simple" capital accumulation that version 2 allows. Moreover version 2 not only allows capital accumulation as one of the two building elements, but can make it the essence of the product design if we choose the "pre-defined level" to be zero.

The problem with the traditional definition of insurance is not that it was created before the appearance of unit linked insurance, when the partitioning type 1 of endowment insurance was self evident and the possibility of partitioning type 2 could easily be neglected, but that although the definition doesn't allow partitioning 2, it is still in use. This means that currently there is a kind of mismatch between the definition and the practice of life insurance.

This seems like a simple definition problem, but if we consider that the regulation and distinction of financial markets is institutional, we can almost be sure that these institutions are/were in permanent fight with each other over the "frontiers". These fights have mostly ended by now and have been closed with "peace" or "ceasefire". Their results are definitions that precisely draw the boundary lines of the exclusive territories of each institution. Until the point when something happens. Then the fights start over again.

In case of the unit linked insurance we are presently witnessing the fight starting over, caused by the unclearness of the definition. Based on this some full-blooded representatives of the banking business accuse insurance companies of "unlicensed deposit collection"[108] in relation to unit linked insurance, saying that according to the definition, unit linked cannot be considered insurance. A similar standpoint can be read from the international accounting standards, too.

What can the solution be to these – somewhat scholastic – problems?

1. First it is important to point out that life insurance has always contained the independent (independent from the specific "insurance" risk) savings option ("simple capital accumulation") in a latent way, but this latency only became apparent with unit linked (this is exactly why these products are attacked). The principle of "pure" long term accumulation/saving is not exterior to insurance, moreover, for a long time insurance companies were the only institutions providing this type of service to wide classes of the population. This way when unit linked insurance is attacked, it is attacked on home territory.

[107] International Accounting Standard Board
[108] This is the "cease fire" definition of the bank industry!

2. We have to take into account that due to the – above described – change in need for safety the savings motive is becoming more and more independent, to which the insurance business has to respond.
3. The definition arguments also point out that the traditional institutional regulation and the institution-based separation of activities is currently totally outdated and mostly unjustified. Practice is putting a continuous strain on these borders set by the “old wars”, and sooner or later it will enforce activity-based regulation, when phrasing the question as definition-based will have little significance.

On the other had, until this state is reached, it is purposeful to complement the definition of insurance with the option of long-term precautionary savings.

6.5. Major Changes Brought by Unit Linked Insurance

6.5.1. Changes Regarding the Client

The most important changes that unit linked brought to clients can be summarized in the following 4 categories.

Compared to traditional insurance – that is fundamentally inflexible, and almost impossible to change them during the term – the unit linked insurance is **flexible**, allows the client a number of options and possibilities to change parameters, that the traditional insurance lacked, namely:

- The client can choose – within the options provided by the insurer – the ratios by which the reserve of the client’s insurance is divided between the different types of assets. The client can reallocate between the different types of assets as many times as he wants, naturally on payment of an extra charge.
- The client can – within certain limits, but with no penalty – freely deviate from the date of premium payments, and may perform irregular top-up payments.
- The client can freely choose – again within a wide range – the minimum sum of all benefits payable in the event of death, and this sum can be changed during the term relatively easily.
- The term itself is also flexible, it can be adjusted to the changing needs of the client. This also means that the distinction between maturity and surrender will gradually disappear.

On the other hand, flexibility doesn’t only bring the freedom of choice. It also means that the insurer can flexibly adjust to developments since the commencement of the policy, e.g. to changes incurred in mortality. Contrary to traditional insurance, where the invariable premium also meant that during the term the insurer didn’t take into account necessary adjustments of the risk-premium due to changes in mortality, in unit linked insurance the insurer reserves the right to change the death-risk premium during the term. The direction of this change – naturally – is not known beforehand, so the client doesn’t know if the change will be favourable or unfavourable. But since in developed countries mortality rates have decreased for decades, this type of flexibility is also to the advantage of the client rather than to the disadvantage.

In exchange of flexibility the client has to pay a certain price. By having the option of selecting the assets of the investment of the reserve, the client **takes over the yield-risk from the insurer**. This frees the insurer from the responsibility of having to achieve a certain yield under all circumstances, which makes it possible to invest in instruments with higher risk, but also higher expected yield. Of course the insurer may undertake a guarantee on the yield of individual investment forms (asset funds) offered to the clients, but this is not yet typical in the market.

Transferring the greater part of investment responsibilities from the insurer to the client supposes in an implicit way the **financial maturity of the client** (as the investment practice of traditional insurance hidden from the eyes of the client supposes the financial immaturity of the client), since only a client informed in the capital markets at least on a basic level can make correct decisions, in accordance with his own risk-bearing “capacity”, regarding the allocation of his reserve. This supposition on the maturity of clients is often not yet justified in the Hungarian practice.

The expense structure of traditional insurance and the magnitude of expenses is invisible to clients, and insurers consciously aimed at this invisibility. Unit linked insurance has brought a great change in this respect, the **expense structure** and the magnitude of expenses is **fundamentally visible** and can be planned by clients. The administration fee, the fund management fee and the bid-offer spread are all openly announced values.

On the other hand, insurers would like to hide certain expenses even in case of unit linked insurance – similarly to the traditional insurance. These are primarily acquisition costs and their coverage. The technique of Initial Units has been created to realise this, which hides the fact that the greater part of the premium of the first (two) years is not accumulated and invested for the client, but taken away to cover acquisition expenses. Since this technique can be applied to mislead clients, its use is forbidden e.g. in the United Kingdom. In Hungary its use is fairly widespread.

The transparency of the expense structure at the same time makes possible a **more even expense loading.** On even loading we mean that expenses are charged on premiums and on the reserve according to the actual services, and not some other factors. In traditional life insurance expenses are usually charged as a certain percentage of the premium, this way loadings are proportional to the sum assured[109], while the actual incurring expenses of the insurer depend on a lot of other factors, so in reality it is not proportional to the sum assured. In unit linked insurance naming separately the different types of expenses and defining them as proportional to different factors closely related to their source makes fairness of a greater degree possible. Maintenance commission and expenses of premium collection are premium income proportional, this way they are covered by the bid-offer spread, fund managing fee is proportional to the reserve, the administration fee is independent from the premium or the sum assured, so it is usually defined as an absolute sum.

Because of all these effects, in unit linked insurance – compared to traditional life insurance – there is much less cross-financing between different groups of clients.

6.5.2. Changes in the Relationship of the Insurer and the Client

The transparent construction, the visible magnitude of expenses at the same time puts pressure on insurers to **decrease expenses**. Visible expenses make it possible for clients to compare also from this point of view – either by themselves, or through an agent arguing beside the product of his company – the offers of individual insurance companies. Insurers automatically take into account this effect, and emphasize more strongly in their pricing that the product should not be much more expensive than competitor products, or that if possible, they should offer the same service cheaper.

Naturally, as we have indicated earlier, greater transparency has its limits (e.g. initial units), insurers try to resist the pressure of decreasing expenses, that they try to accomplish by hiding certain expenses of unit linked insurance, or by presenting them to clients in a special way.

The environmental factors promoting the appearance of unit linked, the – above analysed – product design characteristics of unit linked itself, and the shift in emphasis compared to traditional life insurance have inevitably **changed the systems of arguments** used by insurers – and their representatives – when selling unit linked.

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[109] This is only partly eased by the practice of premium discount for a higher sum assured, that is often applied by traditional life insurance, or the corresponding, but reversed technique, when the premium tariff depends on the sum assured, relatively decreasing.

One of the most emphasized elements in the unit linked design is the choice between different investment options. Correspondingly, this investment characteristic is the most emphasized when selling the product – and in many cases emphasized too strongly by insurance agents – generally on the account of the death coverage. We are witnesses of an interesting phenomenon in relation to this. One reason for the appearance of unit linked insurance throughout the world is the increasing demand for living benefits on the account of the death benefits. In Hungary we see strong under-insurance regarding term assurances – in an international comparison, of historical reasons – but at the same time – similarly to developed countries – unit linked insurance is spreading even on the account of term assurance. Furthermore, the Hungarian mortality rates do not give grounds for Hungarian consumers not to view this risk as important, either. Because of this, some insurance companies see a marketing chance, and incite their agent network through high commission rates to sell high death coverage. In spite of this, there is a certain shift in emphasis in life insurance “rhetoric” from providing for dependants toward self-care.

Because of the above, a number of stock market phrases and connections have been imported into the system of arguments, and agents have to help clients in recognising their own risk-bearing “capacity” and in forming an investment portfolio accordingly.

We have to mark that in Hungary unit linked insurance has been introduced at a very favourable historical moment, in the stock market boom, this way they could become popular, well-known and widespread in a short period of time. This was not the case in every country (Spain is usually mentioned internationally) where due to the bad timing these products are unpopular up to this day.

Traditional life assurance is traditionally sold by agents having an exclusive contract with the insurer, against payment by results (an acquisition commission depending on the term and annual premium of the insurance, payable after the signature of the policy). The cover of the commission is the greater part of the premium of the first (sometimes the first two) years, that is not accumulated in the client’s reserve. From the clients point of view this commission of an unknown sum appears (or would appear if the client had precise information about it) as the fee of the guidance and services of the agent, who in a general sense plays the role of a financial advisor. This guidance and service includes a very general survey of needs and the (insurance) solution to these needs. Traditional insurance is largely standardized, so we can say that these are uniform solutions to uniform needs, and the guidance of the agent is quite simple.

This **traditional marketing technique faces a lot of challenges** – mainly from the new circumstances incorporated by unit linked insurance – to which the insurance industry has not necessarily found the adequate answer yet. The challenges can be characterised by the following contradictions:

- Because of the visible expenses (and the competition of alternative investment options) there is pressure to reduce the largest item, acquisition expenses, while the increasing complexity of needs and products increases the demand for quality counselling. On the other hand – due to the habitude reinforced by the foregoing practice of financial service providers – consumers tend to think of counselling as a free service.
- The diverging and more-and-more unique needs of clients would make it necessary to separate and make independent the costs of counselling and mediation, but these two expense elements are not separated in the current practice of financial providers.
- But the most important contradiction is that although wide classes of society have increasing possibilities and objective needs for long term investments and generally for the formation of long term individual financial strategy, only a minority recognises this fact by himself, the majority has to be persuaded by government instruments (e.g. compulsory pension funds) and through the marketing pressure of financial intermediaries (through agents).

Because of all these, there is a kind of “crisis”, seeking ways and means in the marketing of life assurance throughout the world – in Hungary also –, of which the solution cannot yet be seen, but some factors will probably intensify:

- The marketing of very simple unit linked insurance with low expense ratio, not including counselling fee, only the cover of mediation is increasing through alternative intermediary channels such as the Internet, bank agents and direct mail. Mostly the financially well-informed clients can take advantage of this.
- The need for the counsel and life-cycle planning service of highly qualified advisors independent from insurance companies is probably increasing, and parallel to this the willingness to pay separately for these services will probably increase, too.

6.5.3. Changes in Insurance Technique

One of the most striking characteristic of unit linked insurance is **universality**, since a unit linked insurance can be viewed as a very general life insurance containing all possibilities. The name of the American version (Variable Universal Life) probably tries to imply this through the word “universal”. From this respect we can say that – as we have hinted earlier in this chapter! – that unit linked can be viewed as a generalisation of traditional life insurance, where many restrictions have been resolved and made optional, variable.

Universality also means that the insurer is able to satisfy all insurance needs of an insured within one policy.[110] Internationally there are examples of insurance companies selling only one product, a unit linked insurance. When an insurance company is offering several Unit Linked products, this is probably because of marketing reasons, and is deliberately not exploiting all possibilities provided by the flexibility.

Universality necessarily means also that compared to traditional insurance it has much more features in common with non-insurance financial instruments, so the boundary line between life insurance and other financial areas is starting to fade, not in the least thanks to unit linked insurance.

A more even expense structure brings a kind of **stability** into the expense structure, since the expenses debited to the insurance follow more closely – compared to traditional insurance – the actual incurring costs. At the same time the flexibility of the design brings **a new type of instability** into the cash flow of the product, since the client has more options of premium payment, can choose the timing and sum of premium payment, this way the cash flow into the insurance company is – compared to traditional insurance – much less calculable. Naturally this depends strongly on the type of clientele of the insurance company, if it is middle class with stable income with a disciplined accumulation ethos, or recruited from other classes. Knowing the characteristics of the clientele is many times what motivates insurers not to use the flexible options provided by unit linked, and they deliberately set the premium timing and the sum to be inflexible, this way trying to make their cash flow more calculable. Calculability might be important regarding the commission, too. If the insurer pays large acquisition commission (i.e. uses a traditional commission regulation), that is covered by future expected premiums, then it is important that this cash flow should be as stable as possible.

The death-risk premium variable over the premium term (when calculation is based on a new, more precise mortality table) also brings the insurer closer to a more stabilised cash flow.

It is very important that unit linked insurance can **only** be managed with a modern, **large-capacity computer background** and sophisticated software. This also caused that (and not only because of the lack of a properly developed stock market) in an average Hungarian insurance company this type of insurance product could not be introduced at the beginning

[110] Moreover, even the insured person could be modified, which would principally make a unit linked insurance even inheritable, so it would be able to satisfy the life insurance needs of the insured’s descendants.

of the '90s. The most important requirement from this point of view is the daily valuation of reserves broken down to the level of individual clients.

Insurance companies selling savings type insurance are important players of the financial intermediary system, since a significant part of those having excess money and those who want to use this money meet each other through life insurers. In case of the traditional (savings) life insurance the insurer itself as an institution raised a wall between these two groups, they could only meet with the active participation of the insurer. Although the insurer gains the interest credited to the premium reserve of the savings type life insurance from the investment yield of this reserve, he appears to the policyholder as an individual guarantee undertaker.

Through unit linked those offering and those seeking money meet almost directly, the insurer "only" brings them together, and (generally) doesn't guarantee anything on the performance of those demanding/using the money. In this respect the **financial intermediary position of insurers weakened** compared to the former. This seems to fit into a general tendency: the role of the stock market is increasing – that is the meeting point of the demand money and supply of money – and the role of traditional financial intermediaries is decreasing (banks, life insurance companies). If this tendency truly exists, unit linked insurance can be the forerunner of a life insurance sector with a completely changed function, where the new function of life insurance is much more counselling, individual financial planning and giving aid in the realisation of plans than financial mediation itself.

The weakened financial intermediary role of life insurers means at the same time a stronger and better asset-liability matching, since there is no better matching than when money owners and money borrowers are in direct contact without an institutional filter, since this institution may manage to invest in assets covering its liabilities, or it may not.

In case of traditional life insurance the insurer guarantees all benefit elements, so prospective reserve calculation is very important[111]. On the other hand, in case of unit linked insurance the future maturity benefit is not known, the death benefit is also uncertain (if in case of a policy the value of all funds is higher than the death sum insured), and there is no insurer guarantee on the sum of these elements (excluding the death sum as a minimum death benefit). Because of this, benefits can be calculated **only** based on "historical" data of the policy, that is based on the **retrospective method**.

6.5.4. Expected Further Changes

Naturally nobody can see the future, so regarding the expected changes we have to rely – more or less – on guesses. Based on international experience, existing but unsatisfied needs and simple logic, we can predict certain changes.

Before looking at expected changes in detail, it is important to state that unit linked insurance is not regarded as positively by all players of the insurance market (players in a narrow sense) as it is described in this study. Furthermore, some very developed life insurance markets exist (e.g. the German market), that do not like unit linked insurance at all, and tend to mean only traditional products under the term life insurance. In the Hungarian market it is also said sometimes that unit linked is "not insurance, only an investment", and it can be blamed for "people turning away from life insurance" (and this primarily means term insurance). The big "hit" of year 2000, the single premium unit linked insurance is especially blamed, which is truly pushing the envelope of the traditional concept and traditional "rhetoric" of life insurance.[112]

Unit Linked insurance is also attacked from other sectors (primarily from the side of investment funds), saying that through unit linked insurance companies are in reality doing

[111] See section 12.! This is even stipulated by law.

[112] The main point of this challenge is: Can an instrument of mainly short term, providing no death or other life, accident or health type cover be considered life insurance?

their business (since regulation doesn't handle unit linked funds as investment funds). Although these attacks at present do not seem to be too serious, we cannot predict with certainty if they'll have an effect on unit linked insurance.

Supposing that the development of unit linked insurance will follow a more or less unbroken path, we can sketch a few demands and tendencies:

1. In the options of asset funds there will probably appear those with interest guarantee, or maybe the closed-end funds with fixed interest fixed term. These will especially fit those insurances, where dominant is the death benefit, not the investment characteristic.
2. Products might appear, where the benefits are also declared in units. We have to think about unit linked annuities primarily, that have already gained a significant market share in the United States.
3. The unit linked technique will appear on the boundary lines of life insurance, in Hungary mainly in pension funds.

On the other hand, it is possible that the total life insurance market will change direction sometime in the future. A change in the direction of development doesn't have any significant signs yet, but we can already phrase justifiable questions that cannot be satisfyingly answered with the current organisation and product design of life insurance. It cannot be predicted yet if in reality these questions will have to be phrased or not. A few examples are:

- Will the current sector and product structure of the financial intermediary system stay in its present form, or will it change into totally new sectors and sector boundaries, and totally different product accordingly?
- Is it justified to hide risk type insurances into a savings product?
- Will the insurance sector have to detach from itself the savings products and return to risk type insurances?
- Is it justified that individual savings products are as complicated as some unit linked insurances?
- Is it socially and economically justified to maintain such an expensive insurance intermediary system for long term investment products, or could this be replaced by cheaper media (e.g. banks, other institutions, Internet)?
- Socially isn't it rather the compulsory regulation, savings legislation that should incite long term savings instead of the individual persuasion of agents?
- Shouldn't the role of agents be limited to risk (life and non-life) insurance, and a financial advisor class independent from insurers be built?
- Isn't it bad politics from the part of insurers to spread expenses primarily on saving products and not the risk-type insurances?

7. COMPARING DIFFERENT INVESTMENT FORMS

KEW WORDS

Rules for investment of life premium reserve	Differentiation of clients
Liquidity	Volatility

The most popular life insurance is the so-called savings type insurance. (Here the term “saving” refers to the function of insurance in a wider sense, the same way as the term “pension” in pension insurance, this way we do not deal with it in the categorisation of insurance. This practically means endowment insurance, insurances with an endowment element, term fix insurance and unit linked insurance.) Generally more than 90% of the portfolio of most life insurers consists of such policies. These products are in tight competition from one part with similar products of other insurers, and from the other part with savings forms outside the insurance sector (and primary in this respect). If someone wants to invest money wisely, than he has to consider which option to choose from the numerous possibilities. This chapter provides help in this consideration.

7.1. PRINCIPLES OF COMPARING LIFE INSURANCES

The offered products of life insurers can be compared based on the following questions:

- Precisely what products do the individual insurance companies offer – (viewed from the benefit side)?
- How do they differentiate between the individual groups of client?
- What is the magnitude of expenses accounted by the insurer on the product?
- What kind of inflation handling and profit sharing technique is used?

The above aspects can be applied to both traditional and modern insurances, although sometimes in a different way.

7.1.1.Comparing Benefits

The Hungarian insurance market shows a relatively consolidated picture. This means that regarding the major features, all insurance companies offer the same products, and the products of individual companies differ mostly due to the unique combinations of the different possible element.

According to the above, almost all insurance companies offer term insurance, endowment insurance, term fix insurance, unit linked insurance and single premium, immediate annuity. So the aspect of comparison is how the given insurer guesses the needs of some classes more precisely than rival companies through the right combinations.

Of course there are some insurance companies that are specialised in a certain sales channel (these are primarily banks), and this limits the possible benefits of its product group. E.g. they should emphasize most strongly the savings elements, and smaller risk elements complementary to bank products.

7.1.2. Differentiating Among Customers

The equivalence principle declared in chapter 9 expresses equivalence in two senses:

- In the first sense: it expresses the equivalence of the income and outgo items of the insurer;
- In the second sense: it expresses the equivalence of the unique risk of the insured and the premium he has pays.

We have already discussed the first type of equivalence. The second type of equivalence belongs to the current chapter.

To understand the significance of differentiating between customers, let's look at a fictitious example. Fictitious, because we start from an existing insurance product, but since we know the outcome of the example beforehand, the insurance companies are not in competition with each other in this form.

CSÉB (Group Life and Accident Insurance) was the popular personal insurance product of the '60s and '70s. This is an insurance combination that contained also a life element, and provided (and provides) the same benefits for the same premium for everyone. This is the perfectly non-differentiated state, since it doesn't differentiate between insured and insured in any respect (or rather doesn't differentiate between insured persons of active age, but this is not important at present).

The State Insurance Company could do this because on the Hungarian market it had a monopoly, and could suppose with perfect confidence that everyone will sign this insurance, so it is enough to calculate from averages of the whole population. This meant, e.g. that since the mortality rate of elderly is higher that that of the younger, they paid less than their own risk, while the younger insured paid more. This way only the first type of equivalence could be satisfied, and the second type was only satisfied in case of a small fraction of the population having exactly an "average" mortality rate.

The State Insurance Company – since it was alone in the market – wasn't forced to satisfy the second type equivalence, although in a competitive environment this is inevitable. Regarding the CSÉB, the transition from monopoly to competition could also have happened the following way.[113]

The fist competitive insurance company appears in the market, who observes that while the mortality of men and women differs significantly favouring women, women pay the same premium for CSÉB as men. So if it creates a WOMEN'S CSÉB separately for women, then all women could be attracted by premium reduction to the first rival insurer. This will have the effect that only men remain at the old insurer. Men pay a premium smaller than their own risk, since the premium of CSÉB was calculated so that the premium deficiency of men is compensated by the excess premium of women. This way the old insurer is forced to raise premiums to keep its calculation in order, and loses half of its insured group.

This is when the second insurance company steps into the picture. It recognises that young men pay a premium significantly higher than their own mortality risk. It creates the CSÉB OF YOUNG MEN, and attracts all young men with a premium discount. All old men stay at the old insurance company who pay a premium lower than their own risk, since the premium was calculated so that the premium deficiency is compensated by the excess premium of young men. The old insurer once again lost half of its clients, and is forced to raise premiums.

The third rival insurer experiments with the CSÉB OF NOT TOO YOUNG, BUT NOT TOO OLD MEN, the fourth comes out with the CSÉB OF MIDDLE AGED MEN WITH FAMILIES, and so on... The result will be that the old insurance company gradually looses all of its clients (and those who would stay are frightened away by the constant raise in premiums), while the market only offers CSÉB's that provide the same benefits with premiums differentiated by age and gender.

Naturally – I have to underline once more – the above example is fictitious. CSÉB is a much more complex insurance than one that can be analysed exclusively from the premium side, and this way its extinction (that has more or less happened by now) went on a different path. A element of this is, for example, that the competition of more serious life insurances (called "major life insurance" by some insurance companies) is attracting the best clients from CSÉB.

The lesson of the above example is that in a competitive environment insurers are forced to differentiate in the second sense their portfolio in a deeper and deeper level. Naturally differentiation can have many levels, and the state of a particular market is indicated by what level it has reached in this area. The current Hungarian market is on a level, where every insurer uses differentiation of the premium of new life insurances by age and gender, but by the dimensions of e.g. occupation, smoker/non-smoker, etc. not necessarily. But it is always

[113] This explains that the idea of CSÉB could not have appeared in a market economy.

the insurer first applying further necessary differentiation that is in a winning position. In this respect all Hungarian insurers have significant potential, but no one wants to pass too far the current state of the market, since even differentiating by age and gender has not been used for too long.

Of course differentiating among clients can have its limits. In some countries regulation prohibits insurers from differentiating clients by some characteristics, e.g. religion, race, etc. These regulations are perfectly understandable, and there is no other way to avoid differentiation conflicting such general principles as legal prohibition, since without these prohibitions competition among insurance companies would objectively lead to this kind of differentiation and exclusion. This problem has not yet occurred in Hungary.

7.1.3. Comparison of Expenses

In other words we could phrase the question: What is the total loading, and what are the expenses charged for different purposes?

This is one of the most important aspects by which different insurances can be compared (the other, maybe even more important is the level of profit sharing, that will be discussed under the next point). And this is the question that is most difficult to answer – at least in case of traditional insurance. The expenses of unit linked insurance are mostly transparent, so even an inexperienced client can compare them (an important exception is the expense hidden in the form of initial units, if applied). In case of traditional insurance, on the other hand the client isn't familiar with the level of total loadings, and this cannot be ascertained by simple methods. This way here we can only say negative things.

If someone wants to compare insurance companies by looking at the premium that the same insured has to pay for the same benefits (lets say the insurance of a man of age 42, with 1,000,000 Forints face amount and a term of 20 years), and considers as best offer the one with the smallest premium, he might make a big mistake.

There are namely 3 major factors having an important role in the premium of life insurance:

- the mortality table in use,
- the applied technical interest rate,
- the level of total loadings.

As we see, the level of loadings is only one of these factors, and we cannot see its effect clearly without the effect of the other to factors.

From the effect of the other two factors, it is the second one that is more important, since insurance companies usually use the same mortality table, this way we will look at this factor in more detail.

As we know, the technical interest rate is nothing else, but the guaranteed interest rate of the premium reserve calculated in the premium. Clearly, the higher the calculated rate, the lower the premium will be, so increasing the technical interest rate decreases the premium, if all other factors are unchanged.

From this it follows that the premium tariffs of two insurance companies can only be compared directly if they are using the same technical interest rate[114]. Since if an insurer uses a technical interest rate higher than the other, then it is natural that its premiums will be lower while its offer might not be more favourable, because with the higher technical interest rate – if the interest earned on reserves is the same – the excess yield returned through profit sharing will be lower. (At the same time a higher technical interest rate has the advantage that the insurer has bound himself to crediting the higher interest rate to the client.)

[114] By the reduction of the maximum level of technical interest rates to 4% the range of possible technical interest rates has decreased, this way insurers are using technical interest rates that are closer to each other, which facilitates the comparison of traditional insurances.

This way regarding the expenses we can only state the following: clearly the offer of an insurance company with lower or equal tariffs is more favourable regarding expenses as an insurer applying higher technical interest rate. If, on the other hand an insurer with lower technical interest rate has higher tariffs, then we cannot say anything about the expense loadings compared to another insurer.

7.1.4. Inflation Handling and Profit Sharing

This is the other important factor beside the expense ratio calculated in the premium (loadings) that the client has to take into consideration.

The essence of inflation handling will be discussed in more detail later on, in a separate chapter. Here we will look at how the different methods applied by different insurers can be compared.

Regarding premium increases the question is: which insurer can "estimate" more accurately the rate of increase that the clients can bear. This is an open question that has not yet been answered. But as inflation rates decrease, this question is becoming less and less important, since by low inflation rate (and we can hope that in the years following 2000 this will be typical in Hungary, too) insurers generally do not increase premiums at all.

The method of profit sharing is almost totally the same in case of unit linked insurance, here only the fund management fee and the expected yield can be compared by insurance companies and by funds, of which the expected yield can only be strongly subjective. Regarding the profit sharing of traditional insurances, there are two questions:

1. What is the relative level of profit share offered by the insurer?
2. How big will the profit achieved by the insurer be?

The terms of the policy give the answer to the first question. A typical phrasing of this is:

The percentage of investment profit sharing = (the interest earned by the insurer on the invested premium reserve minus the technical interest rate) multiplied by (profit sharing coefficient).

Example

Interest earned on premium reserve = 8.5%
Technical interest rate = 3.5 %
Coefficient = 0.9
Then the percentage of investment profit share = (8.5% - 3.5%) * 0.9 = 4.5%

Since the technical interest belongs to the insured, the best offer from the insured's point of view is the offer with the largest profit sharing coefficient. This falls usually between 85 % and 95 % (the legal minimum is 80%).

To the second question we cannot give an answer the same way as by unit linked insurance, this depends on how skilled the insurer is in investing the reserve.

When someone takes out a life insurance policy, he frequently asks what the guarantee is that the insurer will pay a high interest rate on the premiums paid. Behind this question there is the memory of the accustomed state guarantee of the past decades. Clients are not yet accustomed to the fact that in competitive economies there is no financial guarantee of the type that earlier the Hungarian state gave on deposits. In an economy, where everything was in the hand of the state this was natural. But presently no insurance company or bank guarantees interests for longer that one year. Even the interest of deposits longer that a year is usually variable.

One of the main reasons of this is that no financial institution can foresee the two most important factors that interest rates depend on in the long term, nobody can predict the inflation rate and the economic productivity (the real interest) for a time period of several years. This means that the only guarantee for the client is if he – the client – chooses wisely between banks and insurers. It is his own responsibility to invest his money wisely.

7.2. Comparing Life Insurance and Other Saving Forms

7.2.1. General Aspects

Table 2.4. contains the comparison of the major investment forms by liquidity, interest and risk. Further remarks:

1. The main aspects that we have to take into account when investing money are the following:

Liquidity. Liquidity means how fast the invested money can be accessed. The most liquid money is cash, and one of the least liquid investment forms is the premium reserve of life insurance.

Interest. It is difficult to compare the interest of the individual investment forms, since in case of stock both the income from capital gain yield and from dividend yield are quite uncertain. It is a usual requirement that the more uncertain the interest (i.e. the more varying, the greater the volatility), the higher it has to be.

Risk. Under the term risk here we mean the investment risk (which means the probability of possible gain or loss), and not the risk that can be decreased through life insurance. This is basically the same as volatility.

Of course, investing may have other aspects, also, but the above three are the most often examined.

2. We have to look at life insurance not as an investment, rather as constant saving. We can only talk about true investment (allocation of a greater capital) in the case of single premium insurances.

3. When comparing investments we can never forget that we also have to pay the element of life insurance that decreases risk (we mean the financial security provided by these elements). This way life insurance and other investment forms are not only alternatives, but also complementary to each other.

4. And finally, we have to mention an advantage of life insurance compared to the other investment forms, that is a little bit paradox. This is namely that it conditions the policyholder to a more disciplined saving attitude, since failure to pay the premium has serious consequences, that is missing from the other investment forms. This way subjectively life insurance could be a better investment form for someone who is undisciplined in savings, because this way in the long term he will accumulate more money with higher probability than if it were his own decision every month to deposit the next payment or not, or to access his accumulated money or not.

7.2.2. Comparing Endowment Insurance and Long Term Deposit

Endowment insurance is the most popular and most widespread savings type insurance, this way the need to compare with other savings forms is most significant regarding this type of insurance. The most obvious question is how the yield of an endowment insurance having the same premium as payments made to a long term savings account (e.g. time deposits, etc.) compare to the interest of this account.

This question can be investigated from several sides:

1. We compare the *expected future value of benefits* of an endowment insurance to the *expected future maturity value* (which is capital + interest) of a time deposit. Depending on how long the client is alive, there is a certain probability that sum insured (together with the profit share accumulated until that time), or the deposit is paid to the beneficiary in year 1, 2, …, n (of course in case of death the deposit is significantly lower than the sum insured).

Clearly enough, money received earlier is worth more, which we take into account by compounding all sums to the end of year n, i.e. we take a “common denominator” of the sums received in different periods. The future value of the benefits of the endowment insurance means that we take the weighted average of these compounded sums, where the weights are provided by the mortality and survival probabilities. We proceed similarly when calculating the future maturity value of the of the time deposit.

At first glance we can say roughly that the two future values *have to be equal*! Why is this? Basically because life insurance can be regarded compared to a bank deposit as a form of instrument where interests have been regrouped in time, so in the event of death close to the beginning of the term benefits paid are increased by the interests of the end of the term, while the maturity value of the insurance is less than that of the time deposit.

When looking at the question from the other side, we also have to investigate some more specific factors:

the expenses of the insurance company are higher than that of the bank due to higher acquisition costs.

the insurance company can generally earn higher interest than a bank, because it invests in longer term (the interest curve has positive steepness)

the possibly different taxation of interest earned by the insurance company and interest credited by a bank.

The first factor decreases, the second clearly, while the third probably increases the expected benefits of the insurer compared to that of the bank. All together we can say that the expected value of benefits paid by the insurance company are higher than that of the bank, but the probability that the sum received from the bank exceeds the benefit paid by the insurance company is lower, since the probability of survival is higher than the probability of death, and the maturity benefit of an insurance is smaller than the maturity value of the time deposit.

2. We take the endowment insurance apart to a pure endowment and a death part, and compare the maturity value of the pure endowment part to the maturity value of a long term bank deposit, to which we pay every month the same sum as the premium of the pure endowment part of the insurance. The principle basis of this comparison is that in the endowment insurance only the pure endowment part serves saving purposes, the function of the risk part is something else (namely the payment of the sum insured in case of death). If we look at the two investment options in this way, then the maturity benefit of the pure endowment insurance will probably be (since in the long term we cannot say anything with certainty) higher than the compounded sum of a time deposit. But it is important to add that we are dealing with a low risk, and consequently low interest bank deposit, since the most important aspect of the investments of an insurance company is security, and the level of interest earned comes a only well after this.

3. Finally, in the most simple (but least correct) comparison we compare the maturity benefit of the endowment insurance to the maturity value of a time deposit with monthly payments equal to the premium of the endowment insurance. Then we see that the maturity benefit of the insurance corresponds to the maturity value of a time deposit with monthly payments equal to the premium of the endowment insurance, but with a yearly interest 2-3% lower than that of similar time deposits. So we can say that the price of the death benefit provided by the endowment insurance is the 2-3% yearly interest loss.

At the same time we can also state that bank deposit and insurance are not exclusive alternatives of each other. It is best if someone possesses both (so the question is not bank or insurance?), since the goal and the function is different. While life insurance is typically a long term investment form (and not only an investment form, but also an insurance), the bank deposit is usually saving for short- or mid-term.

7.2.3. Comparison of Unit Linked Life Insurance and Investment Funds

The most important elements of unit linked insurance are the asset funds, that can basically be viewed as investment funds. The difference between them is mostly regulatory, as one falls under the effect of a different law than the other. If we set aside these – not necessarily justified – regulatory differences then the following main differences remain between the two investment/savings forms:

- unit linked life insurance has a more complex benefit system than an investment fund (insurance benefits)
- in case of the unit linked insurance the insurer offers more choices of funds, and shifting units between funds is relatively simple and cheap.

Unit linked insurance works much more like a savings account than the investment fund, so insurers are well prepared for receiving continuous payments, while investment funds can typically handle one-time or occasional top-up payments well.

7.2.4. Comparison of Interest and Annuity

At times of higher inflation rate (as, for example was the ‘90s in Hungary) the following problem arises very often. If someone wants to decide whether to buy an annuity of 10,000 per month for 1,000,000 Forints, or to deposit the same sum in a bank to receive money, then he often thinks the following way: “If I buy an annuity, then all together I get 120,000 Forints a year. On the other hand, if I deposit the sum in a bank, then regarding the current 26% interest rate I receive 260,000 Forints a year, so I choose the bank deposit and do not trust my money to an insurance company!"

Is the above train of thought correct?

First of all, let’s start by imagining an environment with no inflation, where the real interest rate is 5% (this is a common level of real interest). Then if someone wants to make a living from interest, he can get an interest of 50,000 Forints a year. He can do this every year while his capital, 1,000,000 Forints remains the same (with the same value because of the 0 inflation rate). When he dies, this sum goes to the heir. But what if the owner of the capital didn’t want the heir to receive this money? He might have wanted to consume the whole sum himself.

In the above example the owner of the capital lives on a lower living standard than he could, regarding the volume of his capital. He could also think the following way: “Since my remaining life expectancy is e years, if I draw 120,000 Forints from the bank each year, then due to the continuous interest, my capital will be used up in exactly e years.”

But this plan is very hazardous! Since if he lives longer than e years, he won’t have any money for the remaining years, because his capital gives out after e years. This risk is taken over by the insurance company, who thinks the following way: “People live averagely for another e years, but in reality some live longer and some die earlier. If I collect 1,000,000 Forints from everybody, then I can guarantee the payment of 10,000 Forints monthly for everyone until the end of their life, since the annuity of those who do not die after e years can be financed from the remaining capital of those who die before e years.”

The example shows clearly why it is more favourable to by the annuity compared to the bank deposit.

What is the situation in an inflationary environment? Let’ suppose that the nominal interest provides a real interest of 5% above the inflation rate. Then those who live on the bank interest and want to keep their capital are sensible if they do not draw the total interest from the deposit, only the real interest, that is again 50,000 Forints in the first year, and capitalizes the remaining part of the interest corresponding to inflation, keeping the capital on the same real value. This way the interest of the next year will exceed the 50,000 Forints of the previous year exactly by the inflation rate, and this way inflation has been fended off in case of the interest. The insurance company takes similar steps in respect of annuities, that is, the capital value of annuities is always raised by the inflation rate, and this way the annuity also increases every year bay the inflation rate.

If the nominal interest rate is lower than e.g. the inflation rate, then the situation is more complicated, but the result is only that the monthly annuity benefit doesn't increase according to the inflation rate, and also the capital value of the bank deposit decreases in real value.

Returning to our starting example, it only appears that in inflationary times one can exhaust the total 260,000 Forints as interest, since the greater part of this sum has to be used to keep the real value of the capital. And if we subtract this part from the 260,000 Forints then we receive a yearly interest well below the 120,000 Forints received from the annuity.

7.3. REGULATION CONCERNING THE INVESTMENT OF LIFE INSURANCE

The long insurance term of life insurance requires long term investments. The investment options are regulated by the Act LX. of 2003 on Insurers and Insurance Activities. Regulations concerning the investment of life insurance premium reserve in force in 2004[115] is the following:

PART SIX, CHAPTER III
INVESTMENT RULES

Section 132

(1) Assets covering the insurer's insurance technical reserves must be invested, taking into account the maturity structure of the operated insurance branch and liabilities so that the investments should achieve the highest possible security and profitability and the insurer can preserve its liquidity at all times.

(2) In order to achieve secure investments, an insurer must select several investment forms at the same time and, within a particular investment form, it should also aim at mitigating investment risks and sharing investment risks.

(3) During the investment of its insurance technical reserves, the insurer must comply with the provisions of Annex No. 9, with the exception of reserves specified in Section 117 Paragraph (2) subparagraph i).

(4) With regard to the asset management of portfolio covering reserves defined in Section 117 Paragraph (2) subparagraph i), the portfolio management rules of the Capital Market Act must be applied with exceptions stated in Section 126 Paragraph (2), Sections 127 and 128, Section 133 Paragraph (2) and Section 130 Paragraphs (3)-(4) of the Capital Market Act, with the following differences:

a) Wherever a provision of the Capital Market Act indicates a client, it shall mean the contractual party in the case of unit-linked life assurance contracts;

b) Wherever a provision of the Capital Market Act indicates portfolio, it shall mean separate funds, created from the reserves of unit-linked life assurance contracts;

c) Wherever the Capital Market Act mentions the employment of a subcontractor in the framework of portfolio management activities, it shall mean the outsourcing of all or the majority of the management and investment of assets representing the coverage of reserves of unit-linked life assurance contracts;

d) It is not necessary to have the contractual party's written consent for the outsourcing of all or the majority of management and investment of assets representing the coverage of reserves for unit-linked life assurance contracts, and employment of an intermediary;

e) The purchase of shares issued in the form of a public offer by undertakings having an influential share in the insurer (Capital Market Act Section 5 Paragraph (1) Point 58) life assurance contracts for funds created from the reserves for unit-linked life assurance contracts does not mean the violation of share acquisition prohibition included in Section 294 Paragraph (1) of the Company Act.

Section 133

[115] This form of the law is in effect since 2003.

(1) If an insurer acquires participation in another undertaking, which exceeds 10 per cent of its equity, it must report the acquisition to the Competent Authority within 2 working days.

(2) The insurer's participation in another joint stock company cannot reach 75 per cent of the registered capital of the joint stock company, with the exception of participations in other insurers, credit institutions, financial undertakings, investment undertakings or investment fund managers.

(3) During the investment of insurance technical reserves, the participation of the insurer in other undertakings cannot exceed 25 per cent of the registered capital of the given undertaking.

(4) The insurer cannot invest its assets covering actuary reserves into undertakings of the shareholder having an influential share, not engaged in insurance activities, unless the majority of the activities of such undertakings are directly related to the activities of the insurer.

(5) The investment restrictions contained in Paragraphs (1)-(3) do not apply to undertakings to which the insurer outsources its own activities, including insurance activities, if 75 percent of the annual revenue of the undertaking is from the pursue of its activities for insurers.

(6) With regard to the investment of insurance technical reserves other than actuary reserves and unit-linked life assurance reserves, primarily liquidity aspects should be taken into account.

Section 134

An insurer may keep its insurance technical reserves in the following assets:

A. Investments

a) Debt securities, bonds and other money market and investment instruments representing debt,

b) Loans,

c) Shares, other equity-type securities with variable yield, representing participation,

d) Investment bonds and other collective investment securities,

e) Properties, and rights and titles relating to properties;

B. Receivables

f) Receivables from reinsurers, including also insurance technical reserves allocated by the reinsurer for risks covered by reinsurance,

g) Receivables from security deposits and other receivables from insurance transactions taken for reinsurance,

h) Receivables within 3 months from the insured and insurance brokers originating from insurance and reinsurance transactions,

i) Insurance policy loans,

j) Tax refunds,

k) Recourse claims;

C. Other assets

l) Fixed assets other than property, at a value reduced with depreciation calculated on the basis of the principle of prudence,

m) Cash in current accounts kept by credit institutions and cash in hand, deposits in credit institutions and institutions entitled to collect deposits (receivables),

n) Accrued interest and rent,

o) Accrued acquisition costs.

Section 135

(1) In the case of assets included in the cover of insurance technical reserves, the following principles must be observed:

a) Assets covering insurance technical reserves must be valued in net amounts, deducting all expenses, which incur in relation to the purchase or acquisition of such assets,

b) Loans extended to economic organisations, the state, international organisations, local or regional governments or private individuals can only be included in the cover of insurance technical reserves if the borrower is able to provide sufficient security, in the form of property, lien, bank guarantee, insurance or other security items, with the exception of loans defined in Section 136 Paragraph (2) subparagraph f),

c) Receivables from third parties may be involved in the cover of insurance technical reserves after the deduction of recourse claims,

(2) The cover of insurance technical reserves of the insurer cannot include assets encumbered with a lien, or assets over which the right of disposal is limited.

(3) The cover of insurance technical reserves of the insurer cannot included investment bonds issued by an investment fund investing into derivative transactions defined in Section 278 of the Capital Market Act 278.

Section 136

(1) The cover of insurance technical reserves must be in the territory of the Member States or in assets issued by

An OECD member state,

A local or regional government of an OECD member state,

An economic organisation with a head office in an OECD member state,

An international organisation in which at least one Member State is a member.

(2) With the exception of reserves for unit-linked life assurance, the following limitations apply to assets involved in the cover of insurance technical reserves:

a) Of the items listed in Section 134 subparagraph a), assets not guaranteed by the state and not supported with other security items may represent maximum 25 per cent of the cover of insurance technical reserves,

b) Assets listed in Section 134 subparagraph c) can represent maximum 35 per cent of the cover of insurance technical reserves,

c) Assets listed in Section 134 subparagraph d) may represent 35 per cent of insurance technical reserves, providing that they fall under the scope of the No. 85/611/EEC Directive, otherwise they can be taken into account up to 30 per cent.

d) The insurer can take into account as coverage the value of a given property (land, building) only up to 10 per cent of the total gross insurance technical reserves. The same rule must be applied to several properties situated close to each other, which are considered one investment,

e) The insurer can invest up to 5 per cent of its all gross insurance technical reserves into shares, debt securities issued by the same undertaking, or other money and capital market instrument of the same undertaking, or loans extended to the same undertaking, with the exception of loans given to a Member State, or a local or regional government of a Member State or an international organisation with one or several Member State members. This ratio can be increased up to 10 per cent of the total gross insurance technical reserves, providing that the total amount of securities and loans does not exceed 40 per cent of the gross insurance technical reserves,

f) The insurer may take into account in the coverage of reserves, up to 5 per cent of the total gross insurance technical reserves, loans not covered with any security, except when they are extended to a credit institution, insurer or investment undertaking with a head office in a Member State. This limit shall be equal to 1 per cent of the total gross insurance technical reserves for each loan transaction,

g) Cash in hand can be included in the cover up to 3 per cent of the gross insurance technical reserves,

h) Shares, participations and bonds which have not been introduced to recognised securities market can be taken into account up to 10 per cent of the gross insurance technical reserves,

i) An insurer may invest up to 10 per cent of its gross insurance technical reserves in the investment bonds issued by a particular investment fund, or collective investment securities issued by a particular collective investment in the framework of a collective investment form.

(3) The assets listed in asset categories defined in Section 134. subparagraph h) and l) cannot exceed 5 per cent of the coverage of gross insurance technical reserves.

(4) The not netted off, aggregate market value of derivative transactions, calculated in accordance with the asset valuation procedures and Section 272 Paragraph (5) of the

Capital Market Act cannot exceed 15 per cent of the market value of securities covering the gross insurance technical reserves of the insurer.

(5) The limitations of Paragraph (2) does not apply to the assets issued by international financial institutions in which at least one Member State takes part.

Section 137

(1) Investment bonds issued by closed-end property funds, closed-end collective investment securities representing property investments, investment bonds and open-end collective investment securities representing property investments, issued by open-end property funds, and property together may not exceed 20 per cent of the financial assets covering actuary reserves.

(2) Investment bonds issued by a securities investment fund may not exceed 30 per cent of the financial assets covering actuary reserves. If the insurer acquires investment bonds of a European investment fund, the limit applicable to open-end investment funds shall increase with the amount of investment, but it shall not exceed 35 per cent.

(3) Bonds, other debt securities and loans, not covered with a bank guarantee, insurance, lien or other security items, as well as shares and bonds not introduced to a recognised securities market together cannot exceed 15 per cent of the financial assets covering actuary reserves.

(4) Deposits in credit institutions and other institutions entitled to collect deposits may not exceed 25 per cent of the financial assets covering the actuary reserves.

(5) The non-netted, aggregate market value of derivative transactions, calculated in accordance with the asset valuation procedures and Section 272 Paragraph (5) of the Capital Market Act cannot exceed 5 per cent of the market value of assets covering actuary reserves.

(6) The limits applicable according to Paragraph (5) and Section 136 Paragraph (4) do not include the following:

a) Derivative transactions concluded to reduce exchange rate and interest rate risks;

b) Repo transactions with credit institutions involving government securities.

Section 138

(1) During the valuation of derivative instruments, the principle of prudence must be followed and these assets must be taken into account during the valuation of the base product.

(2) With regard to derivative transactions, the rules laid down in Section 272 Paragraphs (4)-(8) and Section 273 of the Capital Market Act must be applied so that the investment fund manager shall mean the insurer, investment fund shall mean assets representing the coverage for insurance technical reserves, and the fund management regulations mean the asset valuation procedures of the insurer.

(3) The insurer may not enter into transactions, which would result in a short net position applying the netting rules laid down in Section 273 of the Capital Market Act, with the exception of short net positions undertaken for individual risks of securities in accordance with Section 273 Paragraph (6) of the Capital Market Act.

(4) The insurer must have liquid assets corresponding to 100 per cent of the difference between the aggregated transaction price of derivative long positions and already paid in floating deposit in addition to the liquid assets required for its ordinary business activities/operation. In the case of demand deposits or bank deposits fixed for maximum 30 days, the accounted value is identical with the deposit amount. In the case of other liquid assets, the accounted value shall be equal to 85 per cent of the market value of the liquid asset.

Section 139

(1) During the valuation of receivables, the principle of prudence must be followed, taking into account the risk of non-payment. Especially in the case of fixed assets other than property, the value reduced with depreciation calculated on the basis of the principle of prudence may be accepted as the coverage of insurance technical reserves.

(2) During the valuation of all assets involved in the coverage of insurance technical reserves, the principle of prudence must be applied, with special consideration to the risk of non-realisable revenues. Receivables from insurers and insurance brokers, outstanding for more than three months, cannot be taken into account.

(3) In the case of life assurance, the accrued acquisition costs can be involved in the coverage of actuary reserves to the amount of unearned premium reserves, with the exception of cases when the unearned premium reserve is part of actuary reserves.

Section 140

(1) In the case of assets for which no limit is established in Section 136 Paragraph (2)-(3) and Section 137 Paragraphs (1)-(6), the following basic principles shall be applied:

a) In accordance with the provisions of Section 132 Paragraph (2), several investment forms should be selected and risks must be shared during the establishment of an investment portfolio, i.e., the value of any of the investment categories must not be such that it leads to the insurer's dependence,

b) In the case of investment categories where the risk of the asset or issuer is high, the ratios to be taken into account for coverage must be established prudently, with low figures,

c) In the Section 134 subparagraph f), the classification of reinsurance must be taken into account during the definition of receivables from the re-insurer,

d) If a subsidiary of the insurer manages the insurer's investments in part or full, the principles stated in this Section must be applied to the assessment of the assets of the subsidiary too. The same rules must be applied to the assessment of participations in other subsidiaries,

e) The principle of prudence must also be taken into account during the definition of ratio of non-liquid investment assets,

f) Loans extended by those credit institutions and securities issued by those credit institutions may be taken into account into the coverage of investment of insurance technical reserves which have their head office in one of the Member States, are owned only by the Member State and/or its local governments, and the activities of the credit institution consist of loans to the Member State, local governments and public utilities and offices related to them.

(2) With the Competent Authority's permission, the 40 per cent limit established in Section 136 Paragraph (2) subparagraph e) may be increased in the case of mortgage bonds, or whenever the head office of the credit institution issuing the particular securities is in one of the Member States and it is subject to state supervision guaranteed by the law, which has been established in order to protect the interests of holders of such securities. Amounts collected from issue of such securities must be invested in accordance with the law making sure that during the entire term of securities the liabilities represented by the securities can be completed at any time, making sure that the assets serve primarily the payment of principal and its interest.

(3) The insurer may freely select from the investment instruments defined in Section 134, taking into account the various investment limits.

(4) Upon the insurer's request, providing that exceptional conditions prevail, the Competent Authority may permit deviation from the asset categories defined in Section 134 and limits identified in Sections 135-136.

(5) With regard to the issue as to which category a particular instrument belongs from the asset categories specified by law, the Competent Authority shall decide in disputes.

Section 141

(1) The provisions included in Section 136 Paragraphs (2)-(3), Section 137 Paragraphs (1)-(6), Section 140 Paragraph (1) subparagraphs a), b), d), e) and f), and Section 140 Paragraphs (2)-(3) do not apply to the investment of for unit-linked life assurance reserves. The insurer is obliged to invest unit-linked life assurance reserves in the same structure as the structure of its existing liabilities.

(2) If the insurer provides a principal or yield guarantee concerning unit-linked life assurance products, in the case of further reserves allocated for the guarantee, the

provisions of Section 136 Paragraphs (2)-(3), Section 137 Paragraphs (1)-(6), Section 140 Paragraph (1) subparagraphs a), b), d), e) and f) and Section 140 Paragraphs (2)-(3) must also be complied with.

Section 142

(1) If the total of liabilities of the insurer originating from debt exceed 5 per cent of its registered capital, it is obliged to report the main characteristics of the liabilities to the Competent Authority within 2 working days.

(2) The insurer is obliged to prepare quarterly reports on actuary reserves, suspended claim reserves and unit-linked life assurance reserves, and their investment, and submit the reports to the Competent Authority by the 15th day of the month following the quarter, while the report concerning the 4th quarter must be submitted to the Competent Authority by 31 January of the year following the current year.

8. THEORETICAL CONSTRUCTION OF LIFE INSURANCE

KEY WORDS

Elements of life insurance
Bet
Duality

In the following we will introduce the most general "building blocks" of life insurance in order to be able to place all possible (traditional and modern) life insurances in a uniform frame.

8.1. THE MOST IMPORTANT ELEMENTS OF LIFE INSURANCE

If we look at the matter objectively, we see that life insurance in the end is the special case of a wider group of phenomena, namely the "bet". Since there are generally negative associations tied to betting, it is not surprising that insurance studies don't advertise this "relation", or if it is mentioned at all, they try to deny it with all power.

Despite this, the situation is that insurance is a special kind of bet, where the subject of the bet is undoubtedly the incurrence of an event having negative consequences, and where the insured – in a certain sense – makes a bet against himself. The first insurance (that is regarded as an insurance) is supposed to have been formally a bet, where the owner of the ship took his bet on the ship not returning to the harbour with the carried goods. If he lost the bet, then in reality he won, because the ship returned safely, and if he lost his ship, then he won the bet. (Of course the stake of the bet from the part of the owner of the ship was much less than its value. For the other party, on the other hand, it was the ship – loaded with goods – that became the stake.)

In case of life insurance – following the pattern of the two possible life insurance events – two kinds of possible bets can be defined. The insured either makes his bet on that:

- he dies during a certain period, or that
- he doesn't die during a certain period.

We discover one or both of these bets in all possible types of life insurances[116], this way we can look at them as the elements of insurance, that constitute all life insurance products.

The same way as in the bet, there are two parties in an insurance, and the strategies of the two parties are exactly opposite. When the client bets on that he dies within a certain period, then the insurer in contrast, takes the bet on that he doesn't die within a certain period, so his strategy is the other possible outcome. This way, regarding the bet, the position of the insurer and the client is dual – the insurer takes exactly the opposite bet as the client. But it is important to add that the financial position of the insurer and the client is not the same, so only those bets are possible in insurance, where the insurer is the „bookmaker" at the same time, so the client pays his stack in advance to the insurer. Reversed case or third party (including a real bookmaker) doesn't exist.

From the two elemental bets we can build two very simple life insurances:

- A short term, single premium term insurance: I bet I'll die during this period.
- A short term, single premium pure endowment insurance: I bet I won't die within a certain, well defined period.

In the above the premise "short term" is important, because we can only disregard the interest return of the deposited money in the short term. If, on the other hand, we want to take a bet of a longer term, we have to ensure an interest. This way beside the above two elements we need a third element for the construction of life insurance that, for the sake of

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116 Life (and insurance!) sometimes seems to launch an attack against this categorical statement, because insurers sometimes put on the market purely savings products as life insurance. Because of this, we can modify our statement so that every life insurance contains one, two or all three elements – and no other element! – of the these two bets and the third element, that will be discussed shortly.

simplicity we'll call "account". From these three elements all life insurance types can be completely built.

With the introduction of the account our possible range of betting is significantly wider. For example the client can make a contract with the insurer that promises not only one bet, but a series of future bets. This bet-series can be standardised so that they result in the traditional, regular premium term- and pure endowment insurances:

- Regular premium term insurance: the client promises to bet on the same amount of money for a certain term, at certain periods (e.g. annually), and will manage the payment of the stack– that varies due to the changing age of the client – (the premium) from regular payments made to the account and the received interest.
- Regular premium pure endowment insurance: the client promises to make bets at regular intervals with the same expiration date, and always raises the amount at stake with this (for the sake of simplicity only the hoped final sum is highlighted, the continuous increase is not shown).

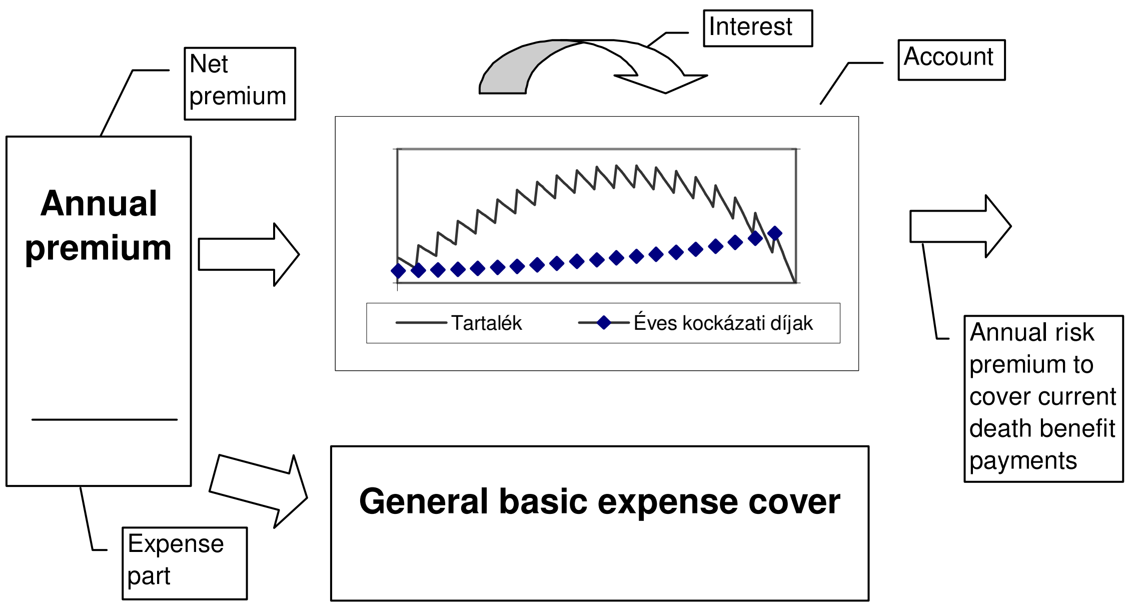

Figure 8.1.: The pattern of regular premium term insurance

Naturally the client can not only promise a series of bets, but can also make several different bets at the same time, e.g. pure endowment bets with different expiration, or different types at the same time. This takes us to the most diverse life insurance designs.

We do not see the above elements directly on the market, but all insurance types we know can be built from them. But we find that until 1997 insurances have been built from these elements in a systematically different way than nowadays.

It is worth to play a little bit with the bet-analogy! We have mentioned that the term and pure endowment bets are dual, in other words opposite pairs of each other, one party of the bet plays one, the other plays the other. All the same, the two bets, or the two betting strategies cannot be exchanged between potential players. One should rather play one, the other should rather play the other strategy (but of course both might have reason to apply both strategies). For the client it is the term bet, for the insurer it is rather the pure endowment bet that is the better strategy. The difference lies in the fact that the client plays one bet, while the insurer plays bets with lots of clients at the same time. The client risks a relatively large stake with the pure endowment bet, for a sum barely higher. It is usually not worth playing this game. But the insurer can make a noticeable profit with the number of small bet profits on the pure endowment bet. Naturally this is closely connected to the different financial position of the two parties.

If we think of life insurance as a cash flow between the client and the insurer, and sometimes reverse this cash flow, then we get an other well known insurance (in this sense these are also each other's duals). For example the reverse of the single premium lifetime annuity is the regular premium whole life insurance. The reverse of the single premium temporary annuity is the regular premium endowment insurance.

8.2. THE CONSTRUCTION OF TRADITIONAL AND MODERN LIFE INSURANCE

In case of traditional life insurance we have to add further properties to the above introduced elemental bets and account, to be able to derive all traditional life insurance types from them.

A typical group of characteristics has been formed in case of traditional life insurance in the 19th century. If we try to find the cause, the root of these, we find it in the administration and computation potential of that time. Since everything had to be calculated and manipulated manually, they aimed at the widest standardisation and to have every possibility played beforehand, and then tried to force all procedures to move on pre-calculated paths.

According to this:

- they fixed the interest rate (this was not a problem, since it was typically stable for long periods of time, as a given parameter),
- they fixed the timing of regular bets (the periods of premium payment) – and tried to handle any deviation from these outside of the system, e.g. by late interests, to be able to handle the premium as if it were paid on time, also in case of late payment,
- the insurer promised the insured not to perform the whole procedure over again in case of the new bets (not to apply underwriting before premium payment, and with this, the insurer took on the risk of anti-selection), but in return demanded stable commitment from the client, in other words, limited the possibilities to quit the insurance,
- with the exclusion of a few rare cases, they never allowed even the slightest modification of the policy – e.g. increasing-decreasing of the premium, or occasional single premium payments during the term,
- by certain insurances the premium has been determined so that by the end of the term the sum of the account reached zero (term insurance), and by other insurances so that it reached the sum of the original bet,
- they promised clients to use the same mortality rates throughout the term, even if mortality changed significantly along the way.

Summarising the above, we can state that the two elemental bets have been combined with an account so that the elemental insurances, the term and the pure endowment insurances were created, and all traditional life insurances could be built from these two elements.

Building **modern insurance** from these three elements is even more easy. Here the account is an instrument returning interest in a wide range, handling premium payments received and benefits paid out flexibly. From the bets – in case of the modern insurances developed so far – we only need the term bet. This naturally doesn't rule out the possibility that later on say, the annuity insurances will be modernized, and insurers offer annuities with benefits defined in units. Then the pure endowment bet can also become in important building block of modern life insurances.

III. THE TECHNIQUE OF LIFE INSURANCE PRODUCTS

9. THE PREMIUM OF LIFE INSURANCE

KEW WORDS

Actuary
Provisions for adverse deviation
Insurance premium
Insured period
Gross premium
Frequency of premium payment
Equivalence principle
Risk premium part
Unearned premium
Net premium
Technical interest rate
Premium loading

9.1. PARTS OF THE PREMIUM

As we know, the premium of non-life insurance is composed of three parts:

1. risk premium,
2. provision for adverse deviation,
3. profit loading.

Provision for adverse deviation serves to cope with the fluctuation of claims. Since life insurance is probably the best “behaving” insurance regarding claims, that is, the volume of claims can be predicted with very little deviation, there usually is no separate provision for adverse deviation applied. According to this, the premium of traditional life insurance consists of two parts:

1. risk premium,
2. premium loading.

The risk premium itself is usually called **net** premium, and the risk premium and the premium loading together are called **gross** premium.

Dividing the premium into two parts indicates that the insurer uses the collected premiums for two fundamentally different purposes. The greater part of the premium, the *risk premium* serves as the cover for the undertaken benefit liabilities. That is, if the insured event occurs, (death or living at the end of the term), then the insurer pays the benefits defined in the policy from the sum accumulated from the payments of this part. The smaller part of the premium, the premium loading serves as the cover of expenses (wages, office rent, profit, commission, etc.) of the insurer.

The premium serves the above two purposes also in the case of unit linked insurance, that is, to cover the expenses of the insurer and the undertaken risks, but dividing the premium into the above two parts is somewhat problematic. The problem comes from the construction of the insurance. The insurance is designed so that the total premium – with the exception of two components – goes to the asset funds. The premium of the death risk and the rider insurances, the administration fee and the fund management fee are from time to time subtracted from the asset funds. The payment of the premium and its timing, the subtraction of the above factors and their timing is different as a main rule, and due to this different timing the value of units changes. This way it is theoretically impossible to define these subtractions as a percentage of the premium – only some kind of subsequent – approximate – calculation is possible after each period. The two above mentioned components, that don’t go into the asset funds are the bid-offer spread, and the value of those initial units created from the given premium payment that will be certainly subtracted. These elements definitely belong to the expense part of the premium, but the expenses of the insurance are higher than these.

Despite the above, the premium of modern life insurance can be understood relatively easily due to its transparent structure, this way here we will focus on the introduction of the premium of traditional life insurance in the following.

9.2. PREMIUM CALCULATION

The insurance premium is calculated from certain basic data by insurance mathematicians (actuaries). Premium calculation has a traditional and a modern method. This mostly follows the traditional insurance – modern insurance division with the addition that traditional insurance can be calculated by modern methods, but not vice versa. Here we will describe the principles and elements of classical premium calculation and will deal with modern premium calculation later. In a certain sense the risk part – premium loading division is a requirement of the classical premium calculation – this kind of division is not absolutely necessary in modern premium calculation methods. But in the classical method the calculation begins with calculating the risk premium, and the calculation of the premium loading is based on this. The basic principle of the calculation of the risk premium is the simple so-called **equivalence principle**:

The present value of expected incomes = the present value of expected payouts.

Based of the so-far discussion we can state that life insurance is a "fair game", because it is not constructed to make the insurer win more, but so that the expected gains are equal.

The term "expected" in the equivalence principle refers to a mathematical (probability-theory) concept, the *"expected value"*, that we got to now earlier.

In life insurance both the income and the outgo items depend on chance (which in the present case means when the insured dies, since up to that point he pays, and after that the insurer), this way we can only predict their expected value, and even that only if we know the *probabilities of death* (mortality rates).

And where do we get these? A technical apparatus has been developed for this several centuries ago, the so-called **mortality table**, that we have also seen already.

In the equation reflecting the equivalence principle it was necessary to put the present value of incomes, because payments due at different times cannot be directly compared, and we know that income will occur continuously (which means that at different points in time), and payouts also don't occur all at the same time. Payments at different times will become comparable if we measure them with a uniform measuring unit. This uniformization can be achieved by calculating present values.

We also know that present value calculations require a discount factor, and this is calculated by aid of an interest rate. This interest rate is always the current interest rate of the market. But we know that this changes all the time, while we might have to calculate the premium of a policy of 45 years based on the equivalence principle, so we can only do discounting with an interest rate that is certain on this time-frame. This can only be the long term real interest rate, that is in a consolidated economy between 2-5%. In the long term, namely, we have to count with the possibility that the inflation rate stops, and the (nominal) interest rate drops to the level of the real interest rate.

Because of this, all insurance companies choose a so-called ***technical interest*** rate, and calculate the premium of life insurance using this interest rate[117]. We use this interest rate in the equivalence principle to calculate present values.

[117] The possible level of the technical interest rate is regulated in a decree of the Ministry of Finance. Currently the highest possible level is 4%, before that it was 5%, and 7% even earlier.

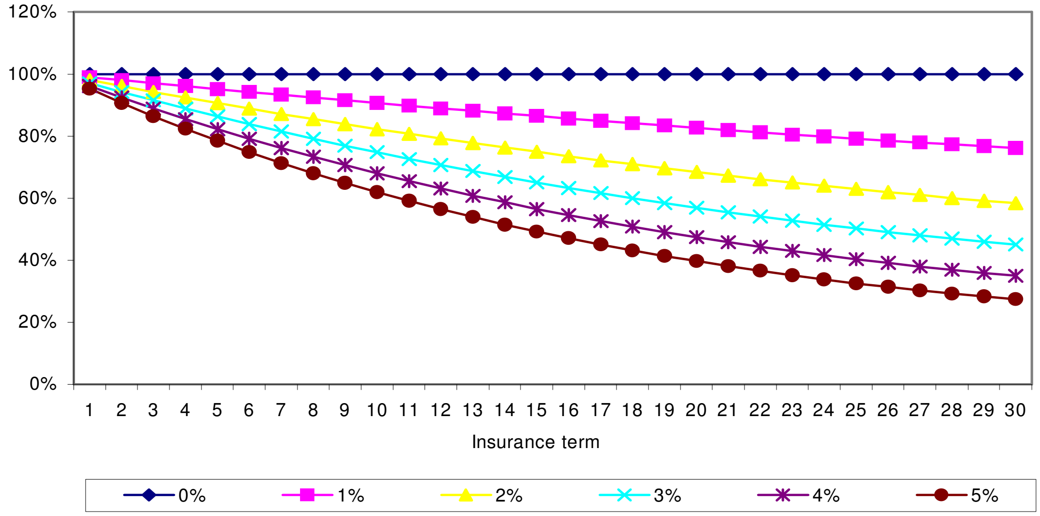

Figure 9.1.: The relative premiums of a single premium endowment insurance by different technical interest rates, compared to the 0% interest rate

The technical interest rate means at the same time also a *guaranteed* interest rate. The insurer guarantees that the investment of the premium reserve will return an interest of at least this level, that will be given back to the policyholder. If the insurer doesn't achieve this yield by investing the premium reserve, he still has to return it to the clients from some other source. The yield of the technical interest rate doesn't appear to the clients in the form of the increase of the sum assured, since it is already calculated in the premium. The higher the technical interest rate, the lower the premium of the insurance will be, as shown on figures 9.1 and 9.2:

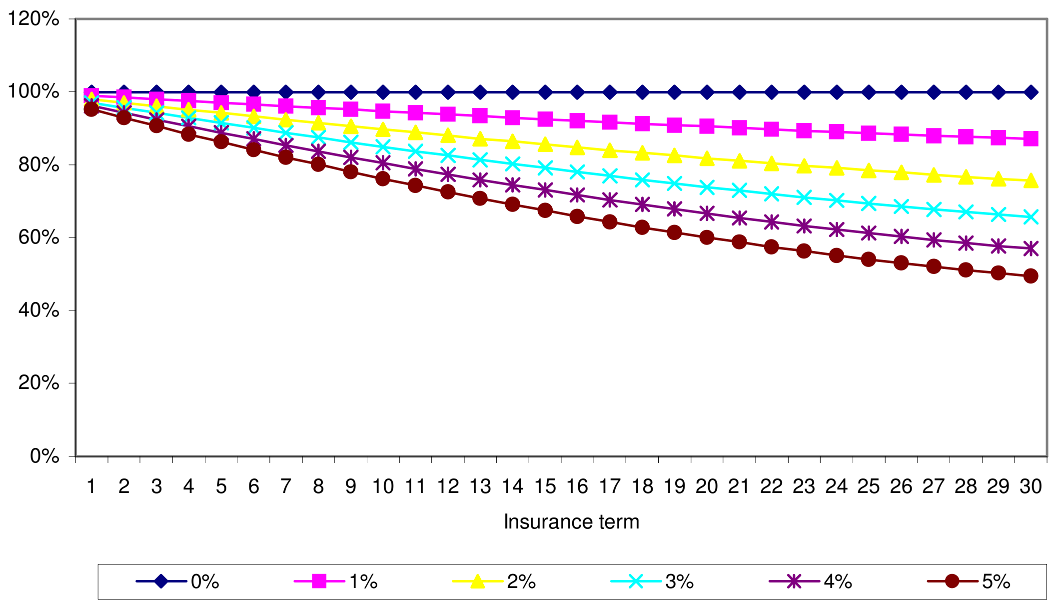

Figure 9.2.: The relative premiums of a regular premium endowment insurance at different technical interest rates, compared to the 0% interest rate

The guaranteed interest of the premium reserve calculated in the premium causes that e.g. in case of an endowment insurance with 1,000,000 Forints sum assured and a term of 30 years the yearly premium is say 24,360 Forints (the insured is a 20 years old man) the client has to pay 24,360*30 = 730,800 at most, while the beneficiary receives 1,000,000 Forints in all cases. (Naturally only if the insured doesn't die during the term and doesn't use the less frequent premium payment options.)

The difference – that is a lot more than 1,000,000-730,800 = 269,200 because of the premium loadings – is covered by the return on the premium reserve corresponding to the technical interest rate.

What is this premium reserve that we have mentioned so often?

Before discussing it in detail in a later chapter, it is important to pin down what it is not.

The premium reserve is **not** all premiums paid until the point of time in question.

10. THE PREMIUM CALCULATION OF LIFE INSURANCE

In the following chapter we'll discuss again what the previous chapter said about the premium calculation of traditional life insurance, but now we phrase the relations with the aid of mathematical formulae.

All notation used in this chapter– with a few exceptions – follow the actuarial standard developed around the turn of the century, since here we are introducing the classical premium calculation methods. Modern premium calculation methods will be discussed later, in chapter 16.

10.1. THE PREMIUM OF SINGLE PREMIUM INSURANCE

10.1.1. The Single Premium of Term Insurance

Let

$$A^{1}_{x:\overline{n|}} \tag{10.1.}$$

mean the following: The net premium of a single premium term insurance with 1 Forint sum assured and a term of *n* years, if the entering age f the insured is x years. E.g. the notation of the single premium of a term insurance with 1 Forint sum assured, 15 years term in case of an insured 45 years old when entering the policy is:

$$A^{1}_{45:\overline{15|}}$$

Since the premium of the insurance is directly proportional to the sum assured (disregarding the potential premium discount from the premium loading depending on the level of the premium, that might be applied by some companies), it is enough to declare the premiums of 1 Forint sum assured, and the premium of the actual sum assured can be calculated very easily from this.

The base of the calculation is the life table. As we know, this shows the number of living at age *x* from a starting population (generally 100,000 lives), as a function of age. In the premium calculation we always start with the simplifying supposition that the number of insured persons of age x, having an insurance of *n* years term is l_x , according to the life table. (This supposition doesn't have any particular significant meaning, it only shows that the risk community doesn't consist of only one person.)

The basis of the calculation is naturally the equivalence principle, which here also means that:

The present value of expected income = the present value of expected payout

(The equivalence principle refers to the net premium now.)

In case of single premium insurances it is very easy to calculate the present value of income, since all premium flows in to the insurer at the beginning of the term. So the expected value equals the actual premium paid, and we do not have to discount it. This way:

$$income = l_x \cdot A^{1}_{x:\overline{n|}} \tag{10.2.}$$

We have to introduce another simplifying supposition to be able to calculate the other side of the equation, the "present value of expected payout". This supposition states that all payouts of a certain insurance year are performed at the end of the insurance year. This supposition, as we'll see, makes our job a lot easier.[118]

As already mentioned, the number of deaths at a given age can be defined based on the life table:

$$d_x = l_x - l_{x+1}. \tag{10.3.}$$

So if the number of insured in the starting year is l_x, from which
the number of deaths during the first insurance year is d_x,
the number of deaths during the second insurance year is: d_{x+1},
...
the number of deaths during the n[th] insurance year is: d_{x+n-1}.

Supposing 1 Forint sum assured the insurer's liability at the end of year t is: $1 \cdot d_{x+t-1}$ Forints. If we discount the expected payouts of the individual years to the beginning of the insurance and add them up, we get the other side of the equivalence equation that we were looking for:

$$d_x \cdot v^1 + d_{x+1} \cdot v^2 + ... + d_{x+n-1} \cdot v^n \tag{10.4.}$$

where

$$v = \frac{1}{1+i}, \tag{10.5.}$$

where

i: is the technical interest rate.

This way the equivalence equation is:

$$l_x \cdot A^{\,1}_{x:\overline{n|}} = d_x \cdot v^1 + d_{x+1} \cdot v^2 + ... + d_{x+n-1} \cdot v^n \tag{10.6.}$$

from this we can get the single premium simply dividing by l_x:

$$A^{\,1}_{x:\overline{n|}} = \frac{d_x \cdot v^1 + d_{x+1} \cdot v^2 + ... + d_{x+n-1} \cdot v^n}{l_x} \tag{10.7.}$$

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[118] **Remark**: This supposition can be further refined the following way: we suppose that in the given insurance year everyone dies at exactly the end of the insurance year, and in the event of death the insurer pays the benefits at once. It is clear that both supposition are unreal. Deaths occur continuously during the year. Because of this, a different, more realistic supposition is sometimes used, that all deaths occur exactly in the middle of the insurance year. This, of course means that insured persons die "averagely" at this time, which is a more realistic supposition, but it somewhat complicates the calculation, so the above, more simple approach is often used in practice.
Supposing payments at the end of the year results a little lower premiums than if we put the payments to the middle of the year, since the liability of the insurer is due later. But the difference is not too big, and the effect is somewhat compensated by the fact that payment usually doesn't happen immediately at death, but generally 1-2 months later. The insurance company needs this time for the assessment of the circumstances of death and the claim, and until that time the sum assured gains interest on the accounts of the insurer, for the insurer.
To eliminate the above problem, usually a correction factor is inserted in the formulae, that we will not discuss in the following.

The obtained result is totally adequate and satisfying, and it is easy to write a computer program based on this, that computes the adequate premiums using an arbitrarily chosen life table and arbitrary technical interest rate.

A few decades ago the option of a computer program was not available for actuaries, this way the above formula was further simplified by introducing new, standard notations.[119] The basis of the simplification was that insurance companies changed life tables and technical interest rates relatively rarely (only in the perspective of decades), so both factors could be considered given at every point of time. From the life tables some standard functions, the so-called commutation functions, or commutation numbers were created, and the values of these functions were given together with the values of the life tables, and calculated in advance.

First of all the "discounted value " of living and dead were introduced, D_x and C_x. These Were defined the following way:

$$D_x = l_x \cdot v^x \,, \tag{10.8.}$$

$$C_x = d_x \cdot v^{x+1} \,. \tag{10.9.}$$

With the aid of these commutation numbers, we can write equation 10.7 of the premium calculation on a different way. Since

$$A^{\,1}_{x:\overline{n}|} = \frac{d_x \cdot v^1 + d_{x+1} \cdot v^2 + \ldots + d_{x+n-1} \cdot v^n}{l_x} \,, \tag{10.10.}$$

from this relation it follows that:

$$A^{\,1}_{x:\overline{n}|} = \frac{d_x \cdot v^{x+1} + d_{x+1} \cdot v^{x+2} + \ldots + d_{x+n-1} \cdot v^{x+n}}{l_x \cdot v^x} \tag{10.11.}$$

If we substitute into 10.11. the new symbols 10.8 and 10.9.:

$$A^{\,1}_{x:\overline{n}|} = \frac{C_x + C_{x+1} + \ldots + C_{x+n-1}}{D_x} \tag{10.12.}$$

(We can see that the asymmetry of multiplying l_x with v^x but d_x with the form of a power one higher was necessary because of the supposition of "death at year end".)

The above equation is a little more simple than the original, but not much. This way a new commutation number was introduced, M_x.

Let

$$M_x = C_x + C_{x+1} + \ldots + C_\omega \,, \tag{10.13.}$$

where ω is the highest age regarded when constructing the life table (in Hungary this is 100 years).

We see that the relation of the above equation gives:

$$C_x + C_{x+1} + \ldots + C_{x+n-1} = M_x - M_{x+n} \tag{10.14.}$$

and so it can also be written in the form

[119] On the other hand, nowadays, when premiums are calculated by computers, these standard symbols are very useful in programming, because they make it possible to design the premium calculation formulae in a "building block-like" manner, and this way the programme becomes structured and clear-cut.

$$A^{1}_{x:\overline{n|}} = \frac{M_x - M_{x+n}}{D_x}\ . \tag{10.15.}$$

Let's look at other single premium insurances!

10.1.2. The Single Premium of Pure Endowment, Pure Endowment With Premium Refund and Endowment Insurance

In respect of the **pure endowment insurance** our suppositions are similar as before. We suppose that l_x number of individuals of age x take out a single premium pure endowment policy of 1 Forint sum assured and n years term. We use similar symbols for the notation of the net single premium of the pure endowment insurance of 1 Forint sum assured and n years term as before:

$$A_{x:\overline{n|}}^{\ \ \ 1}$$

The income side of the equivalence equation now is (because of similar considerations):

$$l_x \cdot A_{x:\overline{n|}}^{\ \ \ 1}\ , \tag{10.16.}$$

Since n years from now l_{x+n} individuals are assumed to be alive from the initial l_x, this way the expected pure endowment benefit payment will be:

$$l_{x+n} \cdot 1\,Ft\ = l_{x+n}\ . \tag{10.17.}$$

Since this payment is due in exactly n years, the present value of the expected payout is:

$$l_{x+n} \cdot v^n\ . \tag{10.18.}$$

So the equivalence equation is:

$$l_x \cdot A_{x:\overline{n|}}^{\ \ \ 1} = l_{x+n} \cdot v^n \tag{10.19.}$$

From this:

$$A_{x:\overline{n|}}^{\ \ \ 1} = \frac{l_{x+n} \cdot v^n}{l_x}\ . \tag{10.20.}$$

From this, similarly as before, multiplying the numerator and the denominator by v^x we can obtain the premium defined with commutation numbers:

$$A_{x:\overline{n|}}^{\ \ \ 1} = \frac{D_{x+n}}{D_x}\ . \tag{10.21.}$$

As we know, from the technical point of view the **endowment insurance** is nothing but a term insurance plus a pure endowment insurance. This way we get the premium also as the sum of these premiums. Let the maturity sum assured be 1 Forint, and the death sum assured h Forints. In other words this means that the ratio of the death sum and the maturity sum is h. (The most common, and almost the only case is when h = 1, so the maturity and the death sum assured are equal.[120])

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[120] Because of this, if not stated otherwise, under endowment insurance we always mean the h=1 type insurance, and only use the special forms if stated in advance.

According to the above, the single net premium of an endowment insurance with 1 Forint death sum assured, *h* Forints maturity sum assured, *n* years term, supposing an insured of age *x* is:

$$A_{x:\overline{n}|} = h \cdot A^{\ \ 1}_{x:\overline{n}|} + A^{1}_{x:\overline{n}|} \tag{10.22.}$$

The same formula using commutation numbers:

$$A_{x:\overline{n}|} = \frac{h \cdot \left(M_x - M_{x+n}\right) + D_{x+n}}{D_x}. \tag{10.23.}$$

The single premium of the pure endowment insurance with premium refund:

$$\begin{aligned} A_{x:\overline{n}|} &= A_{x:\overline{n}|}^{\ \ 1} + P^b_{x:\overline{n}|} \cdot \frac{R_x - R_{x+n} - n \cdot M_{x+n}}{D_x} = \\ &= \frac{D_{x+n} + P^b_{x:\overline{n}|} \cdot \left(R_x - R_{x+n} - n \cdot M_{x+n}\right)}{D_x} \end{aligned} \tag{10.24.}$$

where

$$P^b_{x:\overline{n}|} = P_{x:\overline{n}|} \cdot \left(1 + \lambda_{x:n}\right) \text{ and} \tag{10.25.}$$

$$P_{x:\overline{n}|} = \frac{A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}} = \frac{D_{x+n}}{N_x - N_{x+n} - \left(1 + \lambda_{x:n}\right) \cdot \left(R_x - R_{x+n} - n \cdot M_{x+n}\right)}. \tag{10.26.}$$

Another important formula here is:

$$A_{x+t:\overline{n-t}|} = \frac{D_{x+n} + P^b_{x:\overline{n}|} \cdot \left(t \cdot M_{x+t} + R_{x+t} - R_{x+n} - n \cdot M_{x+n}\right)}{D_{x+t}}, \tag{10.27.}$$

of which the above formula is an obvious sub-case if t=0.

10.1.3. Single Premium Annuity

Annuities can be paid in advance or in arrears. This classification is purely technical. In case of annuity paid in advance the insurer provides the annuity payment at the beginning of each insurance period, in case of the arrears payment it is paid at the end of the insurance period. In advance and in arrears annuity payments only differ when the annuity commences, since the advance payment provides the first payment at once, while the arrears only pays one insurance period later. So if we take away the first payment from the advance version, we get the arrears annuity. Because of this, later on we will only deal with the annuity paid in advance, and if not stated explicitly otherwise, all discussion refers to this case.

10.1.3.1. The Premium of Immediate Lifetime Annuity

As mentioned earlier, this is the most important annuity type in a sense, since many other annuity types can be derived from this one.

Let $\ddot{a}_x$ be the single net premium of the immediate lifetime annuity with 1 Forint annuity payment paid in advance, if the entering age of the insured is *x* years.

When deducting the premium formula, we apply once more the suppositions used earlier in the chapter. According to this the income side of the equation is:

$$l_x \cdot \ddot{a}_x \,, \tag{10.28.}$$

Since the first annuity payment is immediately due to the living, which means all insured, so the expected value of the first payment is:

$$l_x \cdot 1Ft \;=\; l_x Ft \;\;. \tag{10.29.}$$

One year later only l_{x+1}, and two years later only l_{x+2} insured will be alive. This way the expected value of the second payment is l_{x+1} Forints, the third is l_{x+2} Forints, etc… Discounting the payments the payout side of the equivalence equation will be:

$$l_x + l_{x+1} \cdot v^1 + l_{x+2} \cdot v^2 + \ldots + l_\omega \cdot v^{\omega - x} \tag{10.30.}$$

From this we get the following relation for the net premium:

$$\ddot{a}_x = \frac{l_x + l_{x+1} \cdot v^1 + l_{x+2} \cdot v^2 + \ldots + l_\omega \cdot v^{\omega - x}}{l_x}. \tag{10.31.}$$

Using commutation numbers and multiplying by v^x we get:

$$\ddot{a}_x = \frac{D_x + D_{x+1} + D_{x+2} + \ldots + D_\omega}{D_x}. \tag{10.32.}$$

Here we have to face the same problem as before, namely that the formula is still too long. This way we introduce the commutation number N_x. The definition is:

$$N_x = D_x + D_{x+1} + D_{x+2} + \ldots + D_\omega \tag{10.33.}$$

Using N_x we can formulate the equation of $\ddot{a}_x$ in a very simple form:

$$\ddot{a}_x = \frac{N_x}{D_x}. \tag{10.34.}$$

10.1.3.2. The Premium of Deferred Lifetime Annuity

The deferred annuity differs from the immediate type in that the first payment is due not at the commencement of the insurance, but after the deferred period of *m* years, if the insured is still alive at that time. (If the insured dies during the deferred phase, the insurance ends without any benefit payment.)

Let ${}_{m|}\ddot{a}_x$ be the single net premium of a lifetime annuity paying 1 Forint yearly in advance, after a deferred phase of *m* years, in case of an insured of age *x*.

The payout side of the equivalence equation differs from the right side of the previous equation in that the first payment will be paid in *m* years, only to l_{x+m} number of insured, who are still alive at that time. So the equation serving as a theoretical basis of the premium calculation can be written in the following form:

$$l_x \cdot {}_{m|}\ddot{a}_x = l_{x+m} \cdot v^m + l_{x+m+1} \cdot v^{m+1} + \ldots + l_\omega \cdot v^{\omega - x} \tag{10.35.}$$

Using commutation numbers:

$$ {}_{m|}\ddot{a}_x = \frac{D_{x+m} + D_{x+m+1} + D_{x+m+2} + \ldots + D_\omega}{D_x}, \tag{10.36.}$$

Or in a different form:

$$ {}_{m|}\ddot{a}_x = \frac{N_{x+m}}{D_x} \tag{10.37.}$$

We could also have derived this result through a different train of thought. According to this, the deferred lifetime annuity is the combination of a pure endowment insurance of n years term, and an immediate annuity (*m* years from now), where the sum assured of the pure endowment insurance serves as the single premium of the immediate lifetime annuity.

This way the sum assured of the pure endowment insurance has to be $\ddot{a}_{x+m}$, since the insured who currently is *x* years old will be *x+m* years old *m* years from now, and the single net premium of the immediate lifetime annuity starting then will be $\ddot{a}_{x+m}$. The single net premium of the pure endowment insurance with 1 Forint sum assured and *m* years term – as we now – is:

$$A_{x:\overset{1}{\overline{m|}}} = \frac{D_{x+m}}{D_x} \tag{10.38.}$$

So we get:

$$ {}_{m|}\ddot{a}_x = A_{x:\overset{1}{\overline{m|}}} \cdot \ddot{a}_{x+m} \,. \tag{10.39.}$$

Since we know that

$$\ddot{a}_{x+m} = \frac{N_{x+m}}{D_{x+m}},$$

substituting into the above equation we get:

$$ {}_{m|}\ddot{a}_{x+m} = \frac{D_{x+m}}{D_x} \cdot \frac{N_{x+m}}{D_{x+m}} \,. \tag{10.40.}$$

We see that the two D_{x+m} factors drop each other out, so we get:

$$ {}_{m|}\ddot{a}_x = \frac{N_{x+m}}{D_x} \tag{10.41.}$$

With the aid of deferred annuities we can form a relation between annuities with in advance and in arrears payment. If we think about it, we discover that the annuity with payment in arrears is an annuity paying in advance with a deferred phase of 1 year. If a_x denotes the single net premium of the annuity paying in arrears, then the relation takes the following form:

$$a_x = {}_{1|}\ddot{a}_x \tag{10.42.}$$

We can also discover that the difference between the annuity deferred for 1 year and the immediate annuity is that the payment of 1 Forint is missing at the beginning of the term. This way:

$$a_x = {}_{1|}ä_x = ä_x - 1 \tag{10.43.}$$

10.1.3.3. The Premium of the Temporary Annuity

The temporary annuity differs from the “simple” lifetime annuity in that payments stop after a certain period under all circumstances, so the insured receives the annuity until alive, but only until the end of the term of *n* years.
Let the single net premium of the temporary annuity with 1 Forint yearly payment and *n* years term in case of an insured of entering age *x* be: $ä_{x:\overline{n}|}$

The temporary annuity can be derived from the “simple” lifetime annuity very easily. Think about it! The temporary annuity paying for *n* years is the difference of an immediate lifetime annuity and a lifetime annuity deferred for *n* years:
The immediate lifetime annuity with *n* years term

Year	0	1	2	...	n-1	n	n+1	n+2	...
Immediate annuity	1	1	1	...	1	1	1	1	...
Annuity deferred for *n* years	-	-	-	...	-	1	1	1	...
Difference: immediate, *n* years term temporary annuity	1	1	1	...	1	-	-	-	...

Table 10.1.: Benefits received during the term by different types of annuities

Based on this we can also formulate the net premium of the temporary annuity for *n* years:

$$ä_{x:\overline{n}|} = ä_x - {}_nä_x \tag{10.44.}$$

The same formula with commutation numbers:

$$ä_{x:\overline{n}|} = \frac{N_x}{D_x} - \frac{N_{x+n}}{D_x} = \frac{N_x - N_{x+n}}{D_x} \tag{10.45.}$$

10.1.3.4. Certain annuities

Certain annuities cannot be considered insurance in a strict sense, since the fundamental feature of every insurance is that the benefits, or the degree of benefits provided by the insurer depend on the occurrence or non-occurrence of some random event. In case of the certain annuity there is no such random event influencing the existence, the degree or the duration of the benefits paid by the insurer. Certain annuity means that for a specified period, an annuity with specified payment or a payment varying according to specified rules will certainly be paid to the insured or the inheritor of the insured (beneficiary). It is important to talk about the certain annuity all the same, because it can be an important complementing element of annuities and other types of insurance.

Most of the above categories can be applied also to the certain annuity. Accordingly we can talk about certain annuities paid in advance or in arrears, immediate or deferred, temporary (paid for a certain period of time) and … not “lifetime” annuity – since the payment does not depend on the fact that the insured is alive or not –, but instead of this “infinite” annuity, which means that theoretically it is to be paid by the insurer forever to somebody (the current owner of the annuity).

It is obvious – and this concerns also the above – that the immediate annuity is a special case of the deferred annuity, where the deferred phase is 0 years. This way we will not distinguish between the two types of annuities, the 0 index will indicate that we are dealing with the immediate annuity. Due to the above, we'll only examine temporary annuities further on.

So there are two types of annuities remaining: the – deferred – temporary annuity and the – deferred – infinite annuity. (In reality the infinite annuity can also be derived from the temporary annuity, with an infinite annuity term.)

Let F denote the net premium of the certain annuity. Within this

${}_{m|}F_n$: is the single net premium of the certain annuity of 1 Forint yearly paid in advance, if the deferred phase is *m* years and the term is *n* years (starting the term after the deferred phase),

${}_{m|}F$: is the single net premium of the certain infinite annuity of 1 Forint yearly paid in advance, with a deferred phase of *m* years.

According to the above discussions it is easy to see that:

$$ {}_{m|}F_n = v^m + v^{m+1} + \ldots + v^{m+n-1} = v^m \cdot \frac{1 - v^n}{1 - v} \tag{10.46.}$$

It can be proved that the infinite annuity is derived from the temporary annuity by increasing its term to infinity, choosing n to be infinite. If $\lim\limits_{n\to\infty} v^n = 0$. So formula 10.46. is transformed to the following form:

$$ {}_{m|}F = v^m + v^{m+1} + \ldots + v^{m+n-1} + \ldots = \frac{v^m}{1 - v} \tag{10.47.}$$

10.1.3.5. Annuity with Guarantee Period

Annuity with guarantee period is the connecting key on the one hand between lifetime annuities and certain annuities, and on the other hand between single life annuities and joint life annuities, which will be discussed in the next chapter. The guarantee period means that for a certain time period the insurer pays the annuity to the inheritor of the insured or another person declared by the policyholder, even if the insured dies during this period. So we can say that the annuity with guarantee period is a joint life annuity concealed in the form of a single life annuity, or that the annuity with guarantee period is a certain type of "widows' annuity" insurance (that will be discussed later).

The guarantee period can theoretically have two forms:

A) The insurer guarantees that starting from the commencement of the annuity for a period of *g* years (the guarantee period) it will be paid certainly, even if the insured dies before the end of year *g*.

B) After the insured's death the insurer will pay the annuity for exactly *g* years.

The guarantee period can be used to loosen certain inhibitions that are in the way of signing the annuity policy. One of these inhibitions is that someone pays 1 Million Forints as the single premium of an annuity, and receives in exchange, say 10 thousand Forints while alive. But if death occurs in the first month, then – looking at it subjectively, from the insured's point of view – 990 thousand Forints remain at the insurer, so this money was needlessly "thrown out the window". But in a case like this a guarantee period of 5 years means that someone (the insured or the declared beneficiary) will certainly receive 60 months of annuity payment, that is, 600 thousand Forints (disregarding interest and discounting) from the insurer. This way the subjective uncertainty feeling of possibly throwing money out the window by purchasing the annuity insurance is considerably subdued.

Let

${}^{j}\ddot{a}_{x}^{g}$: be the single net premium of the immediate lifetime annuity of 1 Forint yearly payment in advance, with a guarantee period of *g* years, supposing an insured of *x* years, where the possible "values" of *j* are naturally *A* and *B*, depending on whether the annuity has a type *A* or type *B* guarantee period.[121]

What are these single premiums?

If we think about it, then the annuity with type A) guarantee period is the sum of an immediate temporary certain annuity with a term of *g* years, and a deferred lifetime annuity with *g* years deferred period, so the premium in question is:

${}^{A}\ddot{a}_{x}^{g}$ = the premium of an immediate certain temporary annuity with term *g* years + the premium of a deferred lifetime annuity with *g* years deferred period

$$ {}^{A}\ddot{a}_{x}^{g} = {}_{0|}F_{g} + {}_{g|}\ddot{a}_{x} \tag{10.48.}$$

Or using commutation numbers:

$$ {}^{A}\ddot{a}_{x}^{g} = \frac{v^{g}-1}{v-1} + \frac{N_{x+g}}{D_{x}} \tag{10.49.}$$

The same way we can discover that the annuity with type B) guarantee period is an immediate certain annuity with term *g* and an "immediate" annuity having the interesting feature that the insurer pays every payment only *g* years after its due date. (So the last payment, that the insured would receive immediately before his death is only paid to the beneficiary *g* years after that.) The role of the certain annuity here naturally is that the insured receives the annuity payments also during the *g* years before actually receiving the first real annuity payment. So the premium calculation:

$$ {}^{B}\ddot{a}_{x}^{g} = {}_{0}F_{g} + v^{g} \cdot \ddot{a}_{x} \tag{10.50.}$$

Using commutation numbers:

$$ {}^{B}\ddot{a}_{x}^{g} = \frac{v^{g}-1}{v-1} + v^{g} \cdot \frac{N_{x}}{D_{x}} \tag{10.51.}$$

In the above formula the *g* year shift of paying the annuity payments is indicated by the v^g discount factor.

Naturally the type B) annuity can also be regarded in a way that the annuity at the end of the term is a kind of death benefit (the death benefit of a whole life insurance!), that the beneficiary receives in the form of an annuity. The present value of this annuity, when it begins is exactly ${}_{0}F_{g}$, so then the annuity with guarantee period is the sum of a "simple" annuity and a whole life insurance:

$$ {}^{B}\ddot{a}_{x}^{g} = \frac{N_{x}}{D_{x}} + \frac{v^{g}-1}{v-1} \cdot \frac{M_{x}}{D_{x}} \tag{10.52.}$$

[121] With the addition that the default value is A, so if the left hand side superscript doesn't appear in the formula, then the guarantee period is supposed to be type A!

Of course 10.51. and 10.52. are equivalent with each other. To prove this we use the following relation between commutation numbers:

$$C_x = d_x \cdot v^{x+1} = (l_x - l_{x+1}) \cdot v^{x+1} = D_x \cdot v - D_{x+1} \tag{10.53.}$$

From this:

$$\begin{aligned} M_x &= \sum_{k=x}^{\omega} C_k = \sum_{k=x}^{\omega} (D_k \cdot v - D_{k+1}) = v \cdot \sum_{k=x}^{\omega} D_k - \sum_{k=x}^{\omega} D_{k+1} = \\ &= v \cdot N_x - N_{x+1} \end{aligned} \tag{10.54.}$$

10.54. can be transformed into a more convenient form:

$$\begin{aligned} M_x &= v \cdot N_x - N_{x+1} = v \cdot N_x - (N_x - D_x) = \\ &= (v-1) \cdot N_x + D_x \end{aligned} \tag{10.55.}$$

Substituting 10.55. into 10.52. we get:

$$\begin{aligned} {}^{B}\ddot{a}_{x}^{g} &= \frac{N_x}{D_x} + \frac{v^g - 1}{v-1} \cdot \frac{(v-1) \cdot N_x + D_x}{D_x} = \\ &= \frac{N_x}{D_x} + (v^g - 1) \cdot \frac{N_x}{D_x} + \frac{v^g - 1}{v-1} = \\ &= v^g \cdot \frac{N_x}{D_x} + \frac{v^g - 1}{v-1} \end{aligned} \tag{10.56.}$$

This is exactly 10.51., which means that 10.51. and 10.52. are indeed equivalent.

10.1.3.6. The Premium of Joint Life Annuities

The same way as in case of other types of insurances, we can also talk about single life and multiple life annuities, that is, insurances, where there is not only one insured, but two or more. In the following we will only consider the premium calculation of one type, the single premium of the two person, immediate annuity.

In case of the two person, immediate annuity, the same way as in the case of other two person insurances, death means the death of the insured to die first. (It may be strange that payments last only until the first death, although it is needed much more after the death of the spouse than before that. But this annuity type only serves technical purposes, and the premium of most of the "real" two person, joint life annuities can be easily derived from this one.) According to this, the insurer pays the annuity payments in case of a two person immediate annuity until both insured persons are alive.

Let $\ddot{a}_{xy}$ denote the single net premium of the two person, immediate annuity with 1 Forint yearly payment supposing that the insured persons are x and y years old.

When calculating $\ddot{a}_{xy}$ we start from the supposition that all possible couples of years x and y take out the above policy, which means $l_x l_y$ number of couples all together. Every couple receives the first annuity payment, so then the payout of the insurer is $l_x l_y$ Forints. The second annuity payment goes only to those couples, where both are still alive a year later, which means $l_{x+1} l_{y+1}$ number of couples, and so on. So the equivalence equation is:

$$l_x \cdot l_y \cdot \ddot{a}_{xy} = l_x \cdot l_y + l_{x+1} \cdot l_{y+1} \cdot v^1 + \dots . \tag{10.57.}$$

From this:

$$\text{ä}_{\text{xy}} = \frac{l_x \cdot l_y + l_{x+1} \cdot l_{y+1} \cdot v^1 + \ldots}{l_x \cdot l_y}\,. \tag{10.58.}$$

If we multiply both sides of the equation with v^x, then we can use the commutation numbers for two lives.

Let D_{xy} simply denote the $D_x \cdot l_y$ multiple.

So the above equation can be written in the form:

$$D_{xy} \cdot \text{ä}_{\text{xy}} = D_{xy} + D_{x+1,\,y+1} + \ldots\,. \tag{10.59.}$$

The above formula can be significantly simplified if we introduce also the *N* commutation number for two lives. The definition of this is:

$$N_{xy} = D_{xy} + D_{x+1,\,y+1} + \ldots\,. \tag{10.60.}$$

Then the above equation takes the following form:

$$D_{xy} \cdot \text{ä}_{\text{xy}} = N_{xy}\,, \tag{10.61.}$$

so

$$\text{ä}_{\text{xy}} = \frac{N_{xy}}{D_{xy}} \tag{10.62.}$$

We see that the joint life annuity is totally analogous to the single life annuity.

10.1.3.7. The Premium of Annuities in p Payments Yearly

In case of all the above derived annuities we supposed that the payment is due once every (insurance) year, in one instalment, at the beginning of the year. But in real life people need annuities where payment is performed not once yearly, but more often, say 12 times (i.e. monthly). So in the following section we show through a few examples how the above premium formulae change if we suppose not 1, but *p* number of instalments yearly. We are still supposing the payment of 1 Forint yearly, but this is performed in *p* number of instalments, so every time the insured receives *1/p* Forints.

Let’s determine the single net premium of an annuity with 1 Forint yearly payment in *p* instalments, for an insured of *x* years! The notation of the single premium in this case is: $\text{ä}_{\text{x}}^{(p)}$.

$\ddot{a}^{(p)}_x$ could be determined exactly if we knew a life table where the “distance” between neighbouring age categories is not 1 year, but *1/p* years. This can naturally be constructed from the existing mortality table by interpolation, but this already puts approximate values in the place of the exact ones. Below we review a method, or a formula of this kind, but first we’ll derive based on simple (not totally correct) logic a simple approximate formula that can be easily and well applied in practice.

The method is based on the analogy of annuities paid in advance and in arrears. As we have seen in 10.43.:

$$\text{ä}_{\text{x}} - a_x = 1\,, \tag{10.63.}$$

so the difference between the single net premium of the in advance and the in arrears case is 1, i.e. the annuity paid in arrears differs from the one paid in advance in that payment starts 1 year later than in the other case. From this we can deduct that

$$\ddot{a}_x - \frac{m}{p} \tag{10.64.}$$

is the single premium of the annuity, where the yearly payment is due *m/p* year(-fraction) after the commencement of the insurance year[122]. It is clear that the possible values of *m* are: 0,1,2,..., *p*-1. If we substitute these *m* values into the above expression one after the other, and then add them up, we get the single premium of an annuity where the insured receives 1 Forint annuity payment at the beginning of every *1/p* year-fraction, so all together *p* Forints in the year. This will be the single premium of an annuity with *p* Forints yearly payment, but paid in *p* number of instalments yearly, so the formula is:

$$\mathrm{p} \cdot \ddot{a}_x^{(p)} = \ddot{a}_x + \left(\ddot{a}_x - \frac{1}{p}\right) + \left(\ddot{a}_x - \frac{2}{p}\right) + \ldots + \left(\ddot{a}_x - \frac{p-1}{p}\right) =$$

$$= p \cdot \ddot{a}_x - \frac{1+2+..+(p-1)}{p} = p \cdot \ddot{a}_x - \frac{1}{2} \cdot (p-1). \tag{10.65.}$$

I.e.:

$$\ddot{a}_x^{(p)} = \ddot{a}_x - \frac{p-1}{2p} \tag{10.66.}$$

So in case of monthly payment (when *p*=12, so the monthly annuity payment is 1/12 Forints) the net premium is:

$$\ddot{a}_x^{(12)} = \ddot{a}_x - \frac{11}{24}, \tag{10.67.}$$

i.e. in case of monthly annuity the premium is somewhat lower (by 11/24 Forints) than the premium of the annuity paying the total 1 Forint yearly payment in one instalment at the beginning of the year.

Now let's look at the case of instalment payments within the year in case of annuities paid for a certain term (temporary annuities).

We know that the net premium of the temporary annuity of *n* years term is

$$\ddot{a}_{x:\overline{n}|} = \ddot{a}_x - {}_n\ddot{a}_x \tag{10.68.}$$

and if we think about it, then the deferred annuity is the combination of a pure endowment insurance and an immediate annuity, so

$${}_n\ddot{a}_x = A_{x:\overline{n}|}^{\;\;1} \cdot \ddot{a}_{x+n} \tag{10.69.}$$

Using commutation numbers it is easy to see that:

$${}_n\ddot{a}_x = \frac{N_{x+n}}{D_x} = \frac{N_{x+n}}{D_{x+n}} \cdot \frac{D_{x+n}}{D_x} = A_{x:\overline{n}|}^{\;\;1} \cdot \ddot{a}_{x+n} \tag{10.70.}$$

So the temporary annuity can be written in the form:

$$\ddot{a}_{x:\overline{n}|} = \ddot{a}_x - A_{x:\overline{n}|}^{\;\;1} \cdot \ddot{a}_{x+n} \tag{10.71.}$$

[122] Naturally this is not the exact supposition, but it is not too far from reality!

From this it follows that if we pay the temporary annuity in *p* number of yearly instalments, then we get the following relation on the net premium:

$$\ddot{a}^{(p)}_{x:\overline{n|}} = \ddot{a}^{(p)}_{x} - \ddot{a}^{(p)}_{x+n} \cdot A_{x:\overline{n|}}^{\;\;\;1}$$

$$= \ddot{a}_{x} - \frac{p-1}{2p} - A_{x:\overline{n|}}^{\;\;\;1} \cdot \left(\ddot{a}_{x+n} - \frac{p-1}{2p} \right) =$$

$$= \ddot{a}_{x} - A_{x:\overline{n|}}^{\;\;\;1} \cdot \ddot{a}_{x+n} - \frac{p-1}{2p} + A_{x:\overline{n|}}^{\;\;\;1} \cdot \frac{p-1}{2p} =$$

$$= \ddot{a}_{x} - {}_{n}\ddot{a}_{x+n} - \frac{p-1}{2p} \cdot \left(1 - A_{x:\overline{n|}}^{\;\;\;1}\right) =$$

$$= \ddot{a}_{x:\overline{n|}} - \frac{p-1}{2p} \cdot \left(1 - A_{x:\overline{n|}}^{\;\;\;1}\right), \tag{10.72.}$$

so

$$\ddot{a}^{(p)}_{x:\overline{n|}} = \ddot{a}_{x:\overline{n|}} - \frac{p-1}{2p} \cdot \left(1 - A_{x:\overline{n|}}^{\;\;\;1}\right). \tag{10.73.}$$

Based on the above pattern the version of instalment payments within the year can be derived in case of other types of annuities, too.

And now let’s look at a more precise approach concretized for the most common value, 12. As we have already stated, the l_x-s within the year are constructed by interpolation:

$$l_{x+\frac{k}{12}} = \left(1 - \frac{k}{12}\right) \cdot l_x + \frac{k}{12} \cdot l_{x+1}. \tag{10.74.}$$

For the sake of precision we are using compound interest within the year.

Then the expected present value of the 12 number of annuity payments in the year *t* for the starting l_x lives at the beginning of year t is:

$$\begin{aligned} E^{(12)}_{x+t} &= \frac{1}{12} \cdot \sum_{k=0}^{11} v^{\frac{k}{12}} \cdot l_{x+t+\frac{k}{12}} = \\ &= \frac{1}{12} \cdot \sum_{k=0}^{11} v^{\frac{k}{12}} \cdot \left(\left(1 - \frac{k}{12}\right) \cdot l_{x+t} + \frac{k}{12} \cdot l_{x+t+1} \right) = \\ &= l_{x+t} \cdot \frac{1}{12} \cdot \sum_{k=0}^{11} v^{\frac{k}{12}} \cdot \left(1 - \frac{k}{12}\right) + l_{x+t+1} \cdot \frac{1}{12} \cdot \sum_{k=0}^{11} v^{\frac{k}{12}} \cdot \frac{k}{12} \end{aligned} \tag{10.75.}$$

If we introduce the following notations (that are used in this sense only here):

$$A = \frac{1}{12} \cdot \sum_{k=0}^{11} v^{\frac{k}{12}} \cdot \left(1 - \frac{k}{12}\right) \text{ and} \tag{10.76.}$$

$$B = \frac{1}{12} \cdot \sum_{k=0}^{11} v^{\frac{k}{12}} \cdot \frac{k}{12}, \tag{10.77.}$$

then the equivalence equation is:

$$\begin{aligned} l_x \cdot \ddot{a}_{x:\overline{n}|}^{(12)} &= \sum_{t=0}^{n-1} E_{x+t}^{(12)} \cdot v^t = \\ &= \sum_{t=0}^{n-1} \left(A \cdot l_{x+t} + B \cdot l_{x+t+1}\right) \cdot v^t = \\ &= A \cdot \sum_{t=0}^{n-1} l_{x+t} \cdot v^t + B \cdot \sum_{t=0}^{n-1} l_{x+t+1} \cdot v^t \end{aligned} \tag{10.78.}$$

Since:

$$\frac{\sum_{t=0}^{n-1} l_{x+t} \cdot v^t}{l_x} = \ddot{a}_{x:\overline{n}|} \text{ and} \tag{10.79.}$$

$$\begin{aligned} \frac{\sum_{t=0}^{n-1} l_{x+t+1} \cdot v^t}{l_x} &= \frac{\sum_{t=0}^{n-1} l_{x+t+1} \cdot v^t}{l_{x+1}} \cdot \frac{l_{x+1} \cdot v}{l_x} \cdot (1+i) = \\ &= \ddot{a}_{x+1:\overline{n}|} \cdot A_{x:1}^{\ \ 1} \cdot (1+i) \end{aligned} \tag{10.80.}$$

this way

$$\ddot{a}_{x:\overline{n}|}^{(12)} = A \cdot \ddot{a}_{x:\overline{n}|} + B \cdot \ddot{a}_{x+1:\overline{n}|} \cdot A_{x:\overline{1}|}^{\ \ 1} \cdot (1+i) \tag{10.81.}$$

The same for the lifetime annuity is:

$$\begin{aligned} l_x \cdot \ddot{a}_x^{(12)} &= \sum_{t=0}^{\omega-x} E_{x+t}^{(12)} \cdot v^t = \sum_{t=0}^{\omega-x} \left(A \cdot l_{x+t} + B \cdot l_{x+t+1}\right) \cdot v^t = \\ &= A \cdot \sum_{t=0}^{\omega-x} l_{x+t} \cdot v^t + B \cdot \sum_{t=0}^{\omega-x} l_{x+t+1} \cdot v^t \end{aligned} \tag{10.82.}$$

where according to 10.32.:

$$\frac{\sum_{t=0}^{\omega-x} l_{x+t} \cdot v^t}{l_x} = \ddot{a}_x \text{ and} \tag{10.83.}$$

$$\begin{aligned} \frac{\sum_{t=0}^{\omega-x} l_{x+t+1} \cdot v^t}{l_x} &= \frac{\sum_{t=0}^{\omega-x} l_{x+t+1} \cdot v^t}{l_{x+1}} \cdot \frac{l_{x+1} \cdot v}{l_x} \cdot (1+i) = \\ &= \ddot{a}_{x+1} \cdot A_{x:\overline{1}|}^{\ \ 1} \cdot (1+i) \end{aligned} \tag{10.84.}$$

So:

$$\ddot{a}_x^{(12)} = A \cdot \ddot{a}_x + B \cdot \ddot{a}_{x+1} \cdot A_{x:\overline{1}|}^{\ \ 1} \cdot (1+i). \tag{10.85.}$$

10.1.3.8. Some Special Annuities

Earlier we have discussed “pension insurance”, “widow’s/widower’s annuity” and “orphan’s annuity”. We have also given a few examples of these in the sub-section on annuities. Now let’s look at what the premium formula of the annuities given as examples of “widow’s/widower’s annuity” would be. In these formulae we will use the premiums deducted in the earlier sub-section.

One of the examples of “widow’s/widower’s annuity” is a symmetric two person annuity. Here the annuity of 1 Forint yearly payment is paid only after the death of the first insured, to the other insured as beneficiary, until the death of this second insured. If this is a single premium product, then the premium can be determined the following way:

$$\ddot{a}'_{xy} = \ddot{a}_x + \ddot{a}_y - 2 \cdot \ddot{a}_{xy} \qquad (10.86.)$$

As we know, $\ddot{a}_{xy}$ denotes the two person annuity of 10.58., which pays 1 Forint to the two insured persons until the first death. The meaning of the above formula is that both insured persons receive 1 Forint from the insurer until they are alive, so during the period in which both are alive, together they receive 2 Forints. But during the time when they are both alive, they pay these 2 Forints back to the insurer, in other words they do not receive anything. Immediately as one insured dies, his 1 Forint payment stops, and the second insured, who is still alive doesn’t have to pay back his 1 Forint to the insurer any more, so from this point on he receives net 1 Forint until his death.

If we want to generalise the above two person annuity and suppose that the insured receive *C* Forints together, and after the death of the other, the insured of age *x* receives *A* Forints, and the insured of age *y* receives *B* Forints, then we get the following formula:

$$\ddot{a}'_{xy} = A \cdot \ddot{a}_x + B \cdot \ddot{a}_y - (A + B - C) \cdot \ddot{a}_{xy}. \qquad (10.87.)$$

The other example is an asymmetric two person annuity. Let the primary insured (whose death “widows” the other insured) be *x* years old, and the secondary insured (the possible widow/widower) be *y* years old. This is a single premium construction. After the death of the primary insured the secondary insured receives 1 Forint yearly annuity until alive, if the secondary insured is alive when first insured dies. If the secondary insured dies before the death of the first insured, the insurance is terminated without any benefit payment. The single premium of this insurance is:

$$\ddot{a}''_{xy} = \ddot{a}_y - \ddot{a}_{xy} \qquad (10.88.)$$

The meaning of the formula is: the secondary insured receives a yearly 1 Forint annuity starting from the commencement of the insurance, but until the primary insured is also alive (i.e. both are alive), they pay this 1 Forint yearly annuity back to the insurer.

Naturally a number of other special annuity types can be imagined beside the above discussed ones.

10.1.3.9. An Annuity Type of Technical Significance

As we have indicated, many types of annuities can be imagined beside the ones introduced here. E.g. all of the above discussed annuities shared the property that the annuity payment was constant in time. But we can very easily imagine annuities with variable payment. In inflationary times it is a natural requirement that the annuity should preserve its real value – at least to a certain degree. The technique of inflation handling doesn’t necessarily take us out of the range of level payment annuities, as we will see in the section dealing with profit sharing. Inflation runs a path that is not known beforehand, so if the annuity is increased due to inflation, we cannot tell beforehand what the payment will be from year to year.

We can very easily construct annuities where the yearly payment increases each year according to a certain rule, e.g. increases or decreases. From these we will only discuss one, the immediate annuity with yearly payments in advance, where the first payment is 1 Forint, the second is (1+*k*), the third (1+2*k*), and so on… These usually serve purely technical purposes, because we can use them to design other types of insurances.

Let $(\text{Iä})_x$ denote the single premium of the above described increasing annuity in case of an insured of *x* years. (The *I* in front of the *ä* notation refers to the term "Increasing".)

In the above discussion we have not yet made use of the discovery that an annuity is a series of pure endowment insurances, where the term of the individual pure endowment insurances is increasing. E.g. the immediate annuity paid in advance is the sum of pure endowment insurance with term 0 years, 1 year, 2 years, etc… It can easily be seen – and so we leave the proof to the reader – that

$$\text{ä}_x = A_{x:\overset{1}{\overline{0|}}} + A_{x:\overset{1}{\overline{1|}}} + A_{x:\overset{1}{\overline{2|}}} + \ldots \,. \tag{10.89.}$$

We can use this train of thought to calculate *(Iä)ₓ*. Here we basically take a pure endowment insurance of 1 Forint sum assured and term 0 years, *(1+k)* Forints sum assured and 1 year term, *(1+2k)* Forints and 2 years, etc…[123], so

$$(\text{Iä})_x = 1 \cdot A_{x:\overset{1}{\overline{0|}}} + (1+k) \cdot A_{x:\overset{1}{\overline{1|}}} + (1+2k) \cdot A_{x:\overset{1}{\overline{2|}}} + \ldots \,. \tag{10.90.}$$

Using commutation numbers we can simplify the above complicated expression:

$$\text{Iä}_x = \frac{D_x + (1+k) \cdot D_{x+1} + (1+2k) \cdot D_{x+2} + \ldots}{D_x} =$$

$$\text{Iä}_x = \frac{N_x + k \cdot N_{x+1} + k \cdot N_{x+2} + \ldots}{D_x} = \tag{10.91.}$$

And now we introduce another – third degree – commutation number, S_x with the definition:

$$S_x = N_x + N_{x+1} + N_{x+2} + \ldots \,. \tag{10.92.}$$

This way

$$(\text{Iä})_x = \frac{(1-k) \cdot N_x + k \cdot S_x}{D_x} \,. \tag{10.93.}$$

10.1.4. Joint Life Single Premium Insurance

Almost all types of insurances have a two or more person version, as we have seen in case of annuities. Now, and in the further discussion we will only deal with two person insurances, and within these only the term insurance and the pure endowment insurance. We are on the opinion that, on the one hand the relations of the other two person insurances – if necessary – can be derived in an analogous way from the relations of single life insurances, and on the other hand from the practical point of view the relations of single life insurances are much more important, since currently mostly these dominate the market.

In case of two lives we simply regard as death the death of either of the two insured persons (i.e. the first death), and as living until maturity if both insured persons are alive at the end of the term.

In case of the **two person term insurance** we try to think the following way: if all possible couples of ages *x* and *y* take out the policy of *n* years term and 1 Forint sum assured, then the expected value of benefits paid by the insurer yearly will be:

[123] k=1 is the most simple model.

$$(l_x \cdot l_y - l_{x+1} \cdot l_{y+1});(l_{x+1} \cdot l_{y+1} - l_{x+2} \cdot l_{y+2});...;(l_{x+n-1} \cdot l_{y+n-1} - l_{x+n} \cdot l_{y+n}) .$$

(10.94.)

If we denote the single premium in question by $A_{\overset{1}{xy}:\overline{n|}}$, then the equivalence equation is:

$$l_x \cdot l_y \cdot A_{\overset{1}{xy}:\overline{n|}} = v^1 \cdot (l_x \cdot l_y - l_{x+1} \cdot l_{y+1}) +$$
$$+ v^2 \cdot (l_{x+1} \cdot l_{y+1} - l_{x+2} \cdot l_{y+2}) + ... +$$
$$+ v^n \cdot (l_{x+n-1} \cdot l_{y+n-1} - l_{x+n} \cdot l_{y+n}) . \qquad (10.95.)$$

Which gives:

$$A_{\overset{1}{xy}:\overline{n|}} = \frac{v^1 \cdot (l_x \cdot l_y - l_{x+1} \cdot l_{y+1}) + v^2 \cdot (l_{x+1} \cdot l_{y+1} - l_{x+2} \cdot l_{y+2}) + ... + v^n \cdot (l_{x+n+1} \cdot l_{y+n+1} - l_{x+n} \cdot l_{y+n})}{l_x \cdot l_y} .$$

(10.96.)

Multiplying by v^x, and using the two person commutation numbers D_{xy} and N_{xy} introduced earlier we get:

$$D_{xy} \cdot A_{\overset{1}{x}:\overline{n|}} = v^1 \cdot (D_{xy} + D_{(x+1)(y+1)} + ... + D_{(x+n-1)(y+n-1)}) -$$

$$-(D_{(x+1)(y+1)} + D_{(x+2)(y+2)} + ... + D_{(x+n)(y+n)}) =$$

$$= v \cdot (N_{xy} - N_{(x+n)(y+n)}) - (N_{(x+1)(y+1)} - N_{(x+n+1)(y+n+1)}) .$$

(10.97.)

From this:

$$A_{\overset{1}{xy}:\overline{n|}} = \frac{v^1 \cdot (N_{xy} - N_{(x+n)(y+n)})}{D_{xy}} - \frac{(N_{(x+1)(y+1)} - N_{(x+n+1)(y+n+1)})}{D_{xy}} = v \cdot \ddot{a}_{xy:\overline{n|}} - {}_1\ddot{a}_{xy:\overline{n+1|}} . \qquad (10.98.)$$

The equivalence equation of the **two person pure endowment insurance** is:

$$l_x \cdot l_y \cdot A_{xy:\overset{1}{\overline{n|}}} = v^n \cdot l_{x+n} \cdot l_{y+n} \qquad (10.99.)$$

So:

$$A_{xy:\overset{1}{\overline{n|}}} = \frac{v^n \cdot l_{x+n} \cdot l_{y+n}}{l_x \cdot l_y} \qquad (10.100.)$$

Using commutation numbers:

$$A_{xy:\overset{1}{\overline{n|}}} = \frac{v^n \cdot l_{x+n} \cdot l_{y+n}}{l_x \cdot l_y} = \frac{D_{(x+n)(y+n)}}{D_{xy}} \tag{10.101.}$$

10.1.5. The Premium Formula of the Generalised Traditional Life Insurance

It is useful to summarize all of the formulae deducted so far in one single formula, the formula of the generalised traditional insurance. To do this, it is necessary to advance the formula of the gross premium that will be discussed later on:

$$P^b_{x:\overline{n|}} = (1+\lambda_{x:n}) \cdot P_{x:\overline{n|}} \tag{10.102.}$$

The generalised premium formula itself is

$$A_{x:\overline{n|}} = SAD \cdot A^1_{x:\overline{n|}} + SAM \cdot A_{x:\overline{n|}}^{\;\;1} + SAT \cdot A_n + SAP \cdot P^b_{x:\overline{n|}} \cdot \frac{R_x - R_{x+n} - n \cdot M_{x+n}}{D_x} + AA \cdot \ddot{a}_{x:\overline{n|}} \tag{10.103.}$$

Where the following notation is used:
SAD: sum assured death
SAM: pure endowment sum assured (sum assured maturity)
SAT: maturity sum assured independent of the insured being alive (sum assured term fix)
SAP: the multiple of paid gross premiums refunded at death (sum assured premium)
AA: yearly annuity payment (annual annuity)

By choosing the right values of the parameters, e.g. the following classical life insurances can be constructed:
Term insurance: SAD = 1, SAM = 0, SAT = 0, SAP = 0, AA = 0,
Endowment insurance: SAD = 1, SAM = 1, SAT = 0, SAP = 0, AA = 0,
Term fix insurance[124]: SAD = 0, SAM = 0, SAT = 1, SAP = 0, AA = 0,
Pure endowment with premium refund: SAD = 0, SAM = 1, SAT = 0, SAP = 1, SAA = 0,
Temporary annuity: SAD = 0, SAM = 0, SAT = 0, SAP = 0, AA = 1,

10.2. The Premium of Regular Premium Payment Insurance

In case of the regular premium insurance the premium is paid not all at once, at the beginning of the term, but spread through the whole insurance term, in instalments. Most life insurances have both a single premium and a regular premium version (an exception is e.g. the immediate annuity, which doesn't have a regular premium version[125], and the term fix insurance, where the single premium version would be problematic[126]).

To keep the matter simple we suppose that the single premium is paid at the beginning of each insurance year, in equal payments, except in the sub-chapter discussing the specialities of premium frequencies other than annual. This annual premium is derived from the single premium of the insurance with the same parameters, but the single premium version. Obviously this cannot be done simply by dividing the single premium with the number of years in the term to get the annual premium. This has two causes:

[124] Term fix insurance can be constructed also as an endowment insurance with varying death sum assured, but then we cannot use the classical formula of the term insurance, so – for the sake of simplicity – I have introduced the SAT sum and the formula of the single premium term fix, although this is redundancy to a certain degree.

[125] Although even this can be imagined if the policyholder and the insured are not the same. Then the policyholder pays the premium in instalments while the insured already receives annuity payments. This construction was used e.g. in Hungary in the 1990s, when the disability pension liability of closed mines was transferred to insurers. On the other hand, these are usually individual annuity constructions, such products are not developed, because people typically take out annuities for themselves. Of course a greater market demand can be imagined, in which case these products would appear!

[126] We will discuss the term fix insurance under 10.2.2.

Compared to the single premium insurance, the insurer suffers **interest loss** in case of the regular premium insurance, since the greater part of the premium is received only years later, and until then the insurer doesn't earn interest after these parts.

The insurer receives the total single premium. But in case of the regular premium the insurer cannot be totally certain about receiving all of the premium payments, because if the **insured dies during the term**, then further premium payment ceases.

Due to these causes the annual premium will be higher than the single premium divided by the number of years in the term.

As we have indicated, the period of premium payment can be equal to the insurance term, but it can also be shorter. If *m* denotes the number of years of premium payment, and the usual *n* denotes the insurance term, then it is always true that:

$$m \leq n$$

In order to be able to derive the annual premium, we have to realise that the annual premium is technically the same as a temporary annuity paid in advance, where the annuity payment is the annual premium, and the term of the annuity equals the premium payment term, i.e. the term equals *m*, only the annuity payment is not paid by the insurer to the insured, but vice versa. This last circumstance does not influence the value of the annuity.

Starting out from the above consideration, the equivalence equation can be written the following way in case of all regular premium insurances, if the annual premium is P:

$$\ddot{a}_{x:\overline{m}|} \cdot P = A \tag{10.104.}$$

since the expected income of the insurer is exactly the same as the value of the annuity paid to the insurer from the client, i.e. it is $\ddot{a}_{x:\overline{m}|} \cdot P$. The expected value of payout is the same as in case of the single premium insurance, since the single and regular payment versions do not differ in this respect. This way:

$$P = \frac{A}{\ddot{a}_{x:\overline{m}|}} \tag{10.105.}$$

Let's apply this general relation to concrete insurances! We are on the opinion that having derived the formulae of the single premiums now it is enough in most cases to just write the concrete formulae.

10.2.1. The Regular Premium of the Term, Pure Endowment and Endowment Insurance

The regular premium of the **term insurance** is:

$$P^{\,1}_{x:\overline{m}|} = \frac{A^{\,1}_{x:\overline{n}|}}{\ddot{a}_{x:\overline{m}|}} \tag{10.106.}$$

Using commutation numbers:

$$P^{\,1}_{x:\overline{m}|} = \frac{A^{\,1}_{x:\overline{n}|}}{\ddot{a}_{x:\overline{m}|}} = \frac{\dfrac{M_x - M_{x+n}}{D_x}}{\dfrac{N_x - N_{x+m}}{D_x}} = \frac{M_x - M_{x+n}}{N_x - N_{x+m}} \tag{10.107.}$$

The regular premium of the **pure endowment insurance** is:

$$P_{x:\overline{m}|}^{\ 1} = \frac{A_{x:\overline{n}|}^{\ 1}}{\ddot{a}_{x:\overline{m}|}} \tag{10.108.}$$

Using commutation numbers:

$$P_{x:\overline{m}|}^{\ 1} = \frac{A_{x:\overline{n}|}^{\ 1}}{\ddot{a}_{x:\overline{m}|}} = \frac{\dfrac{D_{x+n}}{D_x}}{\dfrac{N_x - N_{x+m}}{D_x}} = \frac{D_{x+n}}{N_x - N_{x+m}} \tag{10.109.}$$

The regular premium of the **endowment insurance** is:

$$P_{x:\overline{m}|} = h \cdot P^{1}_{x:\overline{m}|} + P_{x:\overline{m}|}^{\ 1} = \frac{h \cdot A^{1}_{x:\overline{n}|}}{\ddot{a}_{x:\overline{m}|}} + \frac{A_{x:\overline{n}|}^{\ 1}}{\ddot{a}_{x:\overline{m}|}} \tag{10.110.}$$

Using commutation numbers:

$$P_{x:\overline{m}|} = h \cdot \frac{M_x - M_{x+n}}{N_x - N_{x+m}} + \frac{D_{x+n}}{N_x - N_{x+m}} = \frac{h \cdot (M_x - M_{x+n}) + D_{x+n}}{N_x - N_{x+m}} \tag{10.111.}$$

10.2.2. The Premium of the Term Fix Insurance

In case of the term fix (or “à terme fix” after the French name) the insurer pays the sum assured at the end of the term under all circumstances, regardless of whether the insured is alive or dead. But premiums are paid until maturity, or until the death of the insured, if this happens earlier. From this we can see why the earlier remark was made concerning the term fix insurance, which stated that the single premium term fix insurance would be problematic, and why it wasn’t discussed with other single premium insurances. The single premium term fix wouldn’t have any random element: the insured pays the whole premium at the beginning of the term, and the beneficiary receives the sum assured at the end of the term. The premium of the single premium term fix insurance would be the following:

$$A_n = v^n \tag{10.112.}$$

so we see that it doesn’t depend on the age of the insured, the insurance works simply as a deposit. But we can use this “single premium“ in the calculation of the regular premium version:

$$P_{x:\overline{m}|} = \frac{A_n}{\ddot{a}_{x:\overline{m}|}} = \frac{v^n}{\ddot{a}_{x:\overline{m}|}} \tag{10.113.}$$

Using commutation numbers:

$$P_{x:\overline{m}|} = \frac{A_n}{\ddot{a}_{x:\overline{m}|}} = \frac{v^n}{\dfrac{N_x - N_{x+m}}{D_x}} = \frac{v^n \cdot D_x}{N_x - N_{x+m}} \tag{10.114.}$$

10.2.3. The Premium of Regular Premium Annuities

We can imagine many kinds of regular premium annuities. We won’t discuss all possible types, only one (the one that appears most often).

Regular premium annuities – as we have already mentioned – can only be imagined in case of deferred annuities, or two person annuities, where the annuity is payable starting from the death of one of the insured persons. (Since the immediate regular premium annuity – disregarding a few special cases – is meaningless.)

The deferred annuity can have a version, where the policyholder accumulates during the deferment period the capital value of the annuity commencing thereafter. The annual premium payment in this case is:

$$P_{x:\overline{m}|} = \frac{{}_m\ddot{a}_x}{\ddot{a}_{x:\overline{m}|}} \tag{10.115.}$$

Using commutation numbers:

$$P_{x:\overline{m}|} = \frac{{}_m\ddot{a}_x}{\ddot{a}_{x:\overline{m}|}} = \frac{\dfrac{N_{x+m}}{D_x}}{\dfrac{N_x - N_{x+m}}{D_x}} = \frac{N_{x+m}}{N_x - N_{x+m}} \tag{10.116.}$$

With the aid of commutation numbers this formula can be transformed into the following form:

$$P_{x:\overline{m}|} = \frac{{}_m\ddot{a}_x}{\ddot{a}_{x:\overline{m}|}} = \frac{\dfrac{N_{x+m}}{D_{x+m}} \cdot \dfrac{D_{x+m}}{D_x}}{\ddot{a}_{x:\overline{m}|}} = \ddot{a}_{x+m} \cdot P_{x:\overset{1}{\overline{m}|}} \tag{10.117.}$$

So – as we see – the annual premium of this insurance is the same as the annual premium of a regular premium pure endowment insurance with $\ddot{a}_{x+m}$ sum assured. In other words, this insurance works like a regular premium pure endowment insurance during the deferred phase, and at maturity the policyholder purchases from the received sum assured an immediate annuity.

10.2.4. The Premium of Joint Life Regular Premium Insurances

The annual net premium of the two person term assurance obviously is:

$$P_{\overset{1}{xy}:\overline{n}|} = \frac{A_{\overset{1}{xy}:\overline{n}|}}{\ddot{a}_{xy:\overline{n}|}} \tag{10.118.}$$

And the annual net premium of the two person pure endowment insurance is:

$$P_{xy:\overset{1}{\overline{n}|}} = \frac{A_{xy:\overset{1}{\overline{n}|}}}{\ddot{a}_{xy:\overline{n}|}} \tag{10.119.}$$

10.3. Calculation of Gross Premiums

The above calculated premiums were all net premiums. These cover only the liabilities undertaken by the insurer in the insurance policy. But the insurer also has expenses related to the insurance, and would like to achieve a certain profit through this activity. This way clients have to pay a gross premium for the insurance that is higher than the net premiums calculated so far.

The gross premium is traditionally calculated from the net premium by adding a loading factor. The loading factor itself is calculated from data concerning the expenses. Expenses related to the insurance can be divided into three groups:

α - expenses: expenses related to the signature of the policy, above all the commission and the expenses of underwriting (e.g. medical examination). This expense part is usually regarded as proportional to the gross premium of the insurance in case of (single premium) annuities, but as proportional to the sum assured in case of the other types of insurance. This factor is denoted by α.

β - expenses: expenses related to the collection of the premium belong to this group. It is usually considered as proportional to the gross premium, and the factor is denoted by β. These expenses arise during the premium term (usually *n*, but if it is different, then *m*) in case of regular premium insurances.

γ - expenses: other expenses of the insured related to the insurance (e.g. wages, maintenance expenses, data-processing expenses, etc…). This factor is considered proportional to the (sum assured, single net premium of the (temporary) annuity) multiple. The factor is denoted by γ. These expenses are due throughout the whole insurance term in case of regular premium insurances.

In case of the α-expenses the basis of defining the ratio as percentage of the sum assured is that the commission is usually also defined as a percentage of the sum assured, and this is the largest of the α expenses. The purpose of defining the commission as a percentage of the sum assured principally is that it works for the agents as an incentive of longer term insurances (that are more favourable to the insurer), since with the same annual premium the client can purchase a policy of higher sum assured if the term is longer, so the agent receives higher commission. On the other hand, the commission is not always defined as a percentage of the sum assured, but also as a percentage of the premium, but then the commission rate depends on the insurance term. In these cases it is more complicated to calculate the α factor. Here we will deal with the traditional case.

Let's look at the single premium insurances first!

10.3.1. The Gross Premium of Single Premium Insurances

If the gross premium is denoted by $A^b_{x:n}$, then based on the above formula:

$$A^b_{x:\overline{n}|} = A_{x:\overline{n}|} + \alpha + \beta \cdot A^b_{x:\overline{n}|} + \gamma \cdot \ddot{a}_{x:\overline{n}|} \qquad (10.120.)$$

So:

$$A^b{}_{x:\overline{n}|} = \frac{A_{x:\overline{n}|} + \alpha + \gamma \cdot \ddot{a}_{x:\overline{n}|}}{1 - \beta} \qquad (10.121.)$$

10.3.2. The Gross Premium of Regular Premium Insurances

If the annual gross premium is denoted by $P^b{}_{x:\overline{m}|}$, then the corresponding equations are:

$$P^b{}_{x:\overline{m}|} \cdot \ddot{a}_{x:\overline{m}|} = A_{x:\overline{n}|} + \alpha + \beta \cdot P^b{}_{x:\overline{m}|} \cdot \ddot{a}_{x:\overline{m}|} + \gamma \cdot \ddot{a}_{x:\overline{n}|} \qquad (10.122.)$$

So:

$$P^b{}_{x:\overline{m}|} = \frac{A_{x:\overline{n}|} + \alpha + \gamma \cdot \ddot{a}_{x:\overline{n}|}}{(1-\beta) \cdot \ddot{a}_{x:\overline{m}|}} \qquad (10.123.)$$

10.3.3. The Difference Between Premiums Calculated for Annual and Monthly Premium Payment

Insurers generally calculate premiums for annual payment, which means that they use the above introduced formulae. But in Hungary in most policies the two parties agree in a more frequent premium payment mode, the most common is the monthly payment. This means that the insurer gives a discount to the client, since the monthly frequency is less favourable to the insurer compared to the annual. This discount given to the client naturally has to be compensated in the premium paid by the client. Since the insurer calculates annual premium payment, i.e. supposes that if the insured is still alive, then the premium is received at the beginning of the insurance year all at once. If the insurer gives a discount, than it will have two deficiencies that have to be compensated:

1. Interest- and expense loss, since the greater part of the premium is not received at the beginning of the year, but continuously throughout the year, and premium collection has to be performed 12 times a year, not only once, so its expenses will be higher. Insurers usually try to compensate this loss by raising the 1/12th of the annual premium by a few percentages in case of monthly payment allowance. This increase depends on the current interest rates, since these indicate the degree of interest loss that the insurer suffers. (A practice also exists where the insurer increases the 1/12th part of the annual premium by a fix sum plus a few percentages of the premium, since the expense increase due to the more frequent premium collection doesn't depend on the interest rate.)
2. The premium loss due to mid-year deaths. This arises because in case of annual premium payment the insurer receives the total premium of the year of death, but in case of monthly payment allowance the monthly premiums following the date of death, that are due in that insurance year are not received. The usual solution to this problem is that the insurer stipulates in the policy terms and conditions that the policyholder is obliged to pay to the insurer the total annual premium due in the year of death, or if it is not paid, the insurer subtracts the remaining premium payments from the death benefit.[127]

It is also possible practice that the insurer calculates monthly premiums from the beginning. In these cases the formulae of regular premiums only differ from formulae determined above, that the single premium has to be divided by $\ddot{a}^{(12)}{}_{x:\overline{m}|}$ instead of $\ddot{a}_{x:\overline{m}|}$.

So instead of the above

$$P_{x:\overline{m}|} = \frac{A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{m}|}} \tag{10.124.}$$

annual premium we use

$$P^{(12)}{}_{x:\overline{m}|} = \frac{A_{x:\overline{n}|}}{\ddot{a}^{(12)}{}_{x:\overline{m}|}} = \frac{A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{m}|} - \frac{11}{24}\cdot(1 - A_{x:\overline{m}|}^{\;\;1})}, \tag{10.125.}$$

the so-called "annualised" monthly premium, and 1/12th of this will be the actual monthly premium. (Of course here we can also use the more precise monthly annuity formula in the calculation!) As we see, the "annualised" monthly premium is higher than the annual premium, because the right hand sides of the above two equations only differ in that the denominator of 10.125. is somewhat lower than that of 10.124.

In case of monthly calculation some things change compared to the annual calculation. Here the annual premium payment (or the payment less frequent than monthly) will be the

[127] Nowadays insurers often refrain from this, but then excess loss of this kind has to be included in the so-called "frequency loading" as the losses under point 1.

exception. If the client undertakes the annual premium payment, then he chooses conditions more favourable to the insurer, that the insurer awards with premium discount. The premium discount has the same two sources as previously the premium loading:

1. In case of annual premium payment the insurer receives the greater part of the annual premium earlier than calculated, so it earns interest on the account of the insurer until the premium payment is due according to the calculation. This interest can be returned to the policyholder in the form of premium discount. Naturally the level of this discount also depends on the current interest rates.
2. If the client chooses annual premium payment and dies during the year, then principally he “overpaid” in that year, since according to the calculation, premium payment is only due until the moth of death, and not for the months remaining from the insurance year. This part of the premium, that has arrived to the insurer, but that the insurer is principally not yet entitled to use, the so-called “unearned premium” is to be paid back to the client.

Due to the above findings we can make interesting statements about the movement of annual and monthly calculated premiums compared to each other as the interest rate changes. In case of annually calculated premiums the higher the interest rate, the higher the factor defined as a percentage will be that shows how much higher the monthly premium is than $1/12^{th}$ of the annual premium. In case of monthly calculation the discount given to the client from 12 times the monthly premium for choosing annual payment frequency will be greater and greater. So if we take the difference between the monthly premiums in the two calculation versions, then the higher the interest rate, the greater this difference will be, and vice versa.

What we have stated about the monthly premium – annual premium relation naturally holds – *mutatis mutandis* – for the semi-annual premium – annual premium, quarterly premium – annual premium, etc. relations, too.

An alternative of the above loading and discount based solutions could principally be that the insurer declares separate premium rates for annual, semi-annual, quarterly and monthly premium payment, this way neither the loading, neither the discount is necessary. Of course this has the disadvantage that the tariff book will be much thicker, and the client won’t be able to see through the advantages and disadvantages of the different premium frequencies, this way this method is rarely used by insurers in the practice.

11. THE PREMIUM RESERVE

KEW WORDS

Policy anniversary
Paying up the policy
Premium reserve
Policy loan
Non-forfeiture options
Surrender
Zillmerization

Without any explanation, we have already used the term premium reserve several times. From these references, but also from different reports appearing in the press, it is obvious that this is a term of cardinal importance. The size of life insurance companies is characterized – beside premium income figures – by the size of the premium reserve. It can easily happen that the annual premium income of an insurance company and the premium reserve that it handles differ by several orders of magnitude to the advantage of the premium reserve. But what exactly is this premium reserve?

The premium reserve – referred to as mathematical reserve in the official terminology – is the sum of money accumulated by the insurer from the premiums paid by the members of the risk community to cover later benefits paid to the members of the risk community who have suffered loss. The premium reserve is primarily interpreted on the whole risk community, but because of practical purposes it is accounted on individual policies, and so the total premium reserve is the sum of the premium reserves of all individual policies. This is also due to the fact that the premium reserve is created from individual payments. Summarizing, we can say that the premium reserve is the money of the risk community, on which the insurer promised later benefits, but not yet provided any. The individual insured (policyholders) as members of the risk community can be regarded as owners of proportional parts of the premium reserve, but only to the extent that it doesn't interfere with the functioning of the risk community.

The premium reserve is the cause – as we have already referred to it – of dividing insurances into the branches of life and non-life, since in case of life insurance it is typical that the insurer collects (parts of) the premium paid by the client for decades to cover the later benefits. When we were discounting the above formulae, we have tacitly supposed that the decrease in premium payment due to premiums decreased by discounting is compensated by the investment yields of the insurer. But what does the insurer invest? Naturally the premium reserve. The premium reserve cannot be the summarized and compounded value of all premiums paid by the client, since the insurer also has to cover from the received premiums its expenses and the benefits paid during the term. So the premium reserve – although somewhat similar – is more complex than a bank deposit. So let's examine the premium reserve in detail! Since the premium reserve behaves in a significantly different way in case of term insurance and pure endowment insurance, we will discuss these separately in the following.

11.1. THE PREMIUM RESERVE OF THE TERM INSURANCE

Let's take an example!

We suppose that 1,000 persons take out a term insurance of 1 Forint sum assured and n years term at the same time. All are men, x years old and pay the premium annually. How do we determine the necessary premium?

The most simple case would be if everyone would pay q_{x+t} every year (t = 0, 1, …, n-1), where q_{x+t} – as we know – is the probability of a man $x+t$ years old dying within one year. But since q_{x+t} – as we also know! – increases as x and t increase, i.e. as a person becomes older the probability of death is higher, so in case of this type of premium construction the premium increases from year to year. This method can be imagined in case of group insurance, where the policy is renewed yearly. But it has been discovered several hundred years ago that in case of individual policies that are several years long, the premium increase following the increase of mortality rate has a very negative psychological effect, so other kinds of premium construction methods were used.[128]

Namely that the insurer requires the client to pay the same premium every year, which means that in the first years of the term the premium is somewhat higher than the risk, at the middle of the term it is the same and finally at the end of the term it is lower, as shown by *figure 11.1*.

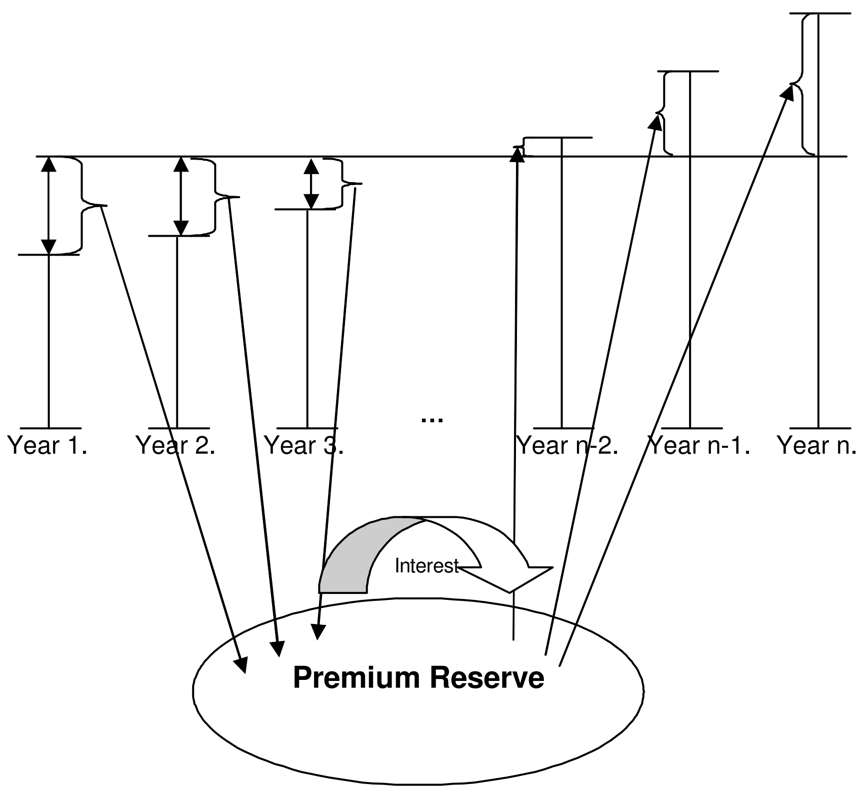

Figure11.1.: The relation of the annual premium needed and the actual premium in case of term insurance

The vertical columns show the value of premiums needed in years 1, 2, … n. The horizontal line shows the value of the level annual premium. The difference of the premium needed and the actual premium paid goes to the premium reserve in the first years of the term, and after that the premium deficiency arising in the later years is gradually supplemented from the reserve.

The insurer accumulates the *excess premium* paid at the beginning of the term, invests it, receives interest for it, and at the end of the term gradually uses it up to cover the premium deficiency calculated for this period. In case of the term insurance this premium accumulated at the beginning of the term is the premium *reserve*. This increases roughly until the middle of the term, and gradually decreases after that, until it becomes zero at exactly the end of the term, as we can see in the following example. The example also shows that the premium reserve of the term insurance never becomes a significant value, and because of this, it generally (at most insurance companies, but not all, and not only because of this) it will not become the base of profit-sharing, neither of non-forfeiture option. (In other words, regular

[128] On the other hand, the yearly renewable term insurance that can be regarded as an element of Unit Linked insurance works this way, so in case of modern insurances this is a possible, although complementary type of construction!

premium term insurance cannot be paid up or surrendered.) We will discuss non-forfeiture options later.

Example: If the client takes out the following term insurance, then the value of the premium reserve on the individual policy anniversaries can be as shown below.

Starting data
Age: 40 years
Insurance term: 20 years
Sum assured: 10,000,000 Forints
Gender: Male
Annual premium: 129,897 Forints

Year	Premium reserve	Year	Premium reserve
0	0	11	484,214
1	68,011	12	483,573
2	132,455	13	473,912
3	193,661	14	454,193
4	251,534	15	422,989
5	305,527	16	377,534
6	354,287	17	315,283
7	396,780	18	233,534
8	431,753	19	129,390
9	458,261	20	0
10	475,794		

Table 11.1.: The premium reserve of a term insurance – before premium payment

The path that the premium reserve of a term insurance runs is shown in figure 11.2. We have also indicated in the figure the effect of premium payment at the beginning of the year, this made the figure "crisscrossed". Normally we only show the premium reserves at anniversaries, before premium payment, which makes the curve more "smooth", as it can be seen on the other figures. We also indicated the level of the annual premium, so the order of the premium reserve can be imagined better (we see that after the first premium payment is the same as the level annual premium, and one year before the end of the term after premium payment it equals exactly the amount needed for the death benefits paid in that year!). The figure also shows what the level of death benefits paid form the premium reserve in each year is in this concrete case.

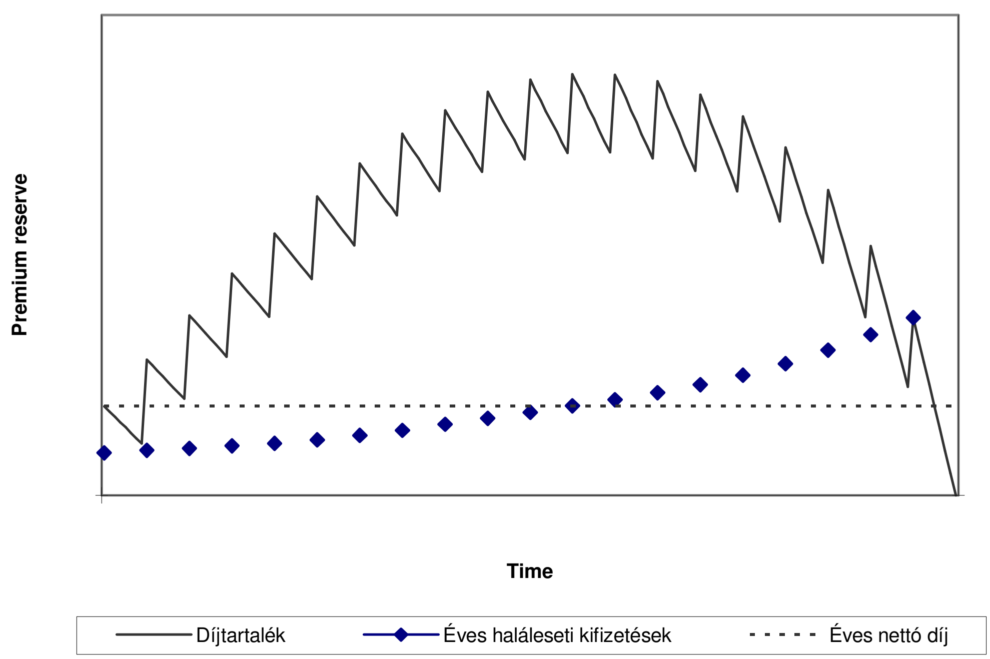

Figure 11.2.: The premium reserve of a term insurance

We should look at a few more things concerning the premium reserve of a term insurance. E.g. – we can declare the following rules concerning its level:

Supposing the same entering age, the longer the term, the higher the maximum of the premium reserve will be, and the closer it will relatively be to the end of the term.

Supposing the same term, the higher the entering age is, the higher the maximum of the premium reserve will be, and the closer it will get to the end of the term.

Both rules have the same cause: mortality rates – as we have seen – increase exponentially with age. This way the gap between the level annual premium and the death premium necessary in the year will increase, which has to be filled by the premium reserve. The difference is especially great at the end of the term, and increases with age exactly here, so a longer part of the term has to accumulate for the “premium deficiency” of the last years.

The above relations are shown on figures 11.3. and 11.4.:

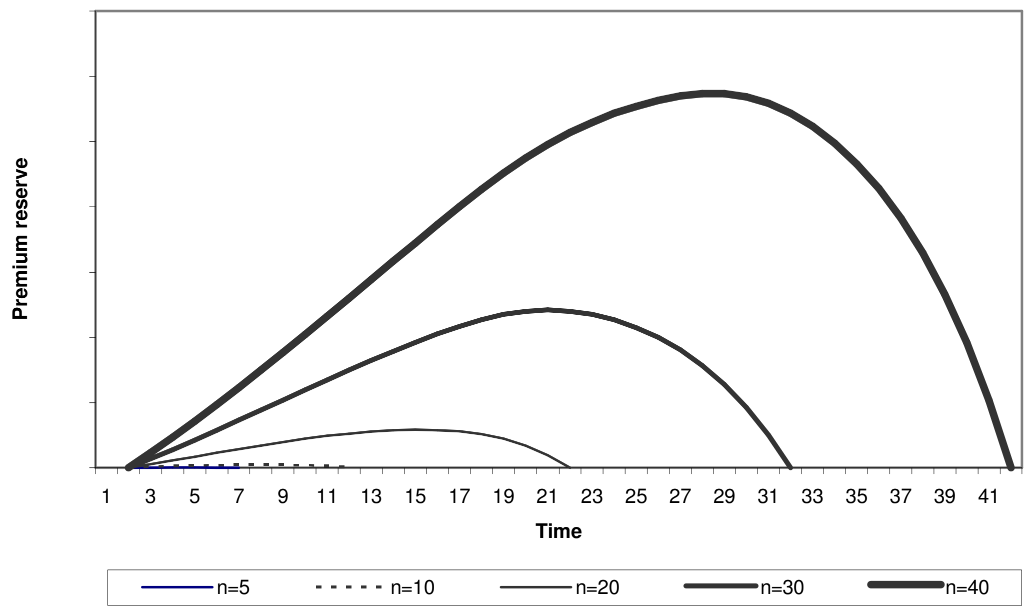

Figure 11.3.: The premium reserve of term insurance with the same entering age and different insurance terms

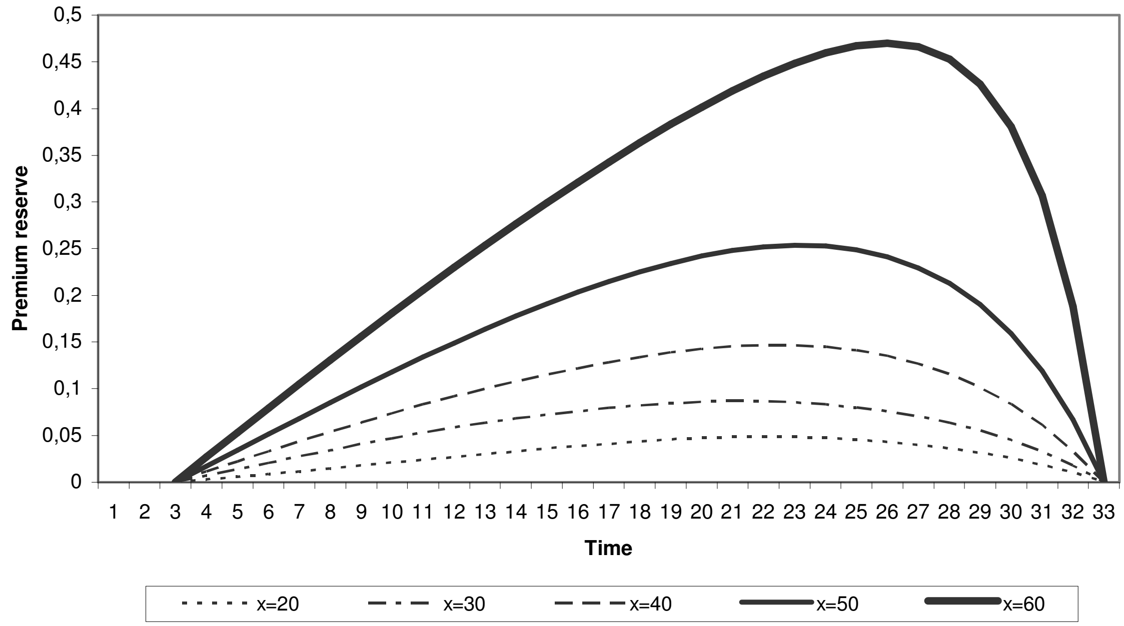

Figure 11.4.: The premium reserve of term insurance with the same insurance term and different entering ages

We also get a very interesting figure if we draw the curve of the term insurance with shortened premium payment term. In figure 11.5. the premium term is shortened to 10 years (the insurance term is 30 years!). After the end of the premium term insurances with shortened premium payment term works like a single premium insurance, so their premium reserve also equals that of the single premium insurance. In the figure the dashed line shows the reserve of the single premium term insurance, and the "filled line" shows that of the term insurance with shortened premium term. It is clear that after the end of the premium term the two curves are the same, and until then the single premium reserve at the end of the premium term has to be accumulated through premium payments.

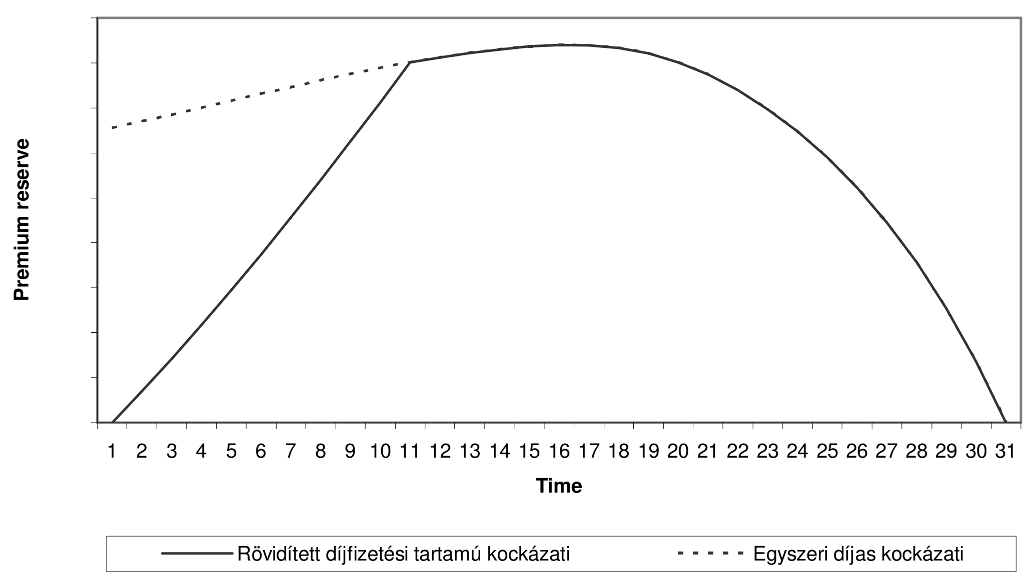

Figure 11.5.: Term insurance with shortened premium term

11.2. PREMIUM RESERVE OF PURE ENDOWMENT AND ENDOWMENT INSURANCE

The premium reserve of the *pure endowment* insurance is different. Here the sum paid to the client at maturity has to be gradually accumulated during the term. (This is why pure endowment, term fix and endowment insurances are called *saving type insurances*.) Here the premium reserve gradually increases, and at the end of the term it equals exactly the sum of benefit payment. The (risk part of) premium payments increase the reserve of the pure endowment insurance similarly as an interest earning deposit. The only difference is that if all other conditions are the same, the premium reserve of the pure endowment insurance increases faster than the deposit account, since the insurer subdivides the payments of those who die during the term among the "accounts" of those who are still alive.

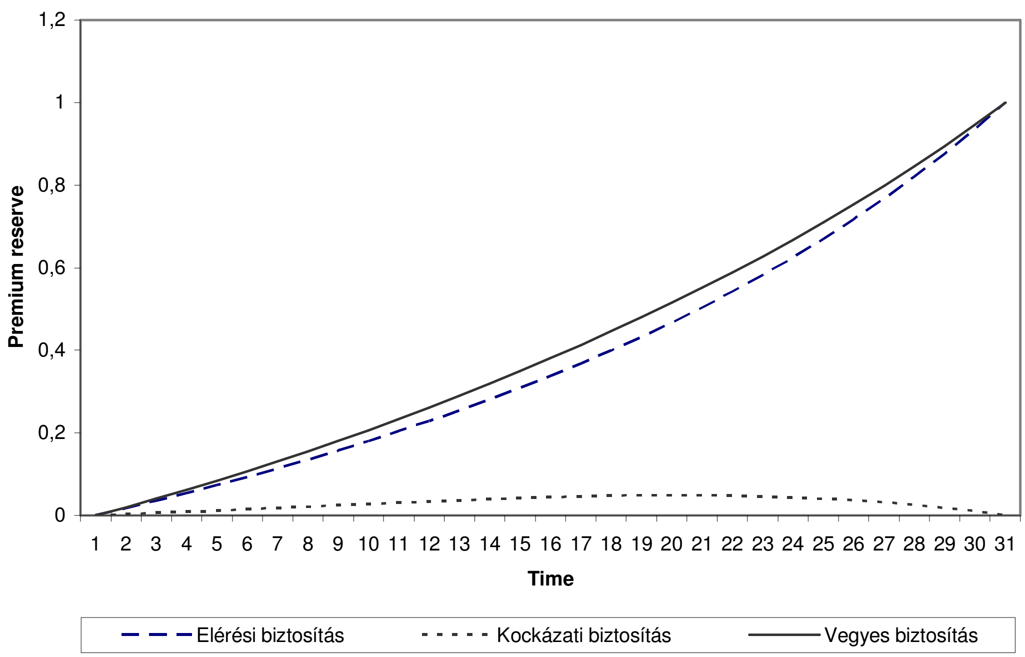

Figure 11.6.: The premium reserve of a regular premium term, pure endowment and endowment insurance

The premium reserve of the *endowment insurance* is the sum of the reserve of a *term insurance* and a *pure endowment insurance*, so it is zero at the beginning of the term, and equals the sum assured at the end of the term. The rate of increase of the premium reserve of an endowment insurance is lower than that of a similar deposit, since this price has to be paid for the security of death benefit payments.

Figure 11.6 shows the curve that a regular premium pure endowment and endowment insurance follows.

The premium reserve of a pure endowment insurance is described by a curve increasing by an accelerated measure. This accelerated increase has the following sources:

- the regularly arriving premium;
- the yield of the premium reserve (to the level of the technical interest rate);
- part of the premium reserve of those who die during the term.

The premium reserve of the single premium pure endowment and endowment insurance is somewhat different from the above. Here we have no premium arriving regularly, but the insurer receives the premium of the whole insurance term (that is, of course is not the same as the term multiplied by the annual premium) at the beginning of the term, so the premium reserve doesn't start from zero. Figure 11.7. illustrates this.

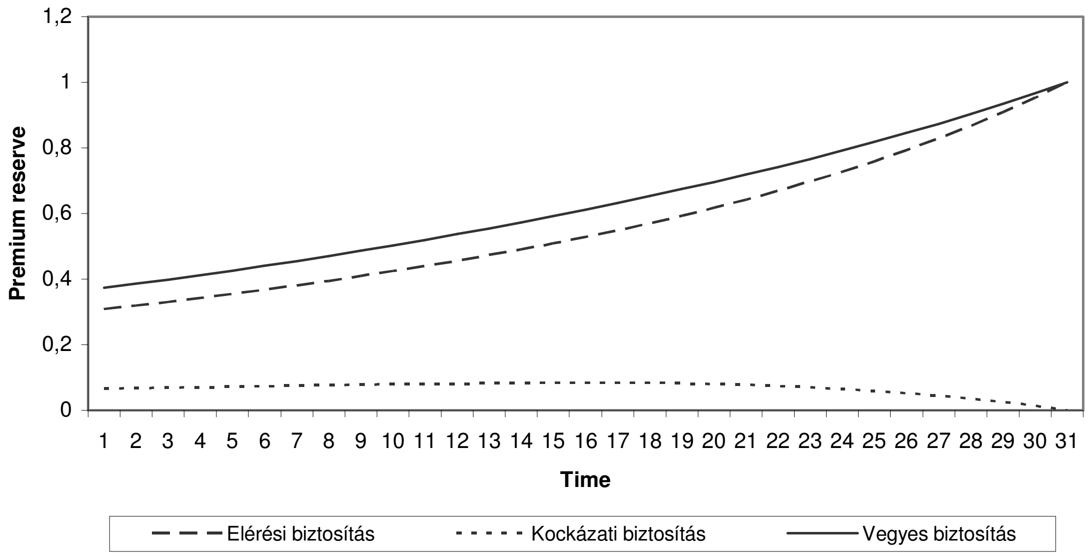

Figure 11.7.: Premium reserve of single premium pure endowment and endowment insurance

We can say the same about the pure endowment (and endowment) insurance with shortened premium term as we have said in case of the term insurance. Here is an example:

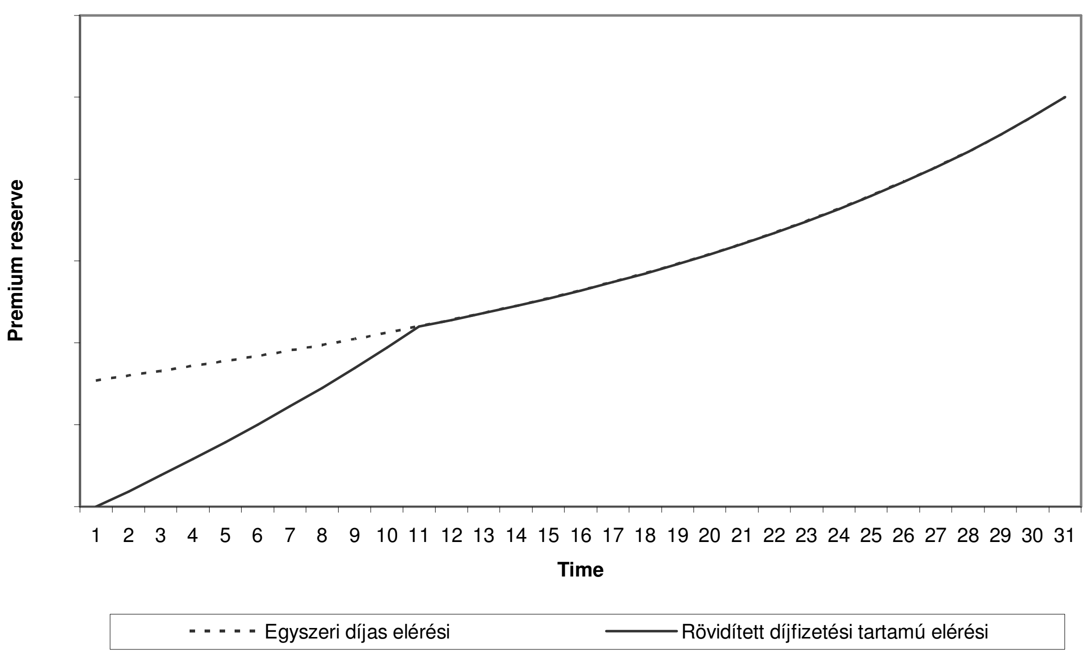

Figure 11.8.: Pure endowment insurance with shortened premium term

We can see by simply analysing the sources that the value of the premium reserve at a given moment cannot be determined by compound interest calculation. This requires special insurance mathematical knowledge. This is even true for the premium reserve of the endowment insurance, that doesn't only depend on the earlier mentioned 3 (increasing) factors, but there is also a constant decreasing effect, namely that the dependents (beneficiaries) of the deceased are also satisfied from the premium reserve.

The calculation of the premium reserve is even more complicated. This is caused by zillmerization, a procedure named after Zillmer, a German actuary.

11.3. Zillmerization[129] and Other Problems

Zillmerization has been created to solve a problem of timing. The cover of the insurer's expenses is the expense loading, that arrives to the insurer (in case of regular premium insurances) gradually throughout the term, in equal payments. There wouldn't be any problem with this, if the expenses of the insurer would also arise this way. But the situation is different. There are expense parts (e.g. premium collection), that arise the same way. But these are the more negligible expense parts. The more significant expenses (contracting, underwriting, and policy issue) arise right at the signature of the policy. These are the expenses of commission, medical examination, administration, etc… that are due when the policy is signed. If the insurer wouldn't zillmerize, then this would have two major consequences:

The insurer would credit acquisition costs – that are to be debited to the policyholder, since they arise because of him – to the policyholder for a long period of time, that the policyholder would pay back gradually through the expense loadings.

If the policyholder surrenders the policy at the beginning of the term, then the insurer couldn't recover these expenses, since further premium payment ceases. This is one of the causes why insurers who do not apply zillmerization define a waiting period for non-forfeiture options.

Zillmerization solves this problem in a way, that the insurer takes the premiums (above the current death benefit payments) of the first 0.5-2.5 years – depending on the insurance term – and uses all of it to cover expenses (i.e. borrows the premium reserve of the first 0.5-2.5 years from the policyholder) and pays this loan back gradually later on from the premium loading. The effect of this is demonstrated by the following example:

Example: The premium reserve of a regular premium pure endowment insurance of 10 years term with and without zillmerization follows the curves of *figure 11.9.* (exaggerating the effect of zillmerization on purpose!):

[129] Chapter 13. discusses zillmerization in detail.

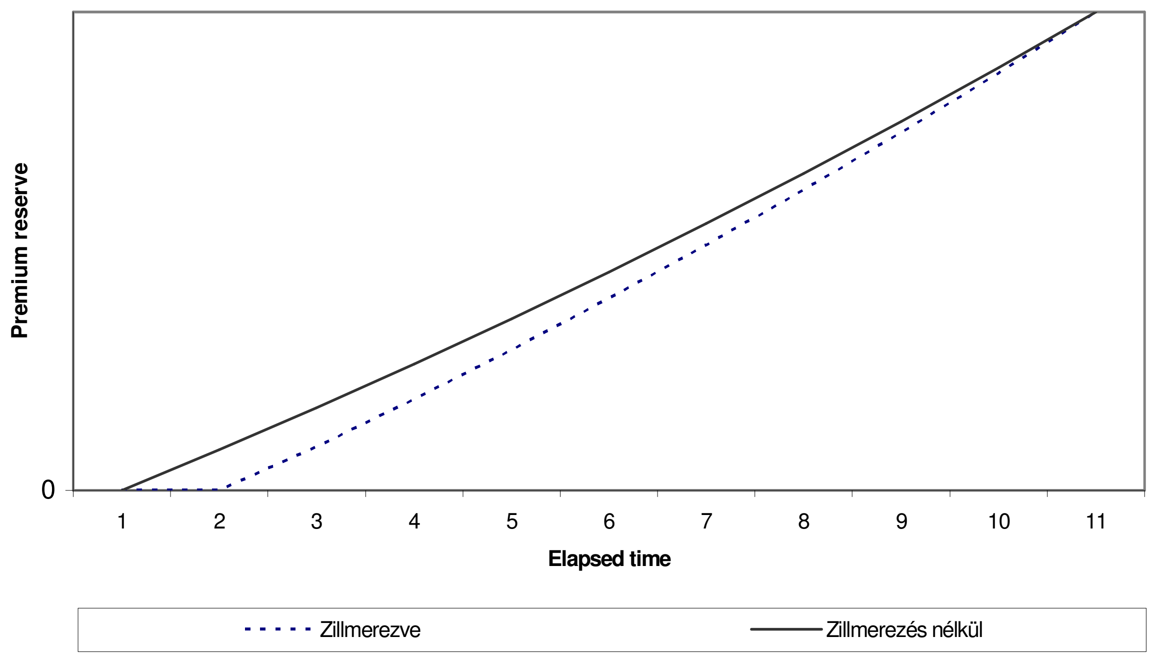

Figure 11.9.: The premium reserve of a regular premium endowment insurance with and without zillmerization

We see that the zillmerized reserve is lower than the non-zillmerized reserve in the whole term. Zillmerization has several consequences. One is that while the premium reserve is zero, there is no profit-sharing. The other is that the value of non-forfeiture options during this period is also zero.

Zillmerization won't result in negative premium reserve, but improper actuarial design can have the effect that the reserve of the product becomes negative somewhere during the term. Since this would mean that the insurer lends money to the client, and the client can surrender the policy any time (contrary to the insurer, who is not allowed to terminate the life insurance policy), these situations have to be avoided under all circumstances!

Non-forfeiture options have appeared several times before, so let's see what these are!

11.4. Non-forfeiture Options and Policy Loan

11.4.1. The Types of Non-forfeiture Options

The basis and the cause of non-forfeiture options is the premium reserve. The premium reserve could also be defined as the sum of money that the insurer has collected to provide some benefits – as stated in the policy – later on. But if the policy is terminated, then this promise of later benefits is gone, and the insurer has to account for the premium reserve. This accounting liability is called from the client's point of view *non-forfeiture options*.

There are several types of non-forfeiture options (and although it is not a non-forfeiture option, it is useful to discuss policy loan in the same section!). When categorising these, let's start from the fact that the insurer would like to keep the client under all circumstances. What does it do then?

First of all, it is possible that the client wants to terminate the policy because of *temporary financial difficulties*, needs a larger sum of money, and is on the opinion that the premium reserve could be used for this purpose. In this case, in order to avoid the termination of the policy, the insurer offers the possibility of *policy loan*. As we have already mentioned, this is not a true non-forfeiture option yet. The base of the loan (and this way also its limit) is the premium reserve. The insurer regards the loan as an investment possibility, and demands an interest on the loan that is the same as interest earned on its other investments. If the insurance company functions properly, then this yield, i.e. the interest of the loan is high, maybe higher than the interest of other loans.

There are a lot of misunderstandings concerning policy loans, that have to be cleared. For one, as we have seen, taking out the policy loan is not a particularly advantageous business for the policyholder. If someone takes out a life insurance policy to be able to take out a policy loan later on, then that person made significant miscalculations, since the sum of the loan remains for a number of years below the sum of premiums paid so-far. (A totally different case is when someone is required by the bank to buy a credit life insurance as a prerequisite of a loan.) Neither is it the best business for the insurer, since investing larger sums of money has less expenses than lending it in small portions. This way the policy loan is only used by insurers if everything else fails.

Secondly: If the client has longer lasting financial difficulties and cannot pay premiums any more (naturally this can only happen in case of regular premium insurances) but doesn't need the premium reserve, then the insurer offers the possibility of *paying up the policy*[130]. The essence of paying up the policy is that the insurer regards the premium reserve accumulated so far as the single premium of an insurance of the same kind, that has a term equal to the remaining years of the original policy's term, and a sum determined by the current age of the insured. This sum assured will naturally be lower than the original sum, since there is no further premium payment.

Thirdly: if the client gets into permanent financial difficulties, that hinder further premium payment, moreover the client needs the money accumulated in the premium reserve, then he *surrenders the policy*[131]. In this case the insurer terminates the policy and gives the premium reserve back to the policyholder. To be precise, not the whole premium reserve in most cases, only the greater part of it. The smaller part, that the insurer "nips off" serves to counterbalance the effect of anti-selection, that is created because the insurance is probably surrendered in a greater portion by those whose health has not deteriorated meanwhile, and have a greater chance of living until maturity. Those, whose health has deteriorated tend to "leave" the greater sum assured to the lower premium reserve in a smaller portion. This "nipping off" is closely related to our general statement, that the premium reserve primarily belongs to the risk community, and only secondly to the concrete client, where the insurer keeps record of it.

11.4.2. Limiting Non-forfeiture Options

The basis of non-forfeiture options is the premium reserve, but not all insurances with premium reserve have non-forfeiture options, or the insurer doesn't usually offer all non-forfeiture options to all policies with premium reserve. This is caused by the *anti-selection* arising due to lapses.

Let's take an example! We start from the case when all non-forfeiture options are offered to all insurances with premium reserve. (Of course this is not true.) E.g. the *term insurance* also has the option of surrender. Let's suppose that two insured persons having a term insurance are thinking about surrendering the policy at the same time. If one is in great shape, and will probably live until maturity, then he will certainly surrender the policy. The other, who is almost dying, will probably not surrender the policy saying that his family will receive much more as sum assured than as surrender value. This way the anti-selection has materialized, which the insurer tries to avoid in case of the term insurance by not offering any non-forfeiture options. (There may be a difference in the health status of the insured persons despite the medical examination and same health status at the beginning of the term,

[130] This is already regarded as a non-forfeiture option by insurance regulation, but if we think about it, in reality this is only a standardized policy transform option!

[131] This is a true non-forfeiture option, and the only one in a sense, since in reality only this case requires "final settlement" between the insurer and the client!

because significant changes might happen compared to the health status of the insured persons at the commencement of the policy.)

The anti-selection realized through non-forfeiture options in case of term insurance is based on the fact that those, whose health status has not deteriorated significantly during the term tend to terminate the policy more. But when calculating the premium, we supposed that healthy people pay the premium through the whole term, and this covers the death benefits of those, whose health has deteriorated. So if healthy people have a chance of leaving the insurance through a non-forfeiture option, this puts the security of the calculation at risk.

This works exactly the opposite way in case of the pure endowment insurance. Here those are tempted to leave the insurance, who know that death is near, and after their death it is not the inheritors who receive the accumulated premium reserve. In this case they surrender the policy if the chance is given. Meanwhile, the pure endowment insurance is naturally calculated in a way that a few insured have to die so that their premium reserve can be subdivided between the remaining insured. Because of this, surrendering or paying up the policy is not allowed in case of the pure endowment insurance. This causes the pure endowment insurance to be a very inflexible construction. If the client doesn't want to pay premiums any more, he loses all payments made so-far. This is why insurance companies don't like to offer this construction.

Because the *immediate annuity* is the series of pure endowment insurances, the same is true for annuities.

In certain cases insurers offer non-forfeiture options to term or pure endowment insurances despite the above. But these have special conditions.

If the business politics of the insurance company is such that the independent term and pure endowment insurance is sold together, quasi as an endowment insurance, then non-forfeiture options similar to that of the endowment insurance can be offered on both policies. But e.g. surrender in case of the pure endowment insurance can only be allowed if surrendering the term insurance at the same time, otherwise the insurer provides a good chance for anti-selection. Namely the client can surrender one policy and keep the other depending on how his health status changes, which he could not do with a united endowment insurance.

A reason for offering non-forfeiture options on a term insurance could possibly be that it is expected to represent a small fraction of the insurers total life portfolio, so the death premium reserve paid out as surrender value is a negligible sum compared to the whole reserve of the portfolio.

Some insurers provide non-forfeiture options in case of pure endowment and annuity insurance subject to medical examination, and only allow the insured in perfect health status to use these options. So the medical examination is a way to ease the offer of non-forfeiture options, but since it is expensive, it is only worth using it in case of large premium reserve, i.e. large sum assured.

12. CALCULATION OF THE PREMIUM RESERVE

We only use the term premium reserve (life insurance mathematical reserve in the official terminology) for the traditional life insurances. We call the reserve of unit linked insurance – that practically has the same function, but requires technically somewhat different handling – the reserve of unit linked insurance, this way separating it from the general life insurance mathematical reserve. Here we will mostly deal with the reserve of traditional insurances, but at the end of the chapter, in a separate sub-section we'll discuss the reserve of unit linked insurance.

12.1. THE CALCULATION OF THE PREMIUM RESERVE GENERALLY

When we deducted the formulae of net premiums in the chapter on premium calculation, then our starting point was the equivalence equation, namely equivalence of the expected present value of all income and all payout. Clearly this equivalence has to hold not only at the commencement of the insurance, but at any point during the whole term (naturally discounting and compounding the appropriate values to this point in time).

Let's suppose that we consider the equivalence equation at the end of the tth year of the insurance term. If at this point we denote the appropriate values the following way:

B^t_1: compounded sum of all income up to the tth point of time (so the income due exactly at the tth point is not included),

K^t_1: compounded sum of all benefit payments up to the tth point of time,

B^t_2: discounted sum of all income expected until maturity,

K^t_2: discounted expected value of benefit payments until maturity,

then according to the equivalence equation:

$$B_1^t + B_2^t = K_1^t + K_2^t \tag{12.1.}$$

From this we get by simple transformation:

$$B_1^t - K_1^t = K_2^t - B_2^t \tag{12.2.}$$

The left hand side of the above equation is the excess of all income received so-far, remaining after the benefits paid so-far, and the right hand side is the excess that indicates how much the expected future benefit payments exceed the expected future income. Both sides show the *premium reserve* of this point in time, and give calculation methods to the computation of this value. So calculation can be performed in two ways:

1. The left hand side (B^t_1 - K^t_1) shows the so-called ***retrospective*** method (looking back at the past), according to which we subtract from the compounded value of income the compounded value of payout, and this way get the excess of accumulated income that can be used for future expected benefit payments.
2. The right hand side (K^t_2 - B^t_2) shows the so-called ***prospective*** method. According to this the value of expected future income has to be subtracted from the value of expected future benefit payments, and this way we get the sum that has to be reserved so that the insurer will be able to cover all future liabilities.

In the practice we always choose the method that is more to the purpose. In most cases this is the prospective method, so in the following we will first give the formulae derived from this method, and only prove in a few cases that the retrospective method would also lead to the same result.[132] Later on we'll also give retrospective formulae, but these – contrary to the prospective ones – will be recursive formulae.

[132] We have to mark that there might be cases when the prospective and the retrospective methods lead to different results. In these cases the principle difference of the two methods becomes important, that the retrospective method gives the reserve calculated from actual premium income and claims payment data of the past, but the prospective method is an estimation of future progressions. If all progressions of the past and the

The retrospective method is useful primarily when calculating the premium reserve of group insurance, when we want to calculate the reserve of a certain year directly from the reserve of the neighbouring year (this is related to the recursive character!). The recursive reserve formulae show better the factors that build the premium reserve, so they have a certain guiding role.

Based on our knowledge acquired so-far, we can determine the values of K^t_2 and B^t_2 more precisely. Namely:

- K^t_2: the discounted value of future benefit payments, as the single premium of an insurance the same type and having the same sum assured as the original, that an insured of *x+t* years could take out for a term of *n-t* years (or a lifetime term). (This statement doesn't hold for those insurances where the sum assured depends on *t*.) So the single premium and K^t_2 basically man the same thing.
- B^t_2: the discounted value of future income in case of regular premium insurances is the capital value of the temporary annuity with payments in advance, which has an annual payment equal to the annual premium of the original insurance, where the insured is *x+t* years old and the term equals the remaining years of the original term, *n-t* years.

In case of single premium insurances our job is easier than in the case of regular premium payment, because here $B^t_2 = 0$, since we do not expect premium income after the single premium payment at the beginning of the term.

The situation is a little more complicated than the above if the sum assured and/or the premium varies as a function of elapsed years, and in the case of premium refund insurances. These require further considerations, naturally keeping the validity of the basic equation $B^t_1 - K^t_1 = K^t_2 - B^t_2$.

We denote the value of the premium reserve at the end of year *t* by

$$V_t\ .$$

The premise of "end of year" means – beside other things – that the changes occurring exactly at the *t* point in time – e.g. insurance payment due in *t* – are not included in V_t. On the other hand, in case of regular premium insurances if it is important to emphasize whether the premium reserve at anniversary indicates the state before or after premium payment, we introduce a further notation (but only use it if necessary!). According to this the annual premium reserve before premium payment (which is the default interpretation if this notation is not used!) is: $\underline{V}_t$, and after premium payment it is: $\overline{V}_t$. Further on we will give these V_t-s for 1 Forint sum assured, as usual.

We have to mention that several insurance companies calculate the premium reserve by the so-called "conservative" method. Under this term we mean that in reserve calculation they suppose the entering age of the insured to be 1 year higher than the actual, or the "technical" age adjusted by increase (due to poorer health status, occupation, etc…) This one year increase doesn't affect the form and logic of the following formulae.

12.2. The Calculation of the Annual Prospective Premium Reserve

Based on the above it is easy to give the formula of single premium insurances.

For the **term** insurance:

$$V_t = A^{\ 1}_{x+t:\overline{n-t}|} \tag{12.3.}$$

so the value of the current reserve equals the single net premium of an insurance having a term of the remaining years *(n-t)* and an insured of age *x+t*.

The **pure endowment** insurance:

---

future follow the original calculation, then there is no difference between the results of the two computatin methods.

$$V_t = A_{x+t:\overline{n-t}|}^{\;\;1} \qquad (12.4\ .)$$

It is obvious that for the **endowment** insurance:

$$V_t = A_{x+t:\overline{n-t}|}^{\;\;1} + A^{1}_{x+t:\overline{n-t}|} \qquad (12.5.)$$

Since the premium formulae of the different annuities are very similar, it will be enough if we look at the prospective reserve formula of the **single premium immediate lifetime annuity**.

$$V_t = \ddot{a}_{x+t} \qquad (12.6.)$$

For regular premium insurances it is enough to determine the formula generally, which is, based on the above, in the prospective case simply:

$$V_t = K_2^t - B_2^t = A_{x+t:\overline{n-t}|} - \ddot{a}_{x+t:\overline{n-t}|} \cdot P_{x:\overline{n}|} \qquad (02.7.)$$

Since this formula is completely unambiguous and we have already derived all necessary sub-results in case of concrete insurances, here we won’t introduce the reserve formulae in more detail.

12.3. The Changing of the Premium Reserve – Recursive, Retrospective Premium Reserve Formulae

We often need to determine the factors that result the change of the premium reserve form one year to the other. This is hidden in the usual prospective formulae, and this way it is necessary to somewhat modify them to making them retrospective (or retrospectively viewed, if that sounds better). Since they are very instructive, we will discuss these recursive, retrospective premium reserve formulae – and their interpretation – in detail. Our method simply is to compute with the aid of the well known prospective formulae their change from year to year, and the result will be a recursive formula of retrospective view. First we transform the single premium reserve formulae, since the reserve of the regular premium version can be derived using these.

12.3.1. The Change of the Premium Reserve of Single Premium Insurances

Here we naturally do not have to distinguish between reserves before and after premium payment! We will examine the change of the reserve of the generalised product introduced earlier, and will analyse its elements one by one. The premium reserve of the generalised product in year (t+1). is:

$$\begin{aligned} V_{t+1} &= A_{x+t+1:\overline{n-t-1}|} = SAD \cdot A^{1}_{x+t+1:\overline{n-t-1}|} + SAM \cdot A_{x+t+1:\overline{n-t-1}|}^{\;\;1} + SAT \cdot A_{n-t-1} + \\ &+ SAP \cdot P^{b}_{x:\overline{n}|} \cdot \frac{(t+1)\cdot M_{x+t+1} + R_{x+t+1} - R_{x+n} - n \cdot M_{x+n}}{D_{x+t+1}} + \\ &+ AA \cdot \ddot{a}_{x+t+1:\overline{n-t-1}|} \end{aligned} \qquad (12.8.)$$

We examine the terms of the change of the premium reserve of the generalised product one by one and interpret them the following way:[133]

In case of the **pure endowment** insurance:

[133] We do not examine separately the change of the reserve of a term fix insurance because of its triviality.

$$
\begin{aligned}
V_{t+1} &= A_{x+t+1:\overline{n-t-1|}}^{\;\;1} = \frac{D_{x+n}}{D_{x+t+1}} = \frac{D_{x+n}}{D_{x+t}} \cdot \frac{D_{x+t}}{D_{x+t+1}} = \\
&= A_{x+t:\overline{n-t|}}^{\;\;1} \cdot \frac{l_{x+t} \cdot v^{x+t}}{l_{x+t+1} \cdot v^{x+t+1}} = \frac{A_{x+t:\overline{n-t|}}^{\;\;1}}{v} \cdot \frac{l_{x+t+1} + d_{x+t}}{l_{x+t+1}} = \\
&= A_{x+t:\overline{n-t|}}^{\;\;1} \cdot (1+i) \cdot \left(1 + \frac{d_{x+t}}{l_{x+t+1}}\right) = \\
&= V_t \cdot (1+i) + \frac{d_{x+t}}{l_{x+t+1}} \cdot V_t \cdot (1+i)
\end{aligned}
\qquad (12.9.)
$$

Interpretation: We know that the reserve of the single premium pure endowment insurance changes from one year to the other due to the following factors:

1. the interest of the existing reserve,
2. a part of the premium reserve of the deceased, that goes to those still living.

According to this, we get the premium reserve of the next year from the previous one by adding the interest of the reserve of the previous year, and then subdividing the reserve of the d_{x+t} number of deceased among the l_{x+t+1} still living.

In case of the **term** insurance:

$$
\begin{aligned}
V_{t+1} &= A^1_{x+t+1:\overline{n-t-1|}} = \frac{M_{x+t+1} - M_{x+n}}{D_{x+t+1}} = \frac{M_{x+t} - M_{x+n} - C_{x+t}}{D_{x+t}} \cdot \frac{D_{x+t}}{D_{x+t+1}} = \\
&= \frac{D_{x+t}}{D_{x+t+1}} \cdot \left(A^1_{x+t:\overline{n-t|}} - \frac{C_{x+t}}{D_{x+t}} \right) = \frac{l_{x+t}}{l_{x+t+1}} \cdot \left(V_t \cdot (1+i) - \frac{d_{x+t} \cdot v^{x+t+\frac{1}{2}}}{l_{x+t} \cdot v^{x+t+1}} \right) = \\
&= \frac{l_{x+t}}{l_{x+t+1}} \cdot \left[V_t \cdot (1+i) - \frac{d_{x+t}}{l_{x+t}} \cdot (1+i)^{\frac{1}{2}} \right] = \\
&= \frac{l_{x+t}}{l_{x+t+1}} \cdot \left[V_t \cdot (1+i) - q_{x+t} \cdot (1+i)^{\frac{1}{2}} \right]
\end{aligned}
\qquad (12.10.)
$$

Interpretation: We get the annual premium reserve of the term insurance from the previous annual reserve by adding its interest until the year-end, and then subtracting the part of death sum that debits this policy compounded from the time of death (middle of the year) to the end of the year, and correcting (increasing) this, since the remaining premium reserve of the deceased (at the end of the year) is subdivided among those still living.

Transforming the previous formula to give it a similar structure as that of the pure endowment insurance:

$$\begin{aligned}
V_{t+1} &= \frac{l_{x+t}}{l_{x+t+1}} \cdot \left[V_t \cdot (1+i) - q_{x+t} \cdot (1+i)^{\frac{1}{2}} \right] = \\
&= \left(1 + \frac{d_{x+t}}{l_{x+t+1}} \right) \cdot \left[V_t \cdot (1+i) - q_{x+t} \cdot (1+i)^{\frac{1}{2}} \right] = \\
&= V_t \cdot (1+i) - q_{x+t} \cdot (1+i)^{\frac{1}{2}} + \frac{d_{x+t}}{l_{x+t+1}} \cdot \left[V_t \cdot (1+i) - q_{x+t} \cdot (1+i)^{\frac{1}{2}} \right] = \\
&= V_t \cdot (1+i) - q_{x+t} \cdot (1+i)^{\frac{1}{2}} \cdot \left(1 - \frac{l_{x+t}}{l_{x+t+1}} \cdot \left[V_t \cdot (1+i)^{\frac{1}{2}} - q_{x+t} \right] \right) = \\
&= V_t \cdot (1+i) - q_{x+t} \cdot (1+i)^{\frac{1}{2}} \cdot \frac{l_{x+t}}{l_{x+t+1}} \cdot \left(1 - V_t \cdot (1+i)^{\frac{1}{2}} \right) = \\
&= V_t \cdot (1+i) - \frac{d_{x+t}}{l_{x+t+1}} \cdot \left(1 - V_t \cdot (1+i)^{\frac{1}{2}} \right) \cdot (1+i)^{\frac{1}{2}}
\end{aligned} \tag{12.11.}$$

Interpretation: compared to the previous formula, here we get the next annual reserve from the compounded previous annual reserve by the addition of a single correction term. The explanation of this added term is: The compounded death benefit of those deceased during the year is subdivided among (subtracted from) the premium reserve of those still living at the end of the year. And the death benefit is the sum assured decreased by the value of the premium reserve of the deceased, compounded to the time of death (since this doesn't have to be covered by the risk community!).

Modifying it a little more, we get the structure similar to that of the pure endowment:

$$\begin{aligned}
V_{t+1} &= V_t \cdot (1+i) - \frac{d_{x+t}}{l_{x+t+1}} \cdot \left(1 - V_t \cdot (1+i)^{\frac{1}{2}} \right) \cdot (1+i)^{\frac{1}{2}} = \\
&= V_t \cdot (1+i) + \frac{d_{x+t}}{l_{x+t+1}} \cdot V_t \cdot (1+i) - \frac{d_{x+t}}{l_{x+t+1}} \cdot (1+i)^{\frac{1}{2}}
\end{aligned} \tag{12.12.}$$

Interpretation: We compound the reserve of the beginning of the year to the year-end. We add to this that part of the beginning-of-year premium reserve of the deceased compounded to the year end that refers to one policy, and then subtract that part of the value of benefits provided to the deceased compounded to the year-end that refers to one policy.

In case of the single premium pure endowment insurance with premium refund:

We have to clarify right at the beginning that this is not the single premium pure endowment insurance with premium refund, but a technical type version of it, from which the regular premium version is derived. So the “premium” that is refunded is not the single premium itself, but a kind of calculated regular gross premium according to the above:

The premium reserve itself is[134]:

$$V_t = A_{x+t:\overline{n-t|}} = \frac{D_{x+n} + P^b_{x:\overline{n|}} \cdot \left(t \cdot M_{x+t} + R_{x+t} - R_{x+n} - n \cdot M_{x+n} \right)}{D_{x+t}} \tag{12.13.}$$

The change of the premium reserve:

[134] To make this meaningful in itself, we not only examine the term that belongs to SAP, but also – and again, in a sense – the term belonging to SAM!

$$\begin{aligned}
&V_{t+1} = A_{x+t+1:\overline{n-t-1}|} = \frac{D_{x+n} + P^b_{x:\overline{n}|} \cdot \left((t+1) \cdot M_{x+t+1} + R_{x+t+1} - R_{x+n} - n \cdot M_{x+n} \right)}{D_{x+t+1}} = \\
&= \frac{D_{x+n} + P^b_{x:\overline{n}|} \cdot \left(t \cdot (M_{x+t} - C_{x+t}) + (M_{x+t} - C_{x+t}) + R_{x+t} - M_{x+t} - R_{x+n} - n \cdot M_{x+n} \right)}{D_{x+t}} \cdot \frac{D_{x+t}}{D_{x+t+1}} = \\
&= \left[\frac{D_{x+n} + P^b_{x:\overline{n}|} \cdot \left(t \cdot M_{x+t} + R_{x+t} - R_{x+n} - n \cdot M_{x+n} \right)}{D_{x+t}} - \frac{P^b_{x:\overline{n}|} \cdot (t+1) \cdot C_{x+t}}{D_{x+t}} \right] \cdot \frac{D_{x+t}}{D_{x+t+1}} = \\
&= \left[A_{x+t:\overline{n-t}|} - \frac{P^b_{x:n} \cdot (t+1) \cdot d_{x+t} \cdot v^{x+t+\frac{1}{2}}}{D_{x+t}} \right] \cdot \frac{D_{x+t}}{D_{x+t+1}} = \\
&= V_t \cdot (1+i) \cdot \left(1 + \frac{d_{x+t}}{l_{x+t+1}} \right) - \frac{P^b_{x:n} \cdot (t+1) \cdot d_{x+t} \cdot v^{x+t+\frac{1}{2}}}{D_{x+t+1}} = \\
&= V_t \cdot (1+i) + \frac{d_{x+t}}{l_{x+t+1}} \cdot V_t \cdot (1+i) - \frac{d_{x+t}}{l_{x+t+1}} \cdot P^b_{x:\overline{n}|} \cdot (t+1) \cdot (1+i)^{\frac{1}{2}}
\end{aligned} \tag{12.14.}$$

Interpretation: We can declare that the result can be interpreted the same way as in case of the term insurance, since the death sum assured after the t^{th} anniversary is (t+1) times the gross premium, and is due at the middle of the year.

Remark: Since the death sum assured depends on whether premium payment has occurred or not, here it could be meaningful to denote separately the state before and after premium payment in the premium reserve. According to this, the above formulae indicate the state before premium payment (since at the anniversary it still shows the old death sum, that jumps exactly then!).

The retrospective reserve formula of the immediate **annuity** paid in advance is:

$$\begin{aligned}
&V_{t+1} = \ddot{a}_{x+t+1:\overline{n-t-1}|} = \frac{N_{x+t+1} - N_{x+n}}{D_{x+t+1}} = \frac{N_{x+t} - N_{x+n} - D_{x+t}}{D_{x+t}} \cdot \frac{D_{x+t}}{D_{x+t+1}} = \\
&= \frac{l_{x+t}}{l_{x+t+1}} \cdot \left(\ddot{a}_{x+t:\overline{n-t}|} - 1 \right) \cdot (1+i) = \left(1 + \frac{d_{x+t}}{l_{x+t+1}} \right) \cdot \left(\ddot{a}_{x+t:\overline{n-t}|} - 1 \right) \cdot (1+i) = \\
&= V_t \cdot (1+i) \cdot \left(1 + \frac{d_{x+t}}{l_{x+t+1}} \right) - (1+i) \cdot \left(1 + \frac{d_{x+t}}{l_{x+t+1}} \right) = \\
&= V_t \cdot (1+i) + \frac{d_{x+t}}{l_{x+t+1}} \cdot V_t \cdot (1+i) - \frac{l_{x+t}}{l_{x+t+1}} \cdot (1+i)
\end{aligned} \tag{12.15.}$$

Interpretation: We compounded the beginning-of-year reserve. The compounded reserve of the deceased is subdivided among those still living. The compounded value of benefits provided to the living is subdivided and subtracted from the reserve of those still alive at the end of the year.

Remark 1: A new element has appeared compared to the previous ones, the living benefit. This only differs from the death benefit in that a different group receives it: those, who are alive at the beginning of the year, and not those, who die during the year; and at a different time: at the beginning of the year and not during of the year.

Remark 2: Although there is no premium payment here, there is something very similar: annuity payment. So the differentiation between pre-annuity payment and post-annuity payment annual reserve could be meaningful here, too. Naturally in the above we examined the pre-annuity payment state, so we could have used the symbol $\underline{V}_t$!

Uniting the above premium reserve formulae we can say that if a single premium insurance contains only pure endowment, term and annuity benefits, then we get the following formula:

$$V_{t+1} = V_t \cdot (1+i) + \frac{d_{x+t}}{l_{x+t+1}} \cdot V_t \cdot (1+i) - \\ - SAD \cdot \frac{d_{x+t}}{l_{x+t+1}} \cdot (1+i)^{\frac{1}{2}} - AA \cdot \frac{l_{x+t}}{l_{x+t+1}} \cdot (1+i) \tag{12.16.}$$

When defining this formula we used the following restrictions:

- SAD is 0 or 1, or something else depending on whether we are dealing with pure endowment, term or endowment insurance, or maybe something else.
- AA is 1 or 0, depending on whether we are dealing with annuity paid in advance[135] or not annuity insurance (or an insurance not containing annuity element).
- The supposed time of death is always the middle of the year[136]
- The time of annuity payment is always the beginning of the year
- The above formula always shows the value of the premium reserve before annuity payment.
- In case of the single premium term fix there naturally is no transfer between reserves, so the formula of this is simply: $V_{t+1} = V_t \cdot (1+i)$.

Interpretation: As a summary of the above: we compound the reserve of the beginning of the year (the value before annuity payment). We add to this that part of the reserve of deceased that refers to one policy. We subdivide the compounded value of the benefits paid to the deceased among those still living at the end of the year. We also subdivide the compounded value of annuity benefit paid to those living at the beginning of the year among those still living at the end of the year.

12.3.2. The Retrospective Formula of the Premium Reserve of Regular Premium Insurances – Without Zillmerization

Without taking the different types of insurances one by one, from the above we can derive the recursive premium reserve formula of regular premium insurances – as yet without zillmerization, that we will examine separately later on.

Since regular premium insurances usually do not provide annuity benefits, this way we leave the factor with AA out of the formula.

The premium payment is an important feature, so in order to emphasize this more strongly we will use the notation of under- and over-lining described earlier. The relation between reserves before and after premium payment is:

$$\underline{V}_t = A_{x+t:\overline{n-t}|} - P \cdot \ddot{a}_{x+t:\overline{n-t}|} \tag{12.17.}$$

$$\overline{V}_t = A_{x+t:\overline{n-t}|} - P \cdot \left(\ddot{a}_{x+t:\overline{n-t}|} - 1 \right) = \underline{V}_t + P \tag{12.18.}$$

Using our general formula of the reserve of single premium insurances and the above describe relations of annuities:

[135] It is not necessary to examine the annuity paid in arrears in our case, or it can usually be derived from the in advance case.

[136] So here we have not followed the earlier practice introduced at the premium calculation, but at the same we have shown the other usual simplifying supposition.

$$\begin{aligned}
\underline{V}_{t+1} &= A_{x+t+1:\overline{n-t-1|}} - P \cdot \ddot{a}_{x+t+1:\overline{n-t-1|}} = \\
&= A_{x+t:\overline{n-t|}} \cdot (1+i) + \frac{d_{x+t}}{l_{x+t+1}} \cdot A_{x+t:\overline{n-t|}} \cdot (1+i) - SA \cdot \frac{d_{x+t}}{l_{x+t+1}} \cdot (1+i)^{\frac{1}{2}} - \\
&- P \cdot \left(\left(\ddot{a}_{x+t:\overline{n-t|}} - 1 \right) + \frac{d_{x+t}}{l_{x+t+1}} \cdot \left(\ddot{a}_{x+t:\overline{n-t|}} - 1 \right) \right) \cdot (1+i) = \\
&= \overline{V}_t \cdot (1+i) + \frac{d_{x+t}}{l_{x+t+1}} \cdot \overline{V}_t \cdot (1+i) - SA \cdot \frac{d_{x+t}}{l_{x+t+1}} \cdot (1+i)^{\frac{1}{2}}
\end{aligned} \qquad (12.19.)$$

So we practically get the same formula as in the single premium case, with the difference that on the left hand side of the equation there are reserve formulae before premium payment, and on the right hand side there are reserve formulae after premium payment. Changing uniformly to reserves before premium payment (using the relation between the two reserves) we get the following formula:

$$\begin{aligned}
\underline{V}_{t+1} &= \underline{V}_t \cdot (1+i) + \frac{d_{x+t}}{l_{x+t+1}} \cdot \underline{V}_t \cdot (1+i) + \\
&+ \frac{l_{x+t}}{l_{x+t+1}} \cdot P \cdot (1+i) - SA \cdot \frac{d_{x+t}}{l_{x+t+1}} \cdot (1+i)^{\frac{1}{2}}
\end{aligned} \qquad (12.20.)$$

So here the reserve before premium payment is derived from the premium reserve before premium payment, but we have to add to the formula the subdivision of the annual premium between the remaining policies.

In a little bit more compact form:

$$\underline{V}_{t+1} = \left(\underline{V}_t + P\right) \cdot (1+i) \cdot \left(1 + \frac{d_{x+t}}{l_{x+t+1}}\right) - SA \cdot \frac{d_{x+t}}{l_{x+t+1}} \cdot (1+i)^{\frac{1}{2}} \qquad (12.21.)$$

Or in an alternative form:

$$\underline{V}_{t+1} = \left(\underline{V}_t + P\right) \cdot (1+i) - \frac{d_{x+t}}{l_{x+t+1}} \cdot (1+i)^{\frac{1}{2}} \cdot \left(SA - \left(\underline{V}_t + P\right) \cdot (1+i)^{\frac{1}{2}} \right) \qquad (12.22.)$$

12.4. The Calculation of Mid-year Premium Reserve

In the previous sections we have learned how the premium reserve can be calculated at the end of the tth insurance year. But we know that the premium reserve changes gradually (the interest is added to the reserve gradually, and deaths also occur spread through the year), so the value of the reserve within the year will be different than at the anniversary. We often need this mid-year reserve for practical calculations. Also because of practical purposes, the mid-year reserve is usually calculated simply as the interpolated value (weighted average) of the annual reserves.

Let’s suppose that we want to know the level of the reserve of single premium insurances within the *(t+1)* insurance year, at the *(t+τ)* point, where $0 < \tau < 1$.

Then the two neighbouring premium reserves will be V_t and V_{t+1}. The weighted average of these is:

$$V_{t+\tau} = (1-\tau) \cdot V_t + \tau \cdot V_{t+1} \qquad (12.23.)$$

In case of regular premium insurances the above picture changes, because we also have to take into account that the next annual premium payment ($P_{x:\overline{n}|}$, or simply *P*) arriving to the insurer at the beginning of the year, that is immediately added to the premium reserve. So at the end of year *t* the premium reserve is V_t, but one moment later, at the beginning of year *t+1* it will be V_t+P. According to this, the mid-year premium reserve will be:

$$V_{t+\tau} = (1-\tau)\cdot(V_t + P) + \tau \cdot V_{t+1} = (1-\tau)\cdot\overline{V}_t + \tau\cdot\underline{V}_{t+1} \tag{12.24.}$$

In case of regular premium insurances we also have to deal with another problem. In the above reserve formula we supposed annual premium payment. But it is possible that the insurer calculated the premium by monthly payment. In this case – if we want to be precise – we have to calculate also the reserve with the monthly value, so instead of the value

$$V_t = A_{x+t:\overline{n-t}|} - \ddot{a}_{x+t:\overline{n-t}|}\cdot P_{x:\overline{n}|} \tag{12.25.}$$

we would have to calculate with

$$V_t = A_{x+t:\overline{n-t}|} - \ddot{a}^{(12)}_{x+t:\overline{n-t}|}\cdot P^{(12)}_{x:\overline{n}|} \quad . \tag{12.26.}$$

If we substitute in the place of $P^{(12)}{}_{x:\overline{n}|}$ the $\dfrac{A_{x:\overline{n}|}}{\ddot{a}^{(12)}{}_{x:\overline{n}|}}$ value that is equal to it, then we get

$$V_t = A_{x+t:\overline{n-t}|} - \ddot{a}^{(12)}{}_{x+t:\overline{n-t}|}\cdot\frac{A_{x:\overline{n}|}}{\ddot{a}^{(12)}{}_{x:\overline{n}|}} = A_{x+t:\overline{n-t}|} - A_{x:\overline{n}|}\cdot\frac{\ddot{a}^{(12)}{}_{x+t:\overline{n-t}|}}{\ddot{a}^{(12)}{}_{x:\overline{n}|}} \tag{12.27.}$$

Since from the practical point of view the difference between $\dfrac{\ddot{a}^{(12)}{}_{x+t:\overline{n-t}|}}{\ddot{a}^{(12)}{}_{x:\overline{n}|}}$ and $\dfrac{\ddot{a}_{x+t:\overline{n-t}|}}{\ddot{a}_{x:\overline{n}|}}$ is negligible, the original value of

$$V_t = A_{x+t:\overline{n-t}|} - \ddot{a}_{x+t:\overline{n-t}|}\cdot P_{x:\overline{n}|} \tag{12.28.}$$

is regarded as annual reserve also in the case of insurances with monthly calculation. So the difference between the two types of insurances (monthly premium payment and annual premium payment) doesn't lie here, but in the mid-year reserves. Because when calculating the mid-year reserve, we principally cannot suppose in the case of monthly calculation that the total annual premium arrives at the beginning of the year. We know that this arrives monthly. This way (and for the sake of simplicity) in cases like this, the mid-year premium reserve is usually not calculated by the

$$V_{t+\tau} = (1-\tau)\cdot(V_t + P) + \tau \cdot V_{t+1} = (1-\tau)\cdot\overline{V}_t + \tau\cdot\underline{V}_{t+1} \tag{12.29.}$$

formula, but with

$$V_{t+\tau} = (1-\tau)\cdot V_t + \tau\cdot V_{t+1}\,. \tag{12.30.}$$

12.5. A NEGATIVE PREMIUM RESERVE

In case of the above discussed products the premium reserve was evidently a positive value. The phrasing ("the part of earlier premium income reserved for later claims payment ") also suggests the non-negative sign. The positivity is natural in case of single premium insurances. But there might be regular premium insurances, where the positivity of the premium reserve doesn't hold. This means – if we think about it – that the insurer has provided more benefits looking at it time-proportionally than the amount of premium that has already been paid, so the client owes the insurer.

Since the client can terminate the policy any time, situations like this have to be avoided, since in case of surrender the insurer might not be able to recover its money, the sum of which is indicated by the negative reserve. This is why we say that it is a **professional mistake** to construct insurances with negative premium reserve, although it happens all the same (or at least it used to happen). A good example of this (and maybe the only one?!) is the so-called "credit life insurance"[137] very popular in the Hungarian market in the '70s and '80s, that was purchased as a cover of loans for OTP apartments. In this product the sum assured decreased from year to year, simulating the sum of the remaining outstanding debt. If the insured dies before the loan is repaid, then the remaining debt is paid by the insurer.

The cause of the negative premium reserve of the credit life insurance was that while the annual premium was fixed, the risk undertaken by the insurer – due to the decreasing sum assured – decreased from year to year. So here – contrary to the term insurance – it is not the deficiency of later premiums lower than the current risk that has to be covered by the excess of earlier years, but the deficiency of earlier premiums lower than the risk is covered by the excess premium of later years. Meanwhile the insurer gives a loan to the client to cover the risk deficiency, and the value of this loan is indicated by the negative reserve.

Let's look at the premium and reserve formulae for 1 Forint sum assured. For the sake of simplicity, we define these right away using commutation numbers.

Let's suppose that during the term the sum assured decreases by *1/n* every year, so the sum assured changes during the years the following way:

$$1, \frac{n-1}{n}, \frac{n-2}{n}, \ldots, \frac{1}{n} \tag{12.31.}$$

Then the single premium is:

$$A_{x:\overline{n}|} = \frac{v \cdot d_x \cdot \frac{n}{n} + v^2 \cdot d_{x+1} \cdot \frac{n-1}{n} + \ldots + v^n \cdot d_{x+n-1} \cdot \frac{1}{n}}{l_x} = \tag{12.32.}$$

$$= \frac{n \cdot C_x + (n-1) \cdot C_{x+1} + \ldots + 1 \cdot C_{x+n-1}}{n \cdot D_x} =$$

$$= \frac{(C_x + C_{x+1} + \ldots + C_{x+n-1}) + (C_x + C_{x+1} + \ldots + C_{x+n-2}) + \ldots + C_x}{n \cdot D_x} =$$

$$= \frac{(M_x - M_{x+n}) + (M_x - M_{x+n-1}) + \ldots + (M_x - M_{x+1})}{n \cdot D_x} = \tag{12.33.}$$

And now we introduce the last important commutation number, R_x. As it can be suspected:

$$R_x = M_x + M_{x+1} + \ldots + M_\omega \tag{12.34.}$$

[137] The term „credit life" naturally refers to generally all life insurances that are used as a coverage of a loan. In case of the product discussed here the term "Credit Life" was the name of this product, at a time when the number of life insurance product offered was quite limited.

Using R_x we can write this single premium in a more simple way:

$$A_{x:\overline{n}|} = \frac{n \cdot M_x - (R_{x+1} - R_{x+n+1})}{n \cdot D_x} \tag{12.35.}$$

The single premium – since the sum assured changes during the term – will look a little different in t years than as we would expect based on our experience, this way we'll give its deduction separately.

$$A_{x+t:\overline{n-t}|} = \frac{v \cdot d_{x+t} \cdot \frac{n-t}{n} + v^2 \cdot d_{x+t+1} \cdot \frac{n-t-1}{n} + \ldots + v^{n-t+1} \cdot d_{x+n-1} \cdot \frac{1}{n}}{l_{x+t}} =$$

$$= \frac{(n-t) \cdot C_{x+t} + (n-t-1) \cdot C_{x+t+1} + \ldots + 1 \cdot C_{x+n-1}}{n \cdot D_{x+t}} =$$

$$= \frac{(n-t) \cdot M_{x+t} - (R_{x+t+1} - R_{x+n+1})}{n \cdot D_{x+t}} \tag{12.36.}$$

The annual net premium, as usual:

$$P_{x:\overline{m}|} = \frac{A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}} \tag{12.37.}$$

and the premium reserve at the end of year t:

$$V_t = A_{x+t:\overline{n-t}|} - \ddot{a}_{x+t:\overline{n-t}|} \cdot P_{x:\overline{n}|} \tag{12.38.}$$

Let's take an example to illustrate the above discussion:

Example
Entering age of the insured: 35 years
Gender: male
Insurance term: 15 years
The initial sum assured of the policy: 100,000 Forints
Mortality table: 1998 Hungarian male population mortality table
Technical interest rate: 3.5%
(The sum assured decreases every year by 100,000/15 = 6666,67 Forints)

Table 12.1 shows the premium reserve at the end of year t:

Table 12.1.: Example of negative premium reserve

t	Premium reserve
0	0
1	27
2	24
3	-11
4	-76
5	-165
6	-264

t	Premium reserve
7	-361
8	-447
9	-511
10	-548
11	-551
12	-512
13	-417
14	-252
15	0

We can draw the conclusion that the premium reserve of the regular premium insurance with decreasing benefit is usually negative, so these constructions should be avoided if possible. Naturally this doesn't mean that we shouldn't develop products of this kind, but that the negative reserve has to be avoided. One possible solution is to decrease the premium term. The shorter the premium term, the higher the curve of the premium reserve will be, that in the end will get totally above the x axis. In the above example the premium reserve becomes everywhere positive already if the premium term is decreased to 12, as indicated by figure 12.1.:

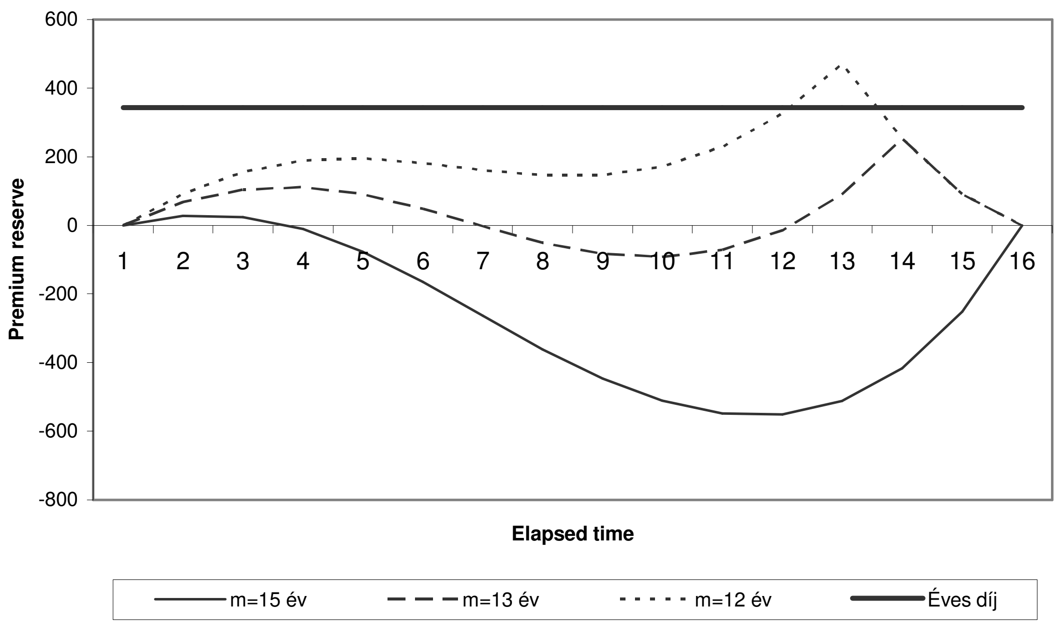

Figure 12.1.: The premium reserve of the "Credit Life insurance" with different premium terms

Figure 12.1 shows clearly that the negative premium reserve can be close to the double of the annual premium, that can cause significant loss to the insurer if the policyholder surrenders the insurance.

Earlier we have mentioned a regular premium orphan's annuity product, where the parent pays the premium regularly until the end of the term, or his (or the later beneficiary's) death. If the beneficiary was alive at the death of the insured, and the insurance term hasn't ended, then until the end of the term, but at most until the death of the (second) insured (beneficiary) the insurer pays him an annuity.

The benefit of this insurance decreases in time, since the longer the parent lives, the shorter the time of annuity paid to the child will be. This way the premium reserve will be negative. In this case this could be avoided e.g. by building in a simple term insurance, so in the event of death of the parent, not only the annuity, but a lump sum is also paid to the child (widow). The positive premium reserve of the term insurance with a correctly chosen sum assured can compensate the negative premium reserve of the orphan's annuity. Naturally the net premium of this insurance is also higher than in the former case of the product resulting a negative premium reserve.

It is a widespread opinion that zillmerization makes the premium reserve negative at the beginning of the term. As we will see in the next chapter discussing zillmerization in detail, this is wrong, the premium reserve will never be negative due to zillmerization. Formulae are sometimes not handled precisely enough in relation to zillmerization, and this might cause the image of negative premium reserve.

12.6. Cash Flows in Unit Linked Insurance

As we have already mentioned, we do not use the term premium reserve for the reserve of unit linked Insurance, but its function is completely analogous to the premium reserve of traditional insurance. This way we will briefly examine this reserve based on the traditional, retrospective reserve formula of endowment insurance.

We have seen that the recursive formula of the premium reserve (before premium payment[138]) of regular (annual) premium endowment insurance – as 12.22. – can also be written in the following useful form:

$$V_{t+1} = (V_t + P)\cdot(1+i) - \frac{d_{x+t}}{l_{x+t+1}}\cdot(1+i)^{\frac{1}{2}}\cdot\left(S-(V_t+P)\cdot(1+i)^{\frac{1}{2}}\right), \qquad (12.39.)$$

Where the definition of S is:

$$V_n = S \qquad (12.40.)$$

We can look at reserve formulae as if they were describing the essential part of the insurance cash flow (But they do not contain the premium part covering expenses, and we also disregard the – by the way relatively simple – modifications/generalizations due to annual premium payment), i.e. as if they were describing the insurance in general. According to this, the most general unit linked insurance construction can also be described by the equation defining its reserve. This reserve-equation can be derived from the equation of the endowment insurance with the following modifications:

- We split the expense part that was until now implicitly handled and united ("premium loadings") into parts and make it partly explicit. The dynamics of the different expense parts from now on will not be the same. The different expense parts are[139]:
  - Administration fee: we express it separately and explicitly in the new cash flow formula. Its increase can depend on how P changes, moreover it can also be defined as an absolute value, independent from P. This means a more even spreading of expenses between insurances, since the administration fee proportional to P (that is customary in traditional insurances) is not justified, since it costs the same to handle policies of larger and smaller sum assured. The same way it changes independent of whether the policyholder has raised the premium meanwhile or not.
  - Fund management fee: it depends on the value of the reserve handled and not the premium, and is subtracted from the gross yield.

[138] And without zillmerization!

[139] Not mentioning conditional expense parts such as the fee of transfer between funds, since this is only due if the client initiates the transfer.

- Bid-offer spread: it is an expense part proportional to the net premium, similar to the premium loading of traditional insurances, that is not included in the cash flow formula, but appears explicitly to the client – contrary to traditional insurances.

- Technical interest rate: instead of the unique, unchanged indicator of investment returns that is known beforehand, we introduce two different kinds: 1. an indicator of investment earnings that can only be known subsequently, 2. a technical type, projected indicator (we will still call this technical interest rate and denote it by *i*!)
- Sum assured: formula (12.40.) concerning the sum assured is changed to a conditional formula: $B_t = \max\left(0; S - (1+i)^{\frac{1}{2}} \cdot (V_t + P - a_t)\right)$, where B_t is the sum assured projected at the beginning of the year, that has to be paid in addition to the value of money in the funds in the event of death of the insured. The formula explicitly includes the administration fee, since in the traditional case P already does not include this, but in case of Unit Linked insurance it does. There is no strict determination of S here, but insurers generally give a maximum value (usually greater than zero) depending on the minimum premium, the term and the insured's age, and the policyholder can freely choose within this.
- Instead of the d_{x+t}/l_{x+t+1} factor insurers generally use its loaded factor, that can be regarded as another hidden expense element. But mortality probability is often decreased due to positive selection.
- Naturally the risk premium is most often subtracted not annually, but e.g. monthly, and due to this, the formula has to be – slightly – altered, but we will disregard this (as we have done in case of traditional insurances!).
- The premium of supplementary risks (insurance riders) is also subtracted from the reserve regularly, but we will disregard this also.
- Hungarian insurers often use the technique of initial units, that is forbidden in several places, that we also do not discuss in the above formula.[140]

According to these, the new formula describing the cash flow of unit linked insurance is (keeping the notation V of the reserve!):

$$V_{t+1} = (V_t + P - a_t) \cdot (1 + (h_t - w)) - \frac{d_{x+t}}{l_{x+t+1}} \cdot (1+p) \cdot (1+i)^{\frac{1}{2}} \cdot B_t, \qquad (12.41.)$$

where we used the following special notation:

$$B_t = \max\left(0; S - (1+i)^{\frac{1}{2}} \cdot (V_t + P - a_t)\right), \qquad (12.42.)$$

and:

h_t: gross yield of year t (earned between the t^th^ and t+1^th^ anniversary)
w: fund management fee, defined as a percentage of the value of the managed fund
p: loading factor used to calculate the death risk premium
a_t: administration fee applied in year *t* and subtracted at the beginning of the year
S: minimum benefit paid upon death.[141] Its value is: $0 \le S \le H(x,t,n,P)$

H(x,t,n,P): limit on the minimum death benefit defined as a function of the insured's entering age, the elapsed time, the term and the annual premium.

B_t: the insurer's projected death benefit above the reserve.

[140] The creation of initial units serves a similar purpose as zillmerization in case of traditional insurances, and we are consistent with ourselves in that neither was zillmerization included in the cash flow formula (12.39) of traditional endowment insurance.

[141] In reality it could have an index to express that S can change. The index was left out to indicate that this change isn't necessarily tied to anniversaries.

The precise value of the above formula can only be determined after t+1. But we already have to know the value of B_t at *t*, because this is when the projected death risk premium is subtracted from the reserve. Due to this, the yield has to be estimated, and *i*, the technical interest rate serves this purpose.

13. ZILLMERIZATION AS A METHOD OF GROSS PREMIUM RESERVE CALCULATION

Due to its outstanding importance we discuss the mathematical relations of zillmerization in detail, in a separate chapter. Zillmerization can be regarded as general practice in Hungary, so when calculating the gross premium reserve of regular premium insurances we cannot leave it out.

At first zillmerization only affected the first year's premium, today it can theoretically affect even that f the past year. This way we'll examine separately the classical ("conservative") approach and today's approach.

13.1. ZILLMERIZATION – IN THE CONSERVATIVE VIEW

Zillmerization means borrowing part of the premium (or the premium reserve) of the first year at the beginning of the term, that will be repaid later on from the premium loadings in equal annual instalments. Because of this, an important element of the conservative approach is that zillmerization concerns strictly only annual premium payment policies.

The "conservative approach" in the title also means that a "natural" limit is set to the borrowing of the premium. According to this, the maximum sum borrowed can be a part of the first year's gross premium so that the remaining part of the premium still covers the continuous expenses[142] and the first year's death benefits. Consequently before premium payment the premium reserve is zero, and after premium payment even at the commencement of the insurance and at the first anniversary it is a positive value, since a part of the first net premium, and the second (greater) net premium totally goes to the reserve, and perhaps even the first premium has a part above the benefit payments that can fill the premium reserve. The limitation is necessary, because in the "modern" practice this limit is not followed, which very often causes the reserve of the first anniversary to be zero (i.e. neither the first, nor the second premium fills the reserve above the part covering the death benefit payments of the given year).

To make the discussion more clear, we define once more the notation used:

z: portion of the sum assured that shows how much more we use of the first year's premium than the cover of continuous expenses (primarily used to cover acquisition commission)

P_1: net premium of the first year, that remains from the gross premium after the first year's expenses

PZ: net premium of the consequent years, that already contains the repayment of zillmerization

P: original net premium

BP: gross premium

VP: original premium loading

FVP: continuous premium loading

p: net premium necessary to cover the risks of the first year

The following equations can easily be written based on the above notation:

$$BP = P + VP = P_1 + z + FVP = PZ + FVP \qquad (13.1.)$$

A further important equation,

$$PZ + FVP = P + z/\ddot{a}_{x:n} + FVP \qquad (13.2.)$$

can be proved starting out from the following.

[142] Under continuous expenses here we mean that the original premium loading is divided into two parts: the repayment of zillmerization and the part covering continuous expenses.

Instead of the original equivalence equation concerning the net premiums,

$$P \cdot \ddot{a}_{x:\overline{n}|} = A_{x:\overline{n}|} \tag{13.3.}$$

it is more correct to write the following in case of zillmerization (while that equation also holds, since we get the value of P from there):

$$P_1 + PZ \cdot \left(\ddot{a}_{x:\overline{n}|} - 1 \right) = A_{x:\overline{n}|} \tag{13.4.}$$

Figure 13.1. summarizes these relations:

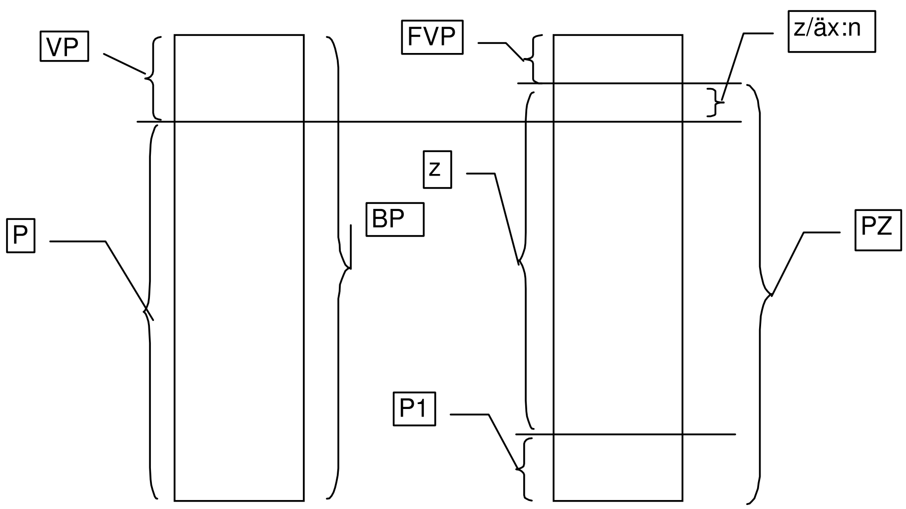

Figure 13.1.: Zillmerization in the conservative approach

The appendix of this chapter contains the proof of the relations.

The premium reserve can be written more precisely according to the above:

$$\underline{V}_0 = A_{x:\overline{n}|} - P_1 - PZ \cdot \left(\ddot{a}_{x:\overline{n}|} - 1 \right) \text{ and} \tag{13.5.}$$

$$\underline{V}_t = A_{x+t:\overline{n-t}|} - PZ \cdot \ddot{a}_{x+t:\overline{n-t}|} \text{, if } 1 \le t \tag{13.6.}$$

Applying the $PZ = P_{x:\overline{n}|} + \frac{z}{\ddot{a}_{x:\overline{n}|}}$ relation based on (13.2.), (13.6.) can also be written in the following form:

$$\underline{V}_t = A_{x+t:\overline{n-t}|} - \ddot{a}_{x+t:\overline{n-t}|} \cdot \left(P_{x:\overline{n}|} + \frac{z}{\ddot{a}_{x:\overline{n}|}} \right), \tag{13.7.}$$

And in the state after premium payment this changes to the following form:

$$\overline{V}_t = A_{x+t:\overline{n-t}|} - \left(\ddot{a}_{x+t:\overline{n-t}|} - 1 \right) \cdot \left(P_{x:\overline{n}|} + \frac{z}{\ddot{a}_{x:\overline{n}|}} \right) \tag{13.8.}$$

It can easily be seen that $\underline{V}_0 = 0$, since we have stated earlier that:

$$P_1 + PZ \cdot \left(\ddot{a}_{x:\overline{n}|} - 1 \right) = A_{x:\overline{n}|} \tag{13.9.}$$

Based on these: $\overline{V}_0 = P_1$

Z can be freely chosen within certain limits. It is naturally positive, and above a certain level P_1 won't be enough to cover the first year's risks. This way, if we denote the net premium necessary to cover the first year's risks, then the following criterion has to be satisfied:

$$P_1 \geq p \tag{13.10.}$$

This holds if z is chosen so that:

$$\frac{A_{x:\overline{n}|} - p}{\ddot{a}_{x:\overline{n}|} - 1} - p = \frac{A_{x:\overline{1}|}^{\ \ 1} \cdot A_{x+1:\overline{n-1}|}}{A_{x:\overline{1}|}^{\ \ 1} \cdot \ddot{a}_{x+1:\overline{n-1}|}} - p = \frac{A_{x+1:\overline{n-1}|}}{\ddot{a}_{x+1:\overline{n-1}|}} - p = P_{x+1:\overline{n-1}|} - p \geq z \tag{13.11.}$$

One of the key relations that we use here is:

$$A_{x:\overline{n}|} - p = A_{x:\overline{1}|}^{\ \ 1} \cdot A_{x+1:\overline{n-1}|} \tag{13.12.}$$

This is also deducted in the appendix of this chapter.

It is useful to calculate the values of the net premium part necessary to cover the first year's risks – p – in case of some traditional insurances. We can do this based on formula (13.12.).

In the **pure endowment** case:

$$p = A_{x:\overline{n}|}^{\ \ 1} - A_{x:\overline{1}|}^{\ \ 1} \cdot A_{x+1:\overline{n-1}|}^{\ \ \ \ 1} = \frac{D_{x+n}}{D_x} - \frac{D_{x+1}}{D_x} \cdot \frac{D_{x+n}}{D_{x+1}} = 0 \tag{13.13.}$$

since in this case there is no benefit payment during the term.

In case of **term** insurance:

$$\begin{aligned} p &= A^1_{x:\overline{n}|} - A_{x:\overline{1}|}^{\ \ 1} \cdot A^1_{x+1:\overline{n-1}|} = \frac{M_x - M_{x+n}}{D_x} - \frac{D_{x+1}}{D_x} \cdot \frac{M_{x+1} - M_{x+n}}{D_{x+1}} = \\ &= \frac{M_x - M_{x+1} - M_{x+n} + M_{x+n}}{D_x} = \\ &= \frac{C_x}{D_x} = q_x \cdot v^{\frac{1}{2}} \end{aligned} \tag{13.14.}$$

Since the **endowment** insurance is the sum of a pure endowment insurance and a term insurance, this way the value of p in case of the endowment insurance is equal to (13.14.).

In case of the **term fix** insurance:

$$p = A_{x:\overline{n}|} - A_{x:\overline{1}|}^{\;1} \cdot A_{x+1:\overline{n-1}|} = v^n - \frac{D_{x+1}}{D_x} \cdot v^{n-1} = v^n \cdot \left(1 - \frac{l_{x+1}}{l_x}\right) = \tag{13.15.}$$
$$= v^n \cdot q_x$$

The result is not surprising if we remember that the term fix insurance can be regarded as an endowment insurance with varying sum assured, which has a death sum assured equal to the value of the maturity benefit discounted to the time of death (and were the death benefit is – according to the policy – immediately changed to a single premium term fix insurance).

In case of the **pure endowment with premium refund** (according to 10.27.):
Since:

$$A_{x+t:\overline{n-t}|} = \frac{D_{x+n} + P^b_{x:\overline{n}|} \cdot \left(t \cdot M_{x+t} + R_{x+t} - R_{x+n} - n \cdot M_{x+n}\right)}{D_{x+t}} \tag{13.16.}$$

this way:

$$\begin{aligned}
p &= \frac{D_{x+n} + P^b_{x:\overline{n}|} \cdot \left(R_x - R_{x+n} - n \cdot M_{x+n}\right)}{D_x} - \\
&- \frac{D_{x+1}}{D_x} \cdot \frac{D_{x+n} + P^b_{x:\overline{n}|} \cdot \left(M_{x+1} + R_{x+1} - R_{x+n} - n \cdot M_{x+n}\right)}{D_{x+1}} = \\
&= \frac{P^b_{x:\overline{n}|} \cdot \left(R_x - R_{x+n} - n \cdot M_{x+n}\right)}{D_x} - \\
&- \frac{P^b_{x:\overline{n}|} \cdot \left(M_{x+1} + R_{x+1} - R_{x+n} - n \cdot M_{x+n}\right)}{D_x} = \\
&= \frac{P^b_{x:\overline{n}|} \cdot \left(R_x - M_{x+1} - R_{x+1}\right)}{D_x} = \frac{P^b_{x:n} \cdot \left(M_x - M_{x+1}\right)}{D_x} = \frac{P^b_{x:\overline{n}|} \cdot C_x}{D_x} = \\
&= P^b_{x:\overline{n}|} \cdot q_x \cdot v^{\frac{1}{2}}
\end{aligned} \tag{13.17.}$$

The values of PZ, $P_{x+1:n-1}$, P_1, and z are closely related to each other.
Theorem 1.: If P_1=p then PZ = $P_{x+1:n-1}$.
Theorem 2.: If PZ = $P_{x+1:n-1}$ then P_1 = p.
Theorem 3.: If $P_{x+1:n-1} - p = z$ then p = P_1.
Theorem 4.: If $P_{x+1:n-1} - p = z$ then $\underline{V}_1 = 0$.
The proof of the theorems is in the appendix of the chapter.

Instead of the above discussed reserve formula the equivalent formula is often applied that can be written in a more simple way (and we have also written these formulae when characterizing the products):

$$\underline{V}_t = A_{x+t:\overline{n-t}|} - PZ \cdot \ddot{a}_{x+t:\overline{n-t}|}, \text{ if } A_{x+t:\overline{n-t}|} - PZ \cdot \ddot{a}_{x+t:\overline{n-t}|} > 0 \tag{13.18.}$$

$$\underline{V}_t = 0 \text{, if } A_{x+t:\overline{n-t}|} - PZ \cdot \ddot{a}_{x+t:\overline{n-t}|} \le 0 \qquad (13.19.)$$

Here the differentiation of the first year's premium is missing, so the formula can even give a negative result, but then we make the reserve equal to 0.

Beside simplicity, that is an advantage of this formula, it also has its disadvantages:

- It is not so obvious to define the reserve after premium payment from the reserve before premium payment, as in case of the "precise" version.
- Some are incited to investigate the mystic meaning of the negative reserve.

As it can be seen from the above train of thought, in reality **negative reserve isn't created by zillmerization**[143], its minimum value is 0, as it can be seen from the precise form. Only the simplified form makes it seem like a negative reserve is created by zillmerization.

Although the "negative reserve" is a misunderstanding, but its value can be given a meaning according to the following. Let's compute the value of the following formula!

$$\begin{aligned}\underline{V}_0 &= A_{x:\overline{n}|} - PZ \cdot \ddot{a}_{x:\overline{n}|} = A_{x:\overline{n}|} - \left(P + \frac{z}{\ddot{a}_{x:\overline{n}|}} \right) \cdot \ddot{a}_{x:\overline{n}|} = \\ &= A_{x:\overline{n}|} - P \cdot \ddot{a}_{x:\overline{n}|} - z = -z\end{aligned} \qquad (13.20.)$$

This is nothing else but the sum (or its negative) that the insurer borrows from the first year's premium, and that the insurer starts to repay already at the beginning of the term, from the first year's premium loading.

This is understandable, since formulated in a different way this means that:

$$\overline{V}_0 = \underline{V}_0 + PZ = -z + PZ = P_1 \qquad (13.21.)$$

which we have already deducted. And we know that the value of the initial reserve after premium payment is lower than the zillmerized net premium, because a part of the first premium is taken out of the reserve (i.e. from the first premium).

Moving on we can generalize the above relation. If for some t

$$\underline{V}_t = A_{x+t:n-t} - PZ \cdot \ddot{a}_{x+t:n-t} < 0 \text{,} \qquad (13.22.)$$

then this means that the value of z has been set so high, that the first year's zillmer premium is not enough to cover it, and parts of the further premiums also have to be subtracted to cover it. How much has to be subtracted after t years (i.e. from the $(t+1)^{th}$ premium) is shown by formula 13.22. of the reserve before premium payment. (Naturally the $(t+1)^{th}$ premium might also not be enough for this, and then another part has to be subtracted from the $(t+2)^{th}$, etc… premiums.)

13.2. ZILLMERIZATION TODAY

The non-conservative approach means that z can exceed the theoretically allowed limit. Another sign of "modernity"[144] is that insurers do not strictly require annual premium payment, but are "satisfied" also with more frequent payment. If z exceeds the conservative limit, then (if we suppose that z stays within the first year's PZ) the division of PZ changes the following way, using new notation according to the following:

e: $PZ - P_{x+1:\overline{n-1}|}$, if this is positive

[143] At the same time the negative reserve – as we have seen – is an existing phenomenon, only not because of zillmerization! We have also seen that the negative reserve generally is a professional mistake.

[144] Naturally the monthly premium payment was not invented now, it also existed earlier, but it has not been so popular as today in Hungary.

h: $p - P_1$, if this is positive

p_i: the net premium part necessary in year i of the term to cover the risks of the given year ($p_1 = p$)

h_i: in year i of the term the part of the PZ of the given year that was not covered from the original z (naturally compounded, etc…, as it appears in the given year) ($h_1 = h$)

z_i: in year i of the term the part of the PZ of the given year that goes to pay z (naturally compounded, etc…, as it appears in the given year)

k: the last year, when the part of the zillmer premium that doesn't cover the risk of the given year goes totally to cover z

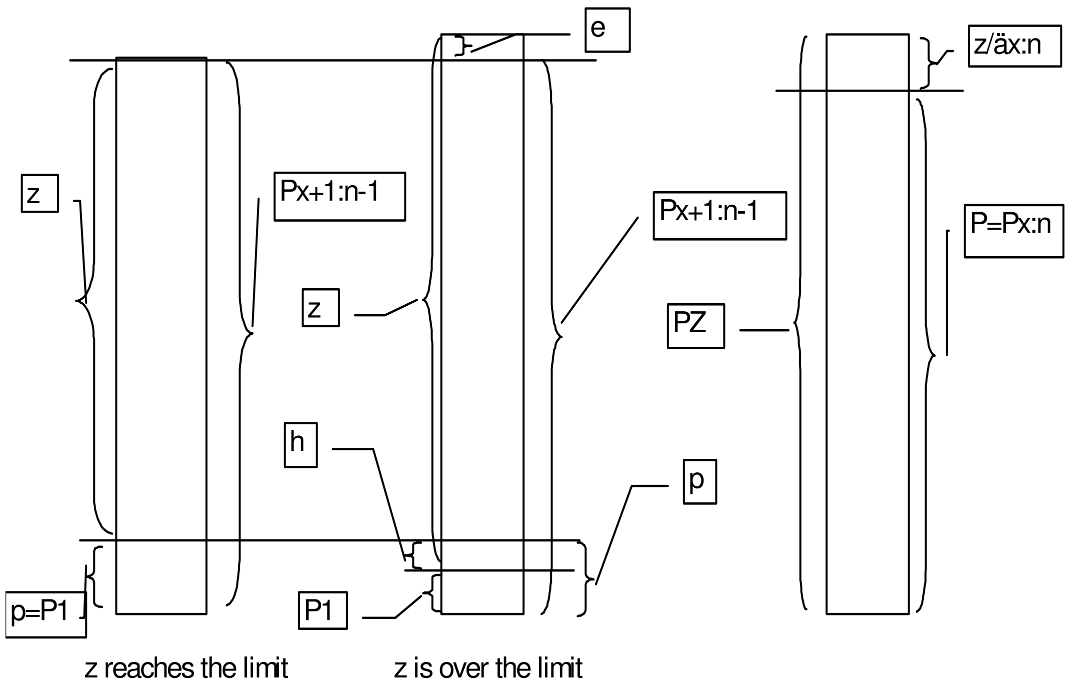

Figure 13.2.: Zillmerization today

Figure 13.2. shows this situation (at least when z doesn't exceed the limit when P_1 still stays positive).

In the first of the two situations indicated by the figure the inequality given for z is satisfied at the limit, so it is an equality, this way it is true that:

$$P_{x+1:\overline{n-1|}} - p = z \quad (13.23.)$$

In the second case this is changed to:

$$P_{x+1:\overline{n-1|}} - p < z\,. \quad (12.24.)$$

In the second case the new PZ exceeds that of the first case, at the limit by "e", so it is true that:

$$e = PZ - P_{x+1:\overline{n-1|}} \quad (13.25.)$$

Using the above, we can easily determine what the value is that has to be subtracted from the second (and maybe further) net (or zillmer) premium(s) for z (since the value PZ-p_1 was not enough). Naturally here we have to take into account that on the first (and further) anniversary already not all insured persons are alive, so the remaining value of z – compounded – has to be subtracted from the premium reserve of the remaining insured. In the computation we use the above interpretation of the “negative reserve”, that is what the insurer “prescribes” to be subtracted from the next premium (if that is enough). According to this, using the “wrong” premium reserve formula we can compute the part that we couldn’t subtract from z in the first year:

$$\begin{aligned}\underline{V}_1 &= A_{x+1:\overline{n-1|}} - PZ \cdot \ddot{a}_{x+1:\overline{n-1|}} = A_{x+1:\overline{n-1|}} - \left(P_{x+1:\overline{n-1|}} + e\right) \cdot \ddot{a}_{x+1:\overline{n-1|}} = \\ &= \left(A_{x+1:\overline{n-1|}} - P_{x+1:\overline{n-1|}} \cdot \ddot{a}_{x+1:\overline{n-1|}}\right) - e \cdot \ddot{a}_{x+1:\overline{n-1|}} = -e \cdot \ddot{a}_{x+1:\overline{n-1|}}\end{aligned} \tag{13.26.}$$

Since

$$h = p - P_1, \tag{13.27.}$$

we can give this the following meaning: from the PZ of the first year a part h less than z can be used for other purposes than to fill the reserve, since p has to go into the reserve under all circumstances. So in the current case P_1 means – contrary to the conservative approach – doesn’t mean the part filling the reserve only the part that remains after z.

Now let’s try to derive (13.26.) (i.e. how much the insurer has to subtract at the first policy anniversary) using h. We use the following relations that are obvious or have been derived earlier:

$$A_{x:\overline{n|}} - p = A_{x:1}{}^{1} \cdot A_{x+1:\overline{n-1|}} \tag{13.28.}$$

$$\ddot{a}_{x:\overline{n|}} - 1 = A_{x:\overline{1|}}{}^{1} \cdot \ddot{a}_{x+1:\overline{n-1|}} \tag{13.29.}$$

$$P_1 = PZ - z = \frac{A_{x:\overline{n|}} + z}{\ddot{a}_{x:\overline{n|}}} - z \tag{13.30.}$$

From (13.28.) and (13.29.):

$$\begin{aligned}p &= A_{x:\overline{n|}} - A_{x:\overline{1|}}{}^{1} \cdot A_{x+1:\overline{n-1|}} = A_{x:\overline{n|}} - \frac{\ddot{a}_{x:\overline{n|}} - 1}{\ddot{a}_{x+1:\overline{n-1|}}} \cdot A_{x+1:\overline{n-1|}} = \\ &= A_{x:\overline{n|}} - \left(\ddot{a}_{x:\overline{n|}} - 1\right) \cdot P_{x+1:\overline{n-1|}}\end{aligned} \tag{13.31.}$$

From (13.27.), (13.30.) and (13.31.):

$$\begin{aligned}h &= p - P_1 = A_{x:\overline{n|}} - \left(\ddot{a}_{x:\overline{n|}} - 1\right) \cdot P_{x+1:\overline{n-1|}} - \frac{A_{x:\overline{n|}} + z}{\ddot{a}_{x:\overline{n|}}} + z = \\ &= A_{x:\overline{n|}} + z - \ddot{a}_{x:\overline{n|}} \cdot P_{x+1:\overline{n-1|}} - \left(\frac{A_{x:\overline{n|}} + z}{\ddot{a}_{x:\overline{n|}}} - P_{x+1:\overline{n-1|}}\right) = \\ &= \left(\frac{A_{x:\overline{n|}} + z}{\ddot{a}_{x:\overline{n|}}} - P_{x+1:\overline{n-1|}}\right) \cdot \left(\ddot{a}_{x:\overline{n|}} - 1\right) \\ &= e \cdot \left(\ddot{a}_{x:\overline{n|}} - 1\right)\end{aligned} \tag{13.32.}$$

So the relation between h and e is:

$$e = \frac{h}{\left(\ddot{a}_{x:\overline{n}|} - 1\right)} \tag{13.33.}$$

Using (13.29.):

$$e = \frac{h}{A_{x:\overline{1}|}^{\;\;1} \cdot \ddot{a}_{x+1:\overline{n-1}|}} \tag{13.34.}$$

Substituting this into (13.26.)[145]:

$$\underline{V}_1 = -e \cdot \ddot{a}_{x+1:\overline{n-1}|} = -\frac{h}{A_{x:\overline{1}|}^{\;\;1} \cdot \ddot{a}_{x+1:\overline{n-1}|}} \cdot \ddot{a}_{x+1:\overline{n-1}|} = -h \cdot \frac{l_x}{l_{x+1}} \cdot (1+i) \tag{13.35.}$$

Interpretation: In the first year the p premium necessary to cover the risks of the first year fills the reserve, so in the next year the h missing from z also has to be subtracted. But in the meantime a year has passed, so h has to be compounded, and since some insured have died during this time, the total missing h (that has probably been spent) has to be subtracted from those still living.

It is useful to determine the risk premiums necessary in the individual years.
If

$$p_1 = A_{x:\overline{n}|} - A_{x:\overline{1}|}^{\;\;1} \cdot A_{x+1:\overline{n-1}|}, \tag{13.36.}$$

then this can easily be generalised the following way:

$$p_2 = A_{x+1:\overline{n-1}|} - A_{x+1:\overline{1}|}^{\;\;1} \cdot A_{x+2:\overline{n-2}|}, \tag{13.37.}$$

…

$$p_k = A_{x+k-1:\overline{n-k+1}|} - A_{x+k-1:\overline{1}|}^{\;\;1} \cdot A_{x+k:\overline{n-k}|}, \tag{13.38.}$$

…

$$p_n = A_{x+n-1:\overline{1}|} \tag{13.39.}$$

(It can be checked the following way: the discounted expected value of necessary premiums has to equal the discounted value of benefits, that is true, since:

$$\begin{aligned} &p_1 + A_{x:\overline{1}|}^{\;\;1} \cdot p_2 + A_{x:\overline{2}|}^{\;\;1} \cdot p_3 + \ldots + A_{x:\overline{n-1}|}^{\;\;1} \cdot p_n = \\ &= A_{x:\overline{n}|} - A_{x:\overline{n}|}^{\;\;1} \cdot A_{x+n:\overline{n-n}|} = A_{x:\overline{n}|} \end{aligned} \tag{13.40.}$$

using the relation on the expected value of necessary premiums derived in the appendix of chapter 13.4.2.)

Let's look at the value of p_i in the concrete case of the 4 classical insurances!

145 That is naturally not the value of the premium reserve at the first anniversary, since that is 0!

Term- and endowment insurance:

$$p_{t+1} = q_{x+t} \cdot v^{\frac{1}{2}} \tag{13.41.}$$

term fix insurance:
Since here $A_{x:\overline{n|}} = v^n$, this way:

$$p_{t+1} = v^{n-t} - \frac{D_{x+t+1}}{D_{x+t}} \cdot v^{n-t-1} = v^{n-t} \cdot \left(1 - p_{x+t}\right) = v^{n-t} \cdot q_{x+t} \tag{13.41.}$$

Pure endowment insurance with premium refund:

$$\begin{aligned}
p_{t+1} &= A_{x+t:\overline{n-t|}} - \frac{D_{x+t+1}}{D_{x+t}} \cdot A_{x+t+1:\overline{n-t-1|}} = \\
&= \frac{D_{x+n} + P^b_{x:\overline{n|}} \cdot \left(t \cdot M_{x+t} + R_{x+t} - R_{x+n} - n \cdot M_{x+n}\right)}{D_{x+t}} - \\
&- \frac{D_{x+t+1}}{D_{x+t}} \cdot \frac{D_{x+n} + P^b_{x:\overline{n|}} \cdot \left(\left(t+1\right) \cdot M_{x+t+1} + R_{x+t+1} - R_{x+n} - n \cdot M_{x+n}\right)}{D_{x+t+1}} = \\
&= \frac{P^b_{x:\overline{n|}} \cdot \left(t \cdot M_{x+t} - \left(t+1\right) \cdot M_{x+t+1} + M_{x+t}\right)}{D_{x+t}} = \\
&= \left(t+1\right) \cdot P^b_{x:\overline{n|}} \cdot \frac{C_{x+t}}{D_{x+t}} = \left(t+1\right) \cdot P^b_{x:\overline{n|}} \cdot q_{x+t} \cdot v^{\frac{1}{2}}
\end{aligned} \tag{13.42.}$$

So:

$$p_{t+1} = \left(t+1\right) \cdot P^b_{x:\overline{n|}} \cdot q_{x+t} \cdot v^{\frac{1}{2}} \qquad 13.43.)$$

The zillmer premium can be split into three parts in the first year (and naturally also in the following years) according to 13.44.:

$$PZ = p_1 + z + \left(-h_1\right), \tag{13.44.}$$

where the sign of h_1 is due to its negative meaning.
If $h_1>0$, then this means that z has been chosen greater than what fits into the zillmer premium of the first year, so payments to repay z have to be made even from the second (and maybe also later) premium(s). We have seen that we do not subtract h_1 from the second years premium, but more than that, since the remaining part that has to be repaid bears interest, and some insured have died and their part also has to be subtracted from the premium of the living. But comparing the compounded value of h_1 divided by the survival probability with the ($PZ-p_2$) value of the next year is equivalent with comparing h_1 to the discounted present value of the next year's maximum cover (i.e. $PZ-p_2$), and so on, if a part of h_1 still remains. According to this, z has to be divided between the following values:
in year 1:

$$PZ - p_1 \tag{13.45.}$$

in year 2:

$$p_x \cdot v \cdot (PZ - p_2) = \frac{l_{x+1}}{l_x} \cdot v \cdot (PZ - p_2) = A_{x:\overline{1}|}^{\;\;1} \cdot (PZ - p_2) \tag{13.46.}$$

in year 3:

$$p_x \cdot p_{x+1} \cdot v^2 \cdot (PZ - p_3) = \frac{l_{x+2}}{l_x} \cdot v^2 \cdot (PZ - p_3) = A_{x:\overline{2}|}^{\;\;1} \cdot (PZ - p_3) \tag{13.47.}$$

etc…

If we do not know at start which year the last one will be (but we know that it is greater than 1) when we subtract from the zillmer premium to cover z, then we can say that we are looking for the k value, that satisfies:

$$z \geq \sum_{j=0}^{k-1} \frac{l_{x+j}}{l_x} \cdot v^j \cdot (PZ - p_{j+1}), \text{ and} \tag{13.48.}$$

$$z < \sum_{j=0}^{k} \frac{l_{x+j}}{l_x} \cdot v^j \cdot (PZ - p_{j+1}) \tag{13.49.}$$

We have derived a formula for this in the appendix of chapter 13.4.1.2., but we can get the same result in a more simple way. Then we are looking for the least k value that satisfies:

$$A_{x+k:\overline{n-k}|} - PZ \cdot \ddot{a}_{x+k:\overline{n-k}|} + PZ - p_{k+1} > 0, \tag{13.50.}$$

so when the reserve is already filled from the new zillmer premium which is not used up completely to repay z (above the part covering the risks of the given year).

On the z part "prescribed" on the further anniversaries and on the premium reserve, we have the following hypothesis:

Hypothesis: Let's suppose that $h_k > 0$, but $h_{k+1} = 0$, where 1 <= k < n. Then for h_i the following holds:

$$h_1 = z + p_1 - PZ \tag{13.51.}$$

$$h_i = \frac{h_{i-1}}{A_{x+i-1:\overline{1}|}^{\;\;1}} + p_i - PZ \text{, where } 1 < i \leq k \tag{13.52.}$$

Then z (taking into account that the missing part of the deceased is paid by the living) arrives to the insurer in k+1 payments according to the following:

$$z_i = PZ - p_i \text{, where } i \leq k \text{ and} \tag{13.53.}$$

$$z_{k+1} = \frac{h_k}{A_{x+k-1:\overline{1}|}^{\;\;1}} \text{, where } z_{k+1} \leq PZ - p_{k+1} \tag{13.54.}$$

To prove the hypothesis we first prove the following lemma:

Lemma: Let's choose a z for a given insurance with an insured of age *x* and a term of *n* years so that $h_1 = z + p_1 - PZ > 0$. Then if everything else is unchanged, if we take an insured of age *x+1* years, a term of *n-1* years and a zillmer percentage of this new insurance, z' so that $z' = \frac{h_1}{A^{\;\;1}_{x:\overline{1}|}}$, then looking at the zillmer premium of this new insurance it will be true that: PZ' = PZ.

Proof:

Using the relation deducted in 13.25. and 13.33.:

$$e = PZ - P_{x+1:\overline{n-1}|} = \frac{h_1}{\left(\ddot{a}_{x:\overline{n}|} - 1\right)} \text{ and the classical formula} \tag{13.55.}$$

$$\ddot{a}_{x:\overline{n}|} - 1 = A^{\;\;1}_{x:\overline{1}|} \cdot \ddot{a}_{x+1:\overline{n-1}|}, \tag{13.56.}$$

$$\begin{aligned} PZ' &= P_{x+1:\overline{n-1}|} + \frac{z'}{\ddot{a}_{x+1:\overline{n-1}|}} = P_{x+1:\overline{n-1}|} + \frac{1}{\ddot{a}_{x+1:\overline{n-1}|}} \cdot \frac{h_1}{A^{\;\;1}_{x:\overline{1}|}} = \\ &= P_{x+1:\overline{n-1}|} + \frac{\left(PZ - P_{x+1:\overline{n-1}|}\right) \cdot \left(\ddot{a}_{x:\overline{n}|} - 1\right)}{\ddot{a}_{x+1:\overline{n-1}|}} \cdot \frac{\ddot{a}_{x+1:\overline{n-1}|}}{\left(\ddot{a}_{x:\overline{n}|} - 1\right)} = \\ &= P_{x+1:\overline{n-1}|} + \left(PZ - P_{x+1:\overline{n-1}|}\right) = PZ \end{aligned} \tag{13.57.}$$

Q.E.D.

From the lemma, and applying it in series follows the proof of the hypothesis.

We summarize the values of the premium reserves at anniversaries in table 13.1. (supposing that as before, there is no premium increase):

Anniversary		0	1	2	..	k-1	k
Premium income (BP=P+VP=PZ+FVP)		BP	BP	BP		BP	BP
From this	Filling the reserve	p_1	p_1	p_3		p_k	$A_{x+k:\overline{n-k}|} - PZ \cdot \ddot{a}_{x+k:\overline{n-k}|} + PZ > p_{k+1}$
	Repaying z	PZ - p_1	PZ – p_2	PZ – p_3		PZ – p_k	$PZ \cdot \ddot{a}_{x+k:\overline{n-k}|} - A_{x+k:\overline{n-k}|}$
	Covering other expenses	FVP	FVP	FVP		FVP	FVP
$\underline{V}_k$		0	0	0		0	0
$\overline{V}_k$		p_1	p_2	p_3		p_k	$A_{x+k:\overline{n-k}|} - PZ \cdot \ddot{a}_{x+k:\overline{n-k}|} + PZ > p_{k+1}$

Table 13.1.: The values of premium reserves at anniversaries

In the non-conservative case we leave the practice that z has to "fit into" the zillmer premium of the first year. The question arises, whether there is an upper limit on z? The answer is yes and no. No, because the zillmer premium increases with z, so it “finances” itself. Yes, because we do not want the gross premium to grow above all limits, we usually define its maximum value compared to the net premium (without zillmerization).

So we have a limit depending on the gross premium. According to this, PZ cannot be higher than the gross premium, i.e.:

$$PZ = P + \frac{z}{\ddot{a}_{x:\overline{n}|}} < P \cdot (1+\lambda) \tag{13.58.}$$

So:

$$\frac{z}{\ddot{a}_{x:\overline{n}|}} < \lambda \cdot P \tag{13.59.}$$

Naturally the λ loading has to exceed the z/ä value by a sum that can bear the necessary continuous premium loadings.

If we want to write the recursive formula in the zillmerized case, then we can say that the recursive formula is the same in the zillmerized case as in the non-zillmerized with the following differences:

In the first k anniversaries we do not use the recursive formula, but table 13.1. with the 0 and p_i values.

In the following years the recursive formula can be used, but naturally with writing PZ in the place of P!

So-far we’ve concentrated on the calculation of the annual reserves, but for most actuarial calculations we need the reserve at any moment. The recursive formulae can be used to determine starting out from the reserve of an anniversary not only the value of the next anniversary, but also the value at any point of time during the year.

Repeating the value of the reserve at the tth anniversary, before premium payment – supposing **annual premium payment** – depends on the sign of 13.50:

0, if $\left(A_{x+t:\overline{n-t}|} - PZ \cdot \ddot{a}_{x+t:\overline{n-t}|}\right) + \left(PZ - p_{t+1}\right) > 0$ is not true and (13.60.)

$A_{x+t:\overline{n-t}|} - PZ \cdot \ddot{a}_{x+t:\overline{n-t}|}$, if it is. (13.61.)

After premium payment (that is in reality very important to us):

p_{t+1}, if it is not true and

$A_{x+t:\overline{n-t}|} - PZ \cdot \ddot{a}_{x+t:\overline{n-t}|} + PZ$, if it is true. (13.62.)

If $\left(A_{x+t:\overline{n-t}|} - PZ \cdot \ddot{a}_{x+t:\overline{n-t}|}\right) + \left(PZ - p_{t+1}\right) > 0$ is true, then the formula of premium reserve in case of not annual premium payment can be determined in a relatively standard way.

If $\left(A_{x+t:\overline{n-t}|} - PZ \cdot \ddot{a}_{x+t:\overline{n-t}|}\right) + \left(PZ - p_{t+1}\right) > 0$ is not true, then before premium payment it is 0, but after premium payment it is:

$$\overline{V}_t = \frac{p_{t+1}}{gy} \cdot k_{gy}\,, \tag{13.63.}$$

where the value of gy depends on the frequency of premium payment (12, 4 or 2), and the calculation of k_{gy}[146] can be found in appendix 13.4.4.

It is worth looking at the mid-year premium reserve in the first few years. We divide the year into gy even segments. Before premium payment it is always 0, and after premium payment

$$\overline{V}_{t+\frac{l}{gy}} = \frac{p_{t+1}}{gy} \cdot k_{gy} \tag{13.64.}$$

(for every 0<l<gy value) is the value of the premium reserve for every dividing point. Between dividing points the premium reserve is the linear interpolation of

$$\overline{V}_t = \frac{p_{t+1}}{gy} \cdot k_{gy} \tag{13.65.}$$

and 0. (Of course this is an estimated value, but it is more than satisfactory). Naturally k_1=1.

13.3. THE INTERPRETATION OF ZILLMERIZATION

What happens if during the determination of the value of z, the insurer hasn't taken into account the limit, that z has to "fit into" the annual gross premium, i.e. z and the loading were set independently? Then naturally the part of z due is subtracted from the gross premium, although the insurer has to fill the missing part from some other source.[147]

Naturally we are not certain that any problem would arise because of exceeding the limit, since it is not certain that the true expenses of the insurer and the determination of z are synchronised. It is possible that the insurer sets z higher than the expense coverage he needs. But the examination of this case would require a separate study!

The question arises what happens if the policy is surrendered in the phase with 0 reserve? This question is especially interesting in the case of premium payment more frequent than annual. The possibility of lapses throws light on the significance of the classical approach and why annual used to be the dominantly supposed premium frequency. Since in these cases the expenses behind z are mostly covered from premium income, so zillmerization actually gave the insurer more sources.

Nowadays, on the other hand, when premium payment is usually not annual, and exceeding the "natural limit" of z can be considered general, zillmerization almost causes more problems than it solves. In case of non-annual premium payment, namely, the insurer is not relieved from having to finance acquisition expenses – at least for a while – from its own capital. This problem is only intensified if its level exceeds the value that the first year's zillmer premium can cover.

Moreover, in case of lapse the insurer not only has to claw back commission paid to the agent for disciplinary purposes, but also because it was fundamentally the insurer's money advanced for acquisition. So in these cases – contrary to the classical approach – the insurer has used up a z that hasn't even arrived yet.

So in cases when the premium payment is not annual, zillmerization cannot fulfil its classical function. Then from the original Zillmerien thoughts the only one remaining is that zillmerization somewhat refines the insurer's cash flow, since a greater part of premium can be used to cover expenses than in the non-zillmerized case. But in these cases other

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[146] A $\frac{k_{gy}}{gy}$ tényező a gy növekedésével gyorsan csökken.

[147] Erre időnként gyakorlati példa is van némelyik biztosítónál! (Jogszabály nem tiltja ezt a gyakorlatot.)

methods have to be applied beside zillmerization to solve the financing of the insurer. This is even more true if z exceeds the classical limit.

Finally let's discuss a little bit how the zillmerized "negative reserve" can be handled from the accounting point of view! Naturally emphasizing that "negative reserve" is not created through zillmerization, we can give an accounting meaning to the negative value calculated by the simplified formula of zillmerization – even more so, since we know the meaning of this value.

The reason of zillmerization: the initial expenses related to the insurance (mainly acquisition expenses) arise already at the beginning of the term, and the cover of these expenses is the "z" part nipped off the first (or first k+1) premium(s).

So the insurer signs the policy, the first year's premium arises and is divided into three parts:

1. The part necessary to cover the first year's risks (or a little more than this) fills the premium reserve
2. $1/z^{th}$ part of the sum assured goes to cover initial expenses
3. The continuous premium loading covers continuous expenses.

If the client terminates the policy during the first year, then the insurer can return the client the current sum of the reserve and still be "at the money", supposing that:

1. Premium payment was annual in reality
2. Z has been determined the conservative way
3. Initial expenses have not exceeded $1/z^{th}$ of the sum insured.

If initial expenses have exceeded $1/z^{th}$ of the sum insured, then the insurer has to bear the consequences, and if they have not exceeded it, then the insurer can be happy. But anyway the insurer can suppose that $1/z^{th}$ of the sum insured describes the initial expenses, even if he knows this is only approximately true.

If, on the other hand the premium payment was not annual and/or zillmerization exceeds the conservative limit, then surrendering the insurance within the first k years could result that the insurance premiums paid do not cover the acquisition costs – regardless of zillmerization! This phenomenon is – in a certain sense – contrary to the principle of zillmerization, which also shows that it has been developed for annual premium payment (and k=1).

It this case the following tactic could be applied (that in the end the insurer uses even if there is no zillmerization):

- we suppose that the initial expenses equal the "negative premium reserve" that can be calculated at the beginning of the term through zillmerization.
- the insurer has paid out this sum in reality, but it has not arrived yet from premium payment (because of the non-annual premium frequency)
- the sum paid out for initial expenses can be considered as recovered if the zillmerized reserve becomes positive "on its own"
- until this premium reserve is negative, and to the extent of its negativity, the negative part is accounted as a profit neutralizing factor (deferred charges), since it is an expense that will soon be covered
- if it should not be recovered (the policy is terminated in the "negative" phase), then the deferred charges on the premium reserve of the terminated policy are resolved (i.e. accounted as a loss)
- the loss can be decreased by efficient commission claw back from the agent.

In the above case the key was that we identified initial expenses (for the sake of simplicity) with the negative premium reserve, and their recovery with the decreasing negativity of the reserve.

If we had not zillmerized, then the initial expenses would have been recovered from the premium loadings in much smaller portions and during a longer time. In this case the deferred charges have to be maintained for a much longer period of time (if this is allowed by accounting regulation).

It is important to mention that the above procedure is justified if our applied reserve formula is sensitive to net premiums actually arriving (and the very general linear approximation is not used during the year, e.g. even in case of annual premium payment).

13.4. Appendix: Relations Concerning Zillmerization, their Proof and Other Addendums

13.4.1. Zillmerization in the Conservative Approach

13.4.1.1. The Division of the Premium Based on the Equivalence Equation

Instead of the equivalence equation of the original net premium

$$P \cdot \ddot{a}_{x:\overline{n}|} = A_{x:\overline{n}|} \tag{13.66.}$$

it is more correct in case of zillmerization to write the following (while this equation also holds, since we get the value of P from this one):

$$P_1 + PZ \cdot \left(\ddot{a}_{x:\overline{n}|} - 1 \right) = A_{x:\overline{n}|} \tag{13.67.}$$

From the modified equivalence equation and from obvious equalities we can deduct the two types of net premiums.

$$P_1 + PZ \cdot \left(\ddot{a}_{x:\overline{n}|} - 1 \right) = A_{x:\overline{n}|} = P \cdot \ddot{a}_{x:\overline{n}|} \tag{13.68.}$$

and

$$P_1 + z = PZ \tag{13.69.}$$

Based on these:

$$P_1 + (P_1 + z) \cdot \left(\ddot{a}_{x:\overline{n}|} - 1 \right) = P \cdot \ddot{a}_{x:\overline{n}|} \tag{13.70.}$$

so:

$$P_1 \cdot \ddot{a}_{x:\overline{n}|} = P \cdot \ddot{a}_{x:\overline{n}|} - z \cdot \left(\ddot{a}_{x:\overline{n}|} - 1 \right) \tag{13.71.}$$

$$P_1 = P - z \cdot \frac{\left(\ddot{a}_{x:\overline{n}|} - 1 \right)}{\ddot{a}_{x:\overline{n}|}} \tag{13.72.}$$

$$PZ = P_1 + z = P - z \cdot \frac{\left(\ddot{a}_{x:\overline{n}|} - 1 \right)}{\ddot{a}_{x:\overline{n}|}} + z = P + \frac{z}{\ddot{a}_{x:\overline{n}|}} \tag{13.73.}$$

13.4.1.2. The Minimum Condition Regarding z

It is the maximum criterion that makes zillmerization “conservative”.

If we denote the net premium necessary to cover the risks of the first year by p, then it is true that:

$$P_1 \geq p\,, \tag{13.74.}$$

so:

$$P_1 = P - z\cdot\frac{\left(\ddot{a}_{x:\overline{n|}}-1\right)}{\ddot{a}_{x:\overline{n|}}} = \frac{A_{x:\overline{n|}}}{\ddot{a}_{x:\overline{n|}}} - z\cdot\frac{\left(\ddot{a}_{x:\overline{n|}}-1\right)}{\ddot{a}_{x:\overline{n|}}} \geq p\,. \tag{13.75.}$$

Because of this:

$$\frac{A_{x:\overline{n|}}}{\ddot{a}_{x:\overline{n|}}} - p \geq z\cdot\frac{\left(\ddot{a}_{x:\overline{n|}}-1\right)}{\ddot{a}_{x:\overline{n|}}} \tag{13.76.}$$

From which it follows that:

$$\left(\frac{A_{x:\overline{n|}}}{\ddot{a}_{x:\overline{n|}}} - p\right)\cdot\frac{\ddot{a}_{x:\overline{n|}}}{\ddot{a}_{x:\overline{n|}}-1} = \frac{A_{x:\overline{n|}}}{\ddot{a}_{x:\overline{n|}}-1} - p\cdot\frac{\ddot{a}_{x:\overline{n|}}}{\ddot{a}_{x:\overline{n|}}-1} = \frac{A_{x:\overline{n|}}-p}{\ddot{a}_{x:\overline{n|}}-1} - p \geq z \tag{13.77.}$$

If we think about it deeper, we can give the following interpretation to the numerator of the last fraction:

$$A_{x:\overline{n|}} - p = A_{x:\overline{1|}}^{\;\;1}\cdot A_{x+1:\overline{n-1|}} \tag{13.78.}$$

i.e.: since p is the premium necessary for the first year's risks, this way if we subtract this from the premium necessary for *n* years, we get a premium that is necessary a year from now for the following *n-1* years – supposing that the policy will still be in effect at that time. And we get this from the multiple of a pure endowment insurance of 1 year and an insurance with an insured of age *x+1* and a term of *n-1*.

Since it is well known (it is practically based on the same principle, and can be easily deducted) that:

$$\ddot{a}_{x:\overline{n|}} - 1 = A_{x:\overline{1|}}^{\;\;1}\cdot \ddot{a}_{x+1:\overline{n-1|}} \tag{13.79.}$$

so:

$$\frac{A_{x:\overline{n|}}-p}{\ddot{a}_{x:\overline{n|}}-1} - p = \frac{A_{x:\overline{1|}}^{\;\;1}\cdot A_{x+1:\overline{n-1|}}}{A_{x:\overline{1|}}^{\;\;1}\cdot \ddot{a}_{x+1:\overline{n-1|}}} - p = \frac{A_{x+1:\overline{n-1|}}}{\ddot{a}_{x+1:\overline{n-1|}}} - p = P_{x+1:\overline{n-1|}} - p \geq z \tag{13.80.}$$

13.4.1.3. The Relation of PZ, $P_{x+1:n-1}$, P_1, p, z and the Premium Reserve at the First Anniversary

Theorem 1.: If P_1=p, then PZ = $P_{x+1:n-1}$.

Proof: It can easily be seen based on the above.

Since it is true for p that:

$$p = A_{x:\overline{n|}} - A_{x:\overline{1}}^{\;\;1}\cdot A_{x+1:\overline{n-1|}} \tag{13.81.}$$

it is true for z that:

$$P_1 = P - z \cdot \frac{\left(\ddot{a}_{x:\overline{n}|} - 1\right)}{\ddot{a}_{x:\overline{n}|}}\text{, from which follows that: } \left(P - P_1\right) \cdot \frac{\ddot{a}_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|} - 1} = z \qquad (13.82.)$$

for PZ it is true that:

$$PZ = P_1 + z \qquad (13.83.)$$

We also use relation 13.79.:

$$\ddot{a}_{x:\overline{n}|} - 1 = A_{x:\overline{1}|}^{\ 1} \cdot \ddot{a}_{x+1:\overline{n-1}|} \qquad (13.84.)$$

Substituting (13.82.) into (13.83.) we get:

$$PZ = P_1 + z = P_1 + \left(P - P_1\right) \cdot \frac{\ddot{a}_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|} - 1} = P \cdot \frac{\ddot{a}_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|} - 1} - P_1 \cdot \frac{1}{\ddot{a}_{x:\overline{n}|} - 1} \qquad (13.85.)$$

In the place of P we write $\frac{A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}}$, in the place of P_1 we put (13.81.) based on the supposition of $P_1 = p$, and substitute (13.84.) in the place of the denominator:

$$\begin{aligned} PZ &= \frac{A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}} \cdot \frac{\ddot{a}_{x:\overline{n}|}}{A_{x:\overline{1}|}^{\ 1} \cdot \ddot{a}_{x+1:\overline{n-1}|}} - \left(A_{x:\overline{n}|} - A_{x:1}^{\ 1} \cdot A_{x+1:\overline{n-1}|}\right) \cdot \frac{1}{A_{x:\overline{1}|}^{\ 1} \cdot \ddot{a}_{x+1:\overline{n-1}|}} = \\ &= \frac{A_{x:\overline{1}|}^{\ 1} \cdot A_{x+1:\overline{n-1}|}}{A_{x:\overline{1}|}^{\ 1} \cdot \ddot{a}_{x+1:\overline{n-1}|}} = \frac{A_{x+1:\overline{n-1}|}}{\ddot{a}_{x+1:\overline{n-1}|}} = P_{x+1:\overline{n-1}|} \end{aligned} \qquad (13.86.)$$

Q.E.D.

What is important: for z we can not only use the part of the original net premium above the cover of the first year's risks, but more than this.

The reverse of Theorem 1 is also true:

Theorem 2.: If PZ = $P_{x+1:n-1}$, then P_1 = p.

Proof: Since we know that

$p = A_{x:\overline{n}|} - A_{x:\overline{1}|}^{\ 1} \cdot A_{x+1:\overline{n-1}|}$ and P_1 = PZ – z, this way we have to prove that

$$PZ - z = A_{x:\overline{n}|} - A_{x:\overline{1}|}^{\ 1} \cdot A_{x+1:\overline{n-1}|}\,. \qquad (13.87.)$$

if

$$PZ = P_{x:\overline{n}|} + \frac{z}{\ddot{a}_{x:\overline{n}|}} = P_{x+1:\overline{n-1}|}\,. \qquad (13.88.)$$

Expressing z from (13.88.):

$$z = \left(P_{x+1:\overline{n-1}|} - P_{x:\overline{n}|}\right) \cdot \ddot{a}_{x:\overline{n}|} \qquad (13.89.)$$

and substituting this into the left hand side of (13.87.):

$$P_{x+1:\overline{n-1|}} - \left(P_{x+1:\overline{n-1|}} - P_{x:\overline{n|}}\right) \cdot \ddot{a}_{x:\overline{n|}} = \ddot{a}_{x:\overline{n|}} \cdot P_{x:\overline{n|}} - \left(\ddot{a}_{x:\overline{n|}} - 1\right) \cdot P_{x+1:\overline{n-1|}} = \\ = A_{x:\overline{n|}} - A_{x:\overline{1|}}^{\;1} \cdot A_{x+1:\overline{n-1|}} \tag{13.90.}$$

holds, which is what we had to prove. Q.E.D.
So Theorem 1 and 2 together state that if PZ = $P_{x+1:n-1}$, then P_1 = p and vice versa.
Theorem 3.: If $P_{x+1:\overline{n-1|}} - p = z$, then p = P_1

Proof: We know that

$$P_1 = P - z \cdot \frac{\left(\ddot{a}_{x:\overline{n|}} - 1\right)}{\ddot{a}_{x:\overline{n|}}} \tag{13.91.}$$

and

$$\frac{A_{x:\overline{n|}} - p}{\ddot{a}_{x:\overline{n|}} - 1} = P_{x+1:\overline{n-1|}} \tag{13.92.}$$

If $P_{x+1:n-1} - p = z$, then from (13.92.) we get

$$z = P_{x+1:\overline{n-1|}} - p = \frac{A_{x:\overline{n|}} - p}{\ddot{a}_{x:\overline{n|}} - 1} - p = \frac{A_{x:\overline{n|}} - \ddot{a}_{x:\overline{n|}} \cdot p}{\ddot{a}_{x:\overline{n|}} - 1} \tag{13.93.}$$

Substituting this into (13.91.):

$$P_1 = P - z \cdot \frac{\left(\ddot{a}_{x:\overline{n|}} - 1\right)}{\ddot{a}_{x:\overline{n|}}} = P - \frac{A_{x:\overline{n|}} - \ddot{a}_{x:\overline{n|}} \cdot p}{\ddot{a}_{x:n} - 1} \cdot \frac{\left(\ddot{a}_{x:\overline{n|}} - 1\right)}{\ddot{a}_{x:n}} = \\ = P - \frac{A_{x:\overline{n|}} - \ddot{a}_{x:\overline{n|}} \cdot p}{\ddot{a}_{x:\overline{n|}}} = P - \frac{A_{x:\overline{n|}}}{\ddot{a}_{x:\overline{n|}}} + p = p \tag{13.94.}$$

So the statement of the theorem is true. Q.E.D.
Theorem 4.: If $P_{x+1:\overline{n-1|}} - p = z$, then $\underline{V}_1 = 0$

Proof: We know that

$$P_1 + z = PZ \tag{13.95.}$$

According to Theorem 3, if $P_{x+1:\overline{n-1|}} - p = z$, then p = P_1, so in this case:

$$P_{x+1:\overline{n-1|}} - P_1 = z \text{ , i.e.: } P_{x+1:\overline{n-1|}} = z + P_1 = PZ \tag{13.96.}$$

$$\underline{V}_1 = A_{x+1:\overline{n-1|}} - PZ \cdot \ddot{a}_{x+1:\overline{n-1|}} = A_{x+1:\overline{n-1|}} - P_{x+1:\overline{n-1|}} \cdot \ddot{a}_{x+1:\overline{n-1|}} = \\ = A_{x+1:\overline{n-1|}} - \frac{A_{x+1:\overline{n-1|}}}{\ddot{a}_{x+1:\overline{n-1|}}} \cdot \ddot{a}_{x+1:\overline{n-1|}} = 0 \tag{13.97.}$$

Q.E.D.

13.4.2. The Discounted Expected Value of Necessary Premiums

If p_i is the risk premium necessary in year *i*, the what is the discounted expected value of premiums necessary in the first *k* years, i.e. what is the following value?

$$p_1 + A^{\;1}_{x:\overline{1}|} \cdot p_2 + A^{\;1}_{x:\overline{2}|} \cdot p_3 + \ldots + A^{\;1}_{x:\overline{k-1}|} \cdot p_k \tag{13.98.}$$

Let's use the following two relations:

$$p_k = A_{x+k-1:\overline{n-k+1}|} - A^{\;1}_{x+k-1:\overline{1}|} \cdot A_{x+k:\overline{n-k}|}\,, \tag{13.99.}$$

and

$$A^{\;1}_{x:\overline{k}|} \cdot A^{\;1}_{x+k:\overline{1}|} = \frac{D_{x+k}}{D_x} \cdot \frac{D_{x+k+1}}{D_{x+k}} = A^{\;1}_{x:\overline{k+1}|} \tag{13.100.}$$

Then:

$$\begin{aligned}
& p_1 + A^{\;1}_{x:\overline{1}|} \cdot p_2 + A^{\;1}_{x:\overline{2}|} \cdot p_3 + \ldots + A^{\;1}_{x:\overline{k-1}|} \cdot p_k = \\
& = \left(A_{x:\overline{n}|} - A^{\;1}_{x:\overline{1}|} \cdot A_{x+1:\overline{n-1}|} \right) + A^{\;1}_{x:\overline{1}|} \cdot \left(A_{x+1:\overline{n-1}|} - A^{\;1}_{x+1:\overline{1}|} \cdot A_{x+2:\overline{n-2}|} \right) + \ldots \\
& + A^{\;1}_{x:\overline{k-1}|} \cdot \left(A_{x+k-1:\overline{n-k+1}|} - A^{\;1}_{x+k-1:\overline{1}|} \cdot A_{x+k:\overline{n-k}|} \right) = \\
& = A_{x:\overline{n}|} - A^{\;1}_{x:\overline{1}|} \cdot A_{x+1:\overline{n-1}|} + A^{\;1}_{x:\overline{1}|} \cdot A_{x+1:\overline{n-1}|} - A^{\;1}_{x:\overline{1}|} \cdot A^{\;1}_{x+1:\overline{1}|} \cdot A_{x+2:n-2} + \ldots \\
& + A^{\;1}_{x:\overline{k-1}|} \cdot A_{x+k-1:\overline{n-k+1}|} - A^{\;1}_{x:\overline{k-1}|} \cdot A^{\;1}_{x+k-1:\overline{1}|} \cdot A_{x+k:\overline{n-k}|} = \\
& = A_{x:\overline{n}|} - A^{\;1}_{x:\overline{1}|} \cdot A_{x+1:\overline{n-1}|} + A^{\;1}_{x:\overline{1}|} \cdot A_{x+1:\overline{n-1}|} - A^{\;1}_{x:\overline{2}|} \cdot A_{x+2:\overline{n-2}|} + \ldots \\
& \ldots + A^{\;1}_{x:\overline{k-1}|} \cdot A_{x+k-1:\overline{n-k+1}|} - A^{\;1}_{x:\overline{k}|} \cdot A_{x+k:\overline{n-k}|} \\
& = A_{x:\overline{n}|} - A^{\;1}_{x:\overline{k}|} \cdot A_{x+k:\overline{n-k}|}
\end{aligned} \tag{13.101.}$$

So the relation is simply:

$$p_1 + A^{\;1}_{x:\overline{1}|} \cdot p_2 + A^{\;1}_{x:\overline{2}|} \cdot p_3 + \ldots + A^{\;1}_{x:\overline{k-1}|} \cdot p_k = A_{x:\overline{n}|} - A^{\;1}_{x:\overline{k}|} \cdot A_{x+k:\overline{n-k}|} \tag{13.102.}$$

13.4.3. Determining the Last Year

We are looking for the *k* that is the last one to satisfy:

$$z \geq \sum_{j=0}^{k-1} \frac{l_{x+j}}{l_x} \cdot v^j \cdot \left(PZ - p_{j+1} \right), \tag{13.103.}$$

In order to get a more simple formula for this search, we have to transform the right hand side of the equation a little bit.

$$\sum_{j=0}^{k-1} \frac{l_{x+j}}{l_x} \cdot v^j \cdot \left(PZ - p_{j+1} \right) = PZ \cdot \sum_{j=0}^{k-1} \frac{l_{x+j}}{l_x} \cdot v^j - \sum_{j=0}^{k-1} \frac{l_{x+j}}{l_x} \cdot v^j \cdot p_{j+1} \tag{13.104.}$$

For further simplification the following can be applied:

$$\sum_{j=0}^{k-1} \frac{l_{x+j}}{l_x} \cdot v^j = \ddot{a}_{x:\overline{k}|}, \tag{13.105.}$$

and the relation 13.102 derived in the appendix of the chapter:

$$\sum_{j=0}^{k-1} \frac{l_{x+j}}{l_x} \cdot v^j \cdot p_{j+1} = A_{x:\overline{n}|} - A_{x:\overline{k}|}^{\ \ 1} \cdot A_{x+k:\overline{n-k}|} \tag{13.106.}$$

So:

$$PZ \cdot \sum_{j=0}^{k-1} \frac{l_{x+j}}{l_x} \cdot v^j - \sum_{j=0}^{k-1} \frac{l_{x+j}}{l_x} \cdot v^j \cdot p_{j+1} = PZ \cdot \ddot{a}_{x:\overline{k}|} - \left(A_{x:\overline{n}|} - A_{x:\overline{k}|}^{\ \ 1} \cdot A_{x+k:\overline{n-k}|} \right) \tag{13.107.}$$

Since:

$$PZ = P + \frac{z}{\ddot{a}_{x:\overline{n}|}} \text{, and } A_{x:\overline{n}|} = P \cdot \ddot{a}_{x:\overline{n}|} \text{, this way:} \tag{13.108.}$$

$$\begin{aligned} & PZ \cdot \ddot{a}_{x:\overline{k}|} - \left(A_{x:\overline{n}|} - A_{x:\overline{k}|}^{\ \ 1} \cdot A_{x+k:\overline{n-k}|} \right) = \\ & = P \cdot \ddot{a}_{x:\overline{k}|} + z \cdot \frac{\ddot{a}_{x:\overline{k}|}}{\ddot{a}_{x:\overline{n}|}} - P \cdot \ddot{a}_{x:\overline{n}|} + A_{x:\overline{k}|}^{\ \ 1} \cdot A_{x+k:\overline{n-k}|} = \\ & = P \cdot \left(\ddot{a}_{x:\overline{k}|} - \ddot{a}_{x:\overline{n}|} \right) + A_{x:\overline{k}|}^{\ \ 1} \cdot A_{x+k:\overline{n-k}|} + z \cdot \frac{\ddot{a}_{x:\overline{k}|}}{\ddot{a}_{x:\overline{n}|}} \end{aligned} \tag{13.109.}$$

Using that:

$$\begin{aligned} & \ddot{a}_{x:\overline{k}|} - \ddot{a}_{x:\overline{n}|} = \frac{\left(N_x - N_{x+k}\right) - \left(N_x - N_{x+n}\right)}{D_x} = -\frac{N_{x+k} - N_{x+n}}{D_x} = \\ & = -\frac{D_{x+k}}{D_x} \cdot \frac{N_{x+k} - N_{x+n}}{D_{x+k}} = -A_{x:\overline{k}|}^{\ \ 1} \cdot \ddot{a}_{x+k:\overline{n-k}|} \end{aligned} \tag{13.110.}$$

$$\begin{aligned} & P \cdot \left(\ddot{a}_{x:\overline{k}|} - \ddot{a}_{x:\overline{n}|} \right) + A_{x:\overline{k}|}^{\ \ 1} \cdot A_{x+k:\overline{n-k}|} + z \cdot \frac{\ddot{a}_{x:\overline{k}|}}{\ddot{a}_{x:\overline{n}|}} = \\ & = A_{x:\overline{k}|}^{\ \ 1} \cdot A_{x+k:\overline{n-k}|} - P \cdot A_{x:\overline{k}|}^{\ \ 1} \cdot \ddot{a}_{x+k:\overline{n-k}|} + z \cdot \frac{\ddot{a}_{x:\overline{k}|}}{\ddot{a}_{x:\overline{n}|}} = \\ & = A_{x:\overline{k}|}^{\ \ 1} \cdot \left(A_{x+k:\overline{n-k}|} - P \cdot \ddot{a}_{x+k:\overline{n-k}|} \right) + z \cdot \frac{\ddot{a}_{x:\overline{k}|}}{\ddot{a}_{x:\overline{n}|}} \end{aligned} \tag{13.111.}$$

Now we can write the inequality in a compact form. So we are looking for the last *k* that satisfies the following:

$$z \ge A_{x:\overline{k|}}^{\;\;1} \cdot \left(A_{x+k:\overline{n-k|}} - P \cdot \ddot{a}_{x+k:\overline{n-k|}} \right) + z \cdot \frac{\ddot{a}_{x:\overline{k|}}}{\ddot{a}_{x:\overline{n|}}} \qquad (13.112.)$$

It is useful to express z:

$$z \cdot \frac{\ddot{a}_{x:\overline{n|}} - \ddot{a}_{x:\overline{k|}}}{\ddot{a}_{x:\overline{n|}}} \ge A_{x:\overline{k|}}^{\;\;1} \cdot \left(A_{x+k:\overline{n-k|}} - P \cdot \ddot{a}_{x+k:\overline{n-k|}} \right) \qquad (13.113.)$$

$$z \ge A_{x:\overline{k|}}^{\;\;1} \cdot \left(A_{x+k:\overline{n-k|}} - P \cdot \ddot{a}_{x+k:\overline{n-k|}} \right) \cdot \frac{\ddot{a}_{x:\overline{n|}}}{\ddot{a}_{x:\overline{n|}} - \ddot{a}_{x:\overline{k|}}} \qquad (13.114.)$$

Since:

$$\begin{aligned} A_{x:\overline{k|}}^{\;\;1} \cdot \frac{\ddot{a}_{x:\overline{n|}}}{\ddot{a}_{x:\overline{n|}} - \ddot{a}_{x:\overline{k|}}} &= \frac{D_{x+k}}{D_x} \cdot \frac{\ddot{a}_{x:\overline{n|}}}{\frac{N_x - N_{x+n}}{D_x} - \frac{N_x - N_{x+k}}{D_x}} = \\ &= \frac{\ddot{a}_{x:\overline{n|}}}{\frac{N_{x+k} - N_{x+n}}{D_{x+k}}} = \frac{\ddot{a}_{x:\overline{n|}}}{\ddot{a}_{x+k:\overline{n-k|}}} \end{aligned}, \qquad (13.115.)$$

this way

$$\begin{aligned} z &\ge A_{x:\overline{k|}}^{\;\;1} \cdot \left(A_{x+k:\overline{n-k|}} - P \cdot \ddot{a}_{x+k:\overline{n-k|}} \right) \cdot \frac{\ddot{a}_{x:\overline{n|}}}{\ddot{a}_{x:\overline{n|}} - \ddot{a}_{x:\overline{k|}}} = \\ &= \left(A_{x+k:\overline{n-k|}} - P \cdot \ddot{a}_{x+k:\overline{n-k|}} \right) \cdot \frac{\ddot{a}_{x:\overline{n|}}}{\ddot{a}_{x+k:\overline{n-k|}}} = \\ &= A_{x+k:\overline{n-k|}} \cdot \frac{\ddot{a}_{x:\overline{n|}}}{\ddot{a}_{x+k:\overline{n-k|}}} - P \cdot \ddot{a}_{x:\overline{n|}} = A_{x+k:\overline{n-k|}} \cdot \frac{\ddot{a}_{x:\overline{n|}}}{\ddot{a}_{x+k:\overline{n-k|}}} - A_{x:\overline{n|}} \end{aligned} \qquad (13.116.)$$

13.4.4. Correction if the Annual Premium is Paid in Instalments, But Arrives with Certainty

In the monthly case: Instead of PZ at the beginning of the year, k_{12}*PZ/12 is paid at the beginning of each month, where k_{12} is (naturally greater than 1) the necessary correction factor. Its value can be determined using the following equality:

$$PZ = \sum_{i=0}^{11} v^{\frac{i}{12}} \cdot \frac{PZ}{12} \cdot k_{12} \text{, from this:} \qquad (13.117.)$$

$$k_{12} = \frac{12}{\sum_{i=0}^{11} v^{\frac{i}{12}}} = \frac{12}{\frac{1-v}{1-v^{\frac{1}{12}}}} = \frac{12 \cdot \left(1 - v^{\frac{1}{12}}\right)}{1-v} \qquad (13.118.)$$

Based on this the quarterly and semi-annual correction factors are simply:

$$k_4 = \frac{4 \cdot \left(1 - v^{\frac{1}{4}}\right)}{1 - v} \tag{13.119.}$$

$$k_2 = \frac{2 \cdot \left(1 - v^{\frac{1}{2}}\right)}{1 - v} \tag{13.120.}$$

Correction done this way supposes that in case of death the remaining part of the annual premium not yet paid is collected.

13.4.5. Correction if the Annual Premium is Paid in Instalments – Depending on Mortality

If in the event of death the remaining parts of the annual premium not yet paid are left out, the correction factor will depend on the current age of the insured (i.e. entering age + time elapsed since entering) as the following.

First we have to compute the **monthly l_x values**. We could also do this by a simple linear interpolation between l_x and l_{x+1}, but we'll rather suppose according to the international practice that the monthly probability of death (and so survival also) within the year is the same, i.e. it is the following equation (series):

$$\frac{l_{x+\frac{1}{12}}}{l_x} = \frac{l_{x+\frac{2}{12}}}{l_{x+\frac{1}{12}}} = \ldots = \frac{l_{x+1}}{l_{x+\frac{11}{12}}} = p_{x(12)}\,, \tag{13.121.}$$

where $p_{x(12)}$ denotes the uniform monthly survival rate within the year *x*.
If we multiply these probabilities together, we get the following equation:

$$p_{x(12)}{}^{12} = \frac{l_{x+\frac{1}{12}}}{l_x} \cdot \frac{l_{x+\frac{2}{12}}}{l_{x+\frac{1}{12}}} \cdot \ldots \cdot \frac{l_{x+1}}{l_{x+\frac{11}{12}}} = \frac{l_{x+1}}{l_x} = p_x\text{, so:} \tag{13.122.}$$

$$p_{x(12)} = \sqrt[12]{p_x} \tag{13.123.}$$

According to this:

$$l_{x+\frac{k}{12}} = l_x \cdot \left(\sqrt[12]{p_x}\right)^k = l_x \cdot \left(\sqrt[12]{\frac{l_{x+1}}{l_x}}\right)^k \tag{13.124.}$$

The correction itself is, in the monthly case, if the entering age of the insured was *x* and *t* years of the insurance have passed (i.e. we are in the (t+1)th year):

$$PZ = \sum_{i=0}^{11} \frac{l_{x+t+\frac{i}{12}}}{l_{x+t}} \cdot v^{\frac{i}{12}} \cdot \frac{PZ}{12} \cdot k_{12}\text{, from this:} \tag{13.125.}$$

$$k_{12} = \frac{12}{\sum_{i=0}^{11} \frac{l_{x+t+\frac{i}{12}}}{l_{x+t}} \cdot v^{\frac{i}{12}}} = \frac{12}{\sum_{i=0}^{11} \left(p_{x+t} \cdot v\right)^{\frac{i}{12}}} = \frac{12 \cdot \left(1 - \left(p_{x+t} \cdot v\right)^{\frac{1}{12}}\right)}{1 - p_{x+t} \cdot v} \qquad (13.126.)$$

According to this the quarterly and semi-annual correction factors are:

$$k_4 = \frac{4 \cdot \left(1 - \left(p_{x+t} \cdot v\right)^{\frac{1}{4}}\right)}{1 - p_{x+t} \cdot v} \qquad (13.127.)$$

$$k_2 = \frac{2 \cdot \left(1 - \left(p_{x+t} \cdot v\right)^{\frac{1}{2}}\right)}{1 - p_{x+t} \cdot v} \qquad (13.128.)$$

14. POSSIBLE METHODS OF HANDLING INFLATION

KEY WORDS

Current sum assured	Value reserving techniques
Current premium reserve	Yield
Premium increase options	Investment profit sharing

Inflation causes many problems, both for the client and the insurance company. The main problem for the client is that inflation deteriorates the insurance benefit undertaken by the insurance company. For the company, the most important problem is that the profit loading calculated gradually becomes inadequate. Problems of fairness also arise. Namely, above we counted with the technical interest rates. As it was pointed out, this is always a relatively low interest rate (generally it is between 2-4%[148]), guaranteed by the insurance company as the yield of the premium reserve.

In times of inflation, however, real yields are significantly higher than the yield that would correspond to the technical interest rate. Who should have this surplus? Considering fairness, the greater part is coming to the client, since it is his money that carried interest. Thus, in times of inflation, the question of ''profit-sharing'' arises emphatically. These are the reasons that make it inevitable to apply techniques of handling (not ceasing, eliminating) inflation.

At first sight we may say that in case of life insurances there are two (parallel) techniques to be applied:

- Premium increase and/or
- Profit sharing.

14.1. PREMIUM INCREASE

In times of inflation, usually every insurance company offers premium increase to its clients. This is not an obligation for the client but an opportunity offered by the insurance company, which has nothing to do with any rises in prices. Premium increase means that the insurance company makes it possible to recalculate the client's insurance – which is deteriorating because of inflation – without exposing him to a new process of underwriting. Premium increase could only be called a rise in prices, if the insurance company increased the premium without increasing the insurance benefit it provides, or, if it calculated the price of the increased insurance benefit on the basis of rates less favourable for the client. Thus, in case of life insurances, a rise in the prices means that the insurance company changes its rates, so it provides the same unit of insurance benefit for higher prices. Premium increase is a different case. This is important to note, because many people who have a life insurance are confused about this difference. It is widely believed, that if an insurance company increases premium by a larger extent than the rate of inflation of the previous year (due to technical reasons that will be discussed later), it deceives its client. In fact, the insurance company does not make the insurance more expensive by increasing premium (if the increase is based on the same rates), even if the extent of this increase is not related at all to the extent of inflation.

The lack of underwriting means a higher level of anti-selection risk for insurance companies, so, according to their best practices, they offer this opportunity only to those clients who regularly accepted premium increase previously, while others, who did not, lose their right to do so. This protects the companies from those who first think of following up inflation rates on their death-beds.

[148] Although in the 1990s in Hungary there were examples of technical interest rates of 7%, and the general rate was 5,5%!

The principle of premium increase is simple: the insurance company considers that for the extra premium (the difference between the increased premium and the premium paid in the previous year), the policyholder buys the same kind of insurance with *regular* premium payment for the rest of the term, calculating with the current age of the insured. The premium of this new insurance is simply added to the previous insurance premium.

It depends on the insurance company to determine the basis for the premium increase. Since the reason of the option is inflation, one of the most frequent solutions is to declare the inflation rate as the basis. For instance, the extent of increase is the same as the inflation rate of the previous year, or 80% of the inflation rate of the previous year, etc. Since inflation only becomes dangerous above a certain level, it is general to set a lower limit to premium increase, e.g. it is not offered if the inflation is below 10%!

Another method of premium increase is the technique of revalorization, where the possible extent of premium increase equals the investment profit of the insurance company in question. The technique of revalorization integrates premium increase and profit sharing systems into one complex technique, which will be discussed later on in more detail.

By what extent will the sum assured rise as a result of the premium increase? There is no general answer to this question, because it depends on the age and sex (of the insured), the insurance term, and the type of insurance. But it is certain that in case of endowment insurances, the sum assured increases less than the premium. This has two reasons:

1. The insured advances in age, and the older he becomes, the higher his probability of death will be, i.e. the more expensive his insurance will become.
2. As time goes by, the remaining term is shorter, which means that the time for saving gets shorter too, and this also results in the increase of the premium.

Thus, as time goes by, premium increase has gradually less impact. This is the reason why a number of insurance companies deny premium increase some years before maturity.

Compared to premium increase, investment profit sharing has a totally different effect.

14.2. Investment Profit Sharing

Investment profit sharing is in relation with the investment nature of savings-type insurances, thus this technique is applied not only to handle inflation. The reason why it is discussed here is that it happens to be an effective method for handling inflation, too.

As it was mentioned before, investment profit is the profit gained on the investment of the premium reserve above the technical interest rate.

There are several modes of investment profit sharing (for instance to draw it among the clients), although only two modes appear in practise. Either it is expended for *raising the insurance benefit*, or it credited to the client as an *interest earning deposit*. (As we will see later on, the technique of revalorization is in general a special case of the first mode.)

When the first mode is applied, the profit corresponding to the technical interest rate is subtracted from the total profit gained (since it was originally calculated into the premium), and then the surplus is shared, in a given ratio, between the insurance company and the client. The client's share then is considered to be the *single premium* of such a life insurance, that has a term equal to the remaining term of the original insurance (so its maturity is the same as that of the original insurance), and the client's entering age is his current age. On the basis of this, the sum assured is calculated, and the original insurance benefit is raised by this amount, the client's share of the investment profit is added to the premium reserve.

The result of the inflation handling technique applied by the insurance company in practice will basically depend on the outcome of the investments made by the insurance company. However, it can never be sure that the economic situation will enable the insurance company to offer elimination of the effect of inflation, in the sense that – resulting from the joint effect of (not compulsory) premium increase and profit share – the sum assured will increase by the same extent as the inflation. Because of this, it is better to talk about *handling inflation* than *following up inflation*.

14.3. THE TECHNIQUE OF REVALORIZATION

As it was already mentioned, techniques of premium increase and investment profit sharing are usually applied together, though their technical transactions are separated. This causes several problems that can be solved by the integrated system of premium increase and investment profit sharing, which is called the technique of revalorization (indexation technique). These problems are the following:

1. It was already mentioned above, that the extent of increase in the sum assured, which results from the premium increase, is gradually seceding from the extent of the premium increase. Although technically this is absolutely reasonable, in the course of time it makes the client less and less interested in increasing the premium.
2. The increase in the sum assured, which is less than that of the premium, is partly compensated by the raising of insurance benefit provided by investment profit sharing, but it is impossible to calculate precisely how the rise in the insurance benefit (resulting from the two techniques together applied together) is related to the extent of premium increase. Natural expectation of the client would be that if the premium increases by e.g. 20%, then the sum assured (in total, together with the profit share) should be increased by 20%, since this seems – at least on the face of it – to be fair, and what is more important, this seems to be practical, since in this way the expenses of the client and the level of the insurance benefit are increased by the same extent.

The increase in the total sum assured resulted in the parallel application of the two techniques is not equal to the premium increase, because they depend on different factors; the rate of premium increase depends on the inflation rate, while investment profit sharing depends on the yields the insurance company could achieve. In order to make the increase of the sum assured resulting from the premium increase precisely supplement the increase caused by investment profit sharing, premium increase has to depend on the extent of profit share.

But how much should the premium increase if the total increase in the sum assured is to be the same as the premium increase?

Before answering this question, let us examine the reason of the fact that if the premium is increased at the policy anniversary by 100% (e.g.), then the sum assured will be increased less? (Although the answer to this question was already given previously, now another approach is shown.)

If somebody wants to take out an endowment insurance for a term of 10 years, and the annual premium is 10,000 Forints for a sum assured of 100,000 Forints, then he will get an insurance of 200,000 Ft-s for an annual premium of 20,000 Forints. Namely, if we double the amount of the premium at the beginning of the term, the sum assured will be increased by the same extent: it is also doubled. But after a year we will have a different situation. By the time of the first policy anniversary, the insurance company accumulates a certain amount of reserve by using the premium paid by the client for the first year. This will be needed for providing the insurance benefit undertaken by the insurance company. The amount of this reserve depends on the sum assured. If the sum assured is 100,000 Forints, then this will need half as much reserve as a sum of 200,000 Forints would need by the time of the first anniversary. Thus, if the client doubles the premium for the rest of the term at the first policy anniversary, then the insurance company cannot double the sum assured, because in the first year it has accumulated a premium reserve which is enough only for one half of the sum assured. So, if the premium reserve and the premium are increased by the same extent, then the sum assured could be increased by that extent, too.

In every year the premium reserve increases – in addition to the pre-calculated amount – in the proportion of the investment profit of the premium reserve, since it is the investment profit of the premium reserve, and its extent is determined in the ratio of that. So, if premium increase follows the extent of investment profit of the premium reserve, then the sum assured will increase precisely by the extent of the investment profit of the premium reserve.

This is the technique of revalorization.

This technique has the advantage of being transparent, thus it needs less registration and computer technology than other techniques of premium increase and investment profit sharing that are used separately. However, the result is the same. The increase in the sum assured can be divided into two parts: the impact of premium increase and the impact of investment profit sharing. If we did so, we would find the same as above, namely that the extent of increase in the sum assured caused by the premium increase is gradually decreasing, while the extent of increase caused by the investment profit sharing is constantly growing.

Revalorization is thus an elegant technique with only one significant disadvantage compared to the separately applied techniques of premium increase and investment profit sharing. Namely, that the extent of premium increase depends on the activity of the insurance company, instead of other objective indicators such as the rate of inflation. However, this is not a problem but an advantage, if clients have greater confidence in the insurance company than in the state (since the rate of inflation is determined by national institutions). Thus it is highly important for insurance companies using this technique to acquire their clients' confidence.

15. THE CALCULATION OF INFLATION PREMIUM INCREASE AND INVESTMENT PROFIT SHARING

15.1. PREMIUM INCREASE INDEPENDENT OF PROFIT SHARING

As it was discussed previously, most insurance companies offer their clients at every policy anniversary the opportunity of increasing the premium – and based on this also the sum assured – by the extent of the inflation of the previous year (or by a given percentage of it). (Previous year hereby means the last calendar year of which there is an official rate of inflation available, thus in January of year x ''previous year'' can also be year x-2.) However, this is usually offered to clients only if the rate of inflation is above 10%. Clearly enough, possibility of premium increase is offered only for clients holding insurances of regular payment.

The extent of premium increase is given. But by how much is the sum assured increased? There are basically two ways for this to be calculated. In both ways, the extra premium is considered to be the annual premium of a new insurance. The difference lies in the tariffs they use to calculate this new insurance. The two possible ways:

- To use a normal tariff, as if the policyholder effected a new insurance;
- To use a preferential tariff. Naturally, the upper limit of the allowance given is that the insurance company provides the increased insurance for a net premium.

Let P_t^b denote the annual premium that has to be paid at the beginning of insurance year *t*.

If *k* denotes the inflation rate in the ''previous'' year, and if it is assumed that this is the rate by which the insurance company lets premium increase, the new premium will be:

$$P_{t+1}^b = P_t^b \cdot (1+k) \tag{15.1.}$$

Thus the extra premium will be:

$$dP_{t+1}^b = P_{t+1}^b - P_t^b = P_t^b \cdot k \tag{15.2.}$$

Assuming that the insurance company uses the first method (does not give any allowance), and $P^b{}_{x:\overline{n}|}$ stands for the gross annual premium of 1 Ft sum assured, then the increase of the sum assured is:

$$dS_{t+1} = \frac{dP_{t+1}^b}{P^b{}_{x+t:\overline{n-t}|}} \tag{15.3.}$$

where $dS_{t+1} = S_{t+1} - S_t$, and S_t is the sum assured valid throughout the insurance year *t*.

If we do not have to use a different reserve formula in a certain part of the term due to zillmerization, then the premium reserve of the end of year *t* (before premium payment) can simply be calculated with the following formula:

$$\underline{V}_t = S_t \cdot (A_{x+t:\overline{n-t}|} - \ddot{a}_{x+t:\overline{n-t}|} \cdot PZ) \tag{15.4.}$$

where *PZ* is the ''reserve-'' or "zillmer"-premium, i.e. the net premium that fills the reserve from year to year.

Guaranteed Insurability Option (GIO) does not belong here due to its subject matter, but because of its similar technical implementation it should be mentioned here. The policyholder possessing such an option has the opportunity from time to time (e.g. every three years) to increase his premium above the inflation premium increase. This does not serve for compensating inflation but for covering his increased insurance needs arising from his permanently improved financial situation. Calculation of premium increase due to GIO is technically the same as it is described above.

15.2. Profit Sharing Independent of Premium Increase

Insurance companies usually give their clients the greater part of the premium reserve's yield over the technical interest rate in the form of profit sharing. The extent of this share varies – as well as the technical interest rate – as it depends on the insurance company in question. If *i* stands for the technical interest rate and *h* denotes the annual yield of the premium reserve, then the investment profit (*h'*) is:

$$h' = h - i \tag{15.5.}$$

For example:

Technical interest rate:	5%
Profit share of the client:	90%
Yield:	10%

Here the extent of the investment profit percentage going to the client (h'') is the following percentage of the average premium reserve of the previous year:

$$h'' = 0{,}9 \cdot (10 - 5) = 4{,}5\% \tag{15.6.}$$

Total from the yield for the insurance company:	0,1·(10 - 5) = 0,5%
Total from the yield for the client:	5% + 4,5% = 9,5%

The insurance company can give this investment profit back to the client in two different ways:

- opens an "account" for the client, where the profit share will be credited from year to year, and where it will carry interest together with the premium reserve,
- considers the profit to be the single premium of a single premium insurance, so it will be added to the premium reserve, thus increasing the sum assured.

The first possibility does not need any mathematical explanation, so here we only discuss the second one, where we promptly face a problem. "Time" for insurances differs from "time" for investments. Namely, if an insurance was effected on the 24th of March, then the "insured year" will last from the 24th of March till the 23rd of March. But results of investments are usually reviewed according to calendar years, and profits are shared between contracts similarly. From this it follows that it is problematic to decide what the basis for investment share should be for a contract? It is usually the average premium reserve of the previous year.

Let's suppose that the insurance had its t^{th} policy anniversary (that was the end of "insured year" number *t*) in the previous calendar year (the investment share of which we are interested in). From the beginning of the previous calendar year until the t^{th} policy anniversary τ (fragment) year elapsed. Then the average annual premium reserve (without investment share) will be the simple arithmetic average of the premium reserve at the beginning and at the end of the calendar year.

The premium reserve at the beginning of the calendar year is:

$$V_{t-\tau} = \tau \cdot \left(\underline{V}_{t-1} + P\right) + (1-\tau) \cdot \underline{V}_t \tag{15.7.}$$

The premium reserve at the end of the calendar year is:

$$V_{t+1-\tau} = \tau \cdot \left(\underline{V}_t + P\right) + (1-\tau) \cdot \underline{V}_{t+1} \tag{15.8.}$$

Thus the average premium reserve of the calendar year is:

$$\frac{V_{t-\tau}+V_{t+1-\tau}}{2} \tag{15.9.}$$

Naturally, above we latently supposed that the contract did not commence, and is not terminated in the calendar year in question, and that its premium is paid regularly, annually (this is why the correction factor *P* is present in the premium reserve at the beginning of the insurance year). Usually in case of the first and the last fragment year a proportional part of the premium reserve is taken, while in case of single premium insurances, *P* factors are simply missing from the above formula. The premium reserve of the investment profit shares that have been distributed before is interpolated in the above described way and added to the basis of profit sharing.

On the basis of the average premium reserve calculated this way the insurance company provides the profit share, the extent of which is <u>*h”*</u>. This means that the investment profit (coming to the client) is:

$$h''\cdot\frac{V_{t-\tau}+V_{t+1-\tau}}{2} \tag{15.10.}$$

But this investment profit is directly added to the client’s premium reserve from the beginning of the following calendar year. However we do not know, how much the sum assured will be increased by?

Insurance companies usually change investment profit to sum assured by using a net tariff, i.e. without charging expenses. But because the number of the years remaining from the term is not a whole number at the beginning of the calendar year, single premiums also have to be interpolated to the beginning of the calendar year. Since the beginning of the given calendar year falls between policy anniversaries *t* and *t+1,* the single premiums $A_{x+t:\overline{n-t}|}$ and $A_{x+t+1:\overline{n-t-1}|}$ have to be interpolated. So, the interpolated single premium is:

$$A_{x+t+1-\tau:\overline{n-t-1+\tau}|}=\tau\cdot A_{x+t:\overline{n-t}|}+(1-\tau)\cdot A_{x+t+1:\overline{n-t-1}|}\ . \tag{15.11.}$$

Accordingly, the increase of the bonus sum assured (dBS) from the new calendar year is:

$$dBS=\frac{h''\cdot(V_{t-\tau}+V_{t+1-\tau})}{2\cdot A_{x+t+1-\tau:\overline{n-t-1+\tau}|}}\,. \tag{15.12.}$$

15.3. Integrated Premium Increase and Investment Profit Sharing System – the Technique of Revalorization

The two methods of managing inflation described above are usually applied together, though technically separately. But another technique was also mentioned, which integrates the two and has the advantage that all important values – premium, premium reserve, sum assured – are increased by the same extent, which extent is the investment profit share (%) returned to the client by the insurance company. This is called the technique of ”revalorization”.

The technique of revalorization is based on the finding that if the premium is raised by *x*% on a policy anniversary, then the sum assured cannot be raised by the same extent, (it can only be increased by a lower degree – e.g. in the case of endowment insurances, where this technique is most often used), because that part of premium reserve that should have been accumulated starting from the beginning of the insurance to cover the *x*% extra sum assured is missing. The shortfall is precisely *x*% of the premium reserve up to that date.

Here comes the technique of revalorization, saying that the percentage of investment profit share shows exactly the extent of premium increase. And if this percentage is also the percentage of the premium increase, then the increase of the sum assured resulting from the premium increase and that resulting from profit sharing will exactly complete each other to the extent of the percentage of the investment profit share.

However, this all seems so simple only at first sight, but when going into details, we find several problems. Thus, not all problems will be discussed in the following, only the basics. We use some simplifying assumptions.

1. In case of every age-term combination the same loadings (determined as a percentage of the net premium) are added to the net premiums, thus the extent of the net premium increase is the same as that of the gross premium increase.
2. The premium reserve is not zillmerized.
3. Investment profit share is allocated according to insurance year instead of calendar year. This brings along technical problems for the insurance company, namely that it needs to know the actual yield at any moment. But a more serious problem, which prevents this technique from being applied without further considerations is the demand of clients that their investment profit shares should be calculated on the basis of authorized investment profit data. Such data are, however, available only per calendar year, some months after closing the calendar year in question.
4. The basis of the investment profit is not the annual average premium reserve, but the premium reserve at the end of the calendar year.

First of all, it has to be proved that, under these conditions, the extent of the increase of the sum assured resulting from the investment profit sharing and that resulting from the premium increase really equal to the extent of the investment profit going to the client (*h’’*).

We prove this for the first increase and profit sharing. If P_1 denotes the net premium of the first year, where – as we know –

$$P_1 = S \cdot \frac{A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}} = S \cdot P_{x:\overline{n}|} \tag{15.13.}$$

then the premium of the second year is:

$$P_2 = P_1 \cdot (1 + h'') \tag{15.14.}$$

Then the increase of the sum assured resulting from the premium increase is (here we can calculate with net premiums, using the assumption that the loadings are equal):

$$\frac{P_2 - P_1}{\dfrac{A_{x+1:\overline{n-1}|}}{\ddot{a}_{x+1:\overline{n-1}|}}} = \frac{h'' \cdot S \cdot P_{x:\overline{n}|}}{\dfrac{A_{x+1:\overline{n-1}|}}{\ddot{a}_{x+1:\overline{n-1}|}}} \tag{15.15.}$$

The increase of the sum assured resulting from the profit share is (here we make use of the assumption that the basis of the profit share is the premium reserve at the end of the year):

$$\frac{V_1 \cdot h''}{A_{x+1:\overline{n-1}|}} = \frac{S \cdot (A_{x+1:\overline{n-1}|} - \ddot{a}_{x+1:\overline{n-1}|} \cdot P_{x:\overline{n}|}) \cdot h''}{A_{x+1:\overline{n-1}|}} \tag{15.16.}$$

What needs to be verified is:

$$\frac{h'' \cdot S \cdot P_{x:\overline{n}|}}{\frac{A_{x+1:\overline{n-1}|}}{\ddot{a}_{x+1:\overline{n-1}|}}} + \frac{S \cdot (A_{x+1:\overline{n-1}|} - \ddot{a}_{x+1:\overline{n-1}|} \cdot P_{x:\overline{n}|}) \cdot h''}{A_{x+1:\overline{n-1}|}} = h'' \cdot S \qquad (15.17.)$$

Dividing this by *S*, breaking up the parenthesis and making simplifications we get the following equation:

$$\frac{h'' \cdot P_{x:\overline{n}|} \cdot \ddot{a}_{x+1:\overline{n-1}|}}{A_{x+1:\overline{n-1}|}} + \frac{A_{x+1:\overline{n-1}|} \cdot h'' - \ddot{a}_{x+1:\overline{n-1}|} \cdot P_{x:\overline{n}|} \cdot h''}{A_{x+1:\overline{n-1}|}} = h'' \qquad (15.18.)$$

Since the positive and negative elements drop out each other, we get the

$$h'' = h''$$

equation, which means that the statement is verified.

Without going into any more details, we only mention what happens if the client refuses the premium increase at a policy anniversary, or if – due to its slight extent – the insurance company does not offer this option. In this case the best solution is to choose the “account method” for handling the generated profit share, because this will not ‘’ruin’’ the future possibility of applying the technique of revalorisation.

16. MODERN PREMIUM AND RESERVE CALCULATION

KEY WORDS

Field of actuarial control
Internal rate of return (IRR)
Yearly renewable term insurance
Level premium
Unbundled products
Net present value
Profit test

In the previous chapter the traditional method of premium calculation was presented. It is characterised by the strict differentiation between net and gross premium, and by its dealing with the impact of relatively few factors. Nowadays modern methods of calculating premium are spreading, existing side by side with the traditional modes. The main feature of these methods is that through computer programme packages they can explicitly consider much more factors effecting the premium. Due o the nature of the subject they do not work with closed formulae, but with the method of trial and error: different assumptions are being varied until an ''acceptable'' and stable premium is calculated. In these models profit is the main outcome-variable, so the method itself is called profit test.

In the followings, we introduce the main points of profit tests, and we show through a case study that using simple assumptions we can make such models for ourselves.

16.1. THE PROFIT TEST[149]

First profit testing programmes were made in actuarial consultative companies[150] in order to check the calculations of clients, but it was soon realised that these are marketable products on their own.

Profit testing programmes appeared at the time when unbundled products were spreading, which do not contain fixed elements (endowment, term fix, etc. – i.e. traditional life insurances as this book calls them), but they are constructed by visible elements (universal life, unit linked – modern life insurances as this book calls them). In these programmes all parts of the premium are visible and are handled like an account. (As we have seen, this is the combination of an account and a yearly renewable term = YRT insurance.) This development gave momentum to new pricing methods, because applying classical methods to these products is rather difficult (since classical methods of premium calculation were developed to ''suit'' traditional products).

The new methods made it necessary, while the development of the environment made it possible for new pricing methods to come into life. Main elements of the development of the environment are:

1. Opportunities provided by information technology.
2. Given conditions (inflation, investment environment: shift from investment in bonds towards investments in shares).
3. A state of competition in which insurance companies ''found'' themselves.

As we have already seen, the change of the environment in itself contributed to and reflected on the appearance of modern insurances.

Modern principles used in the pricing process do not contradict the classic ones, but they are slightly rephrased. In case of modern techniques of premium calculation we do not talk about the principle of equivalence, but about the net present value of the premium being equal to zero, i.e. about the requirement of $NPV(BP)=0$.

One of the most important new concepts is the cash flow, which is the difference of and the time series of incomes and expenditures, and the internal rate of return (IRR), that – we can say – is the same as the technical interest rate in the classical case.

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[149] This sub-chapter is based on Tibor Edvi's lecture on the topic.
[150] E.g. the Prophet, the product of Bacon & Woodrow that became part of Deloite & Touche is the most popular such programme in Hungary today.

From the point of view of modern products, classic methods have the disadvantage that they do not deal with the following factors, problems:

- profit sharing (investments)
- inflation, i.e. the increase of premiums and expenses
- cancellations, policy pay-ups, money withdrawal, partial surrender, other options
- realistic expectations of the owner (return on capital)
- complicated expense structures (e.g. occasional expenses)

In comparison to this, the technique of net present value can be more easily adjusted to reality. The conditions of this are:

1. determining the realistic cash flow (of course, this often means only an expected value both in the area of mortality and other options. However, correlations between certain factors can be taken into consideration.)
2. applying a realistic discount rate

The elements of cash flow are (+ and – represent the direction of the flow of money from the insurer's point of view):

\+ premium income
\+ investment earnings
\- benefit payments (insurance benefits, surrender)
\- expenses/commissions (acquisition, administrative/maintenance)
\- reserving expenditure (change of reserves)
\- solvency (extra immobilisation of capital)

Two methods exist for handling the expected value (which partly means certain numbers by the piece and partly yields):

- deterministic and
- stochastic.

Within the deterministic method (classic premium calculation belongs here) mortality tables and multiple exit tables (Markov chains) are used. The problem is that this contains only the most probable cases (along the most probable picture of future). It is completed with, and made more subtle, by a sensibility test, or maybe several future scenarios are created, and their weighted average used as a result. But this easily leads to inconsistency. Such problems are to be solved by the stochastic methods, which sometimes analyse mortality, but more often yields. Complicated stochastic methods are used only if deterministic methods fail (e.g. their predictions are not verified).

The most widespread technique is the sensibility test, which is the profit test itself.

In a sensibility test it is examined how ''robust'' the product is, i.e. how much its profitability tolerates significant changes of certain parameters. It might be said, that sooner the product's cash flow returns early expenses, the more robust the product is. From this point of view we can say that the goal of zillmerization is to make the product more robust, since it changes the cash flow in the following way (see F*igure 16.1.*):

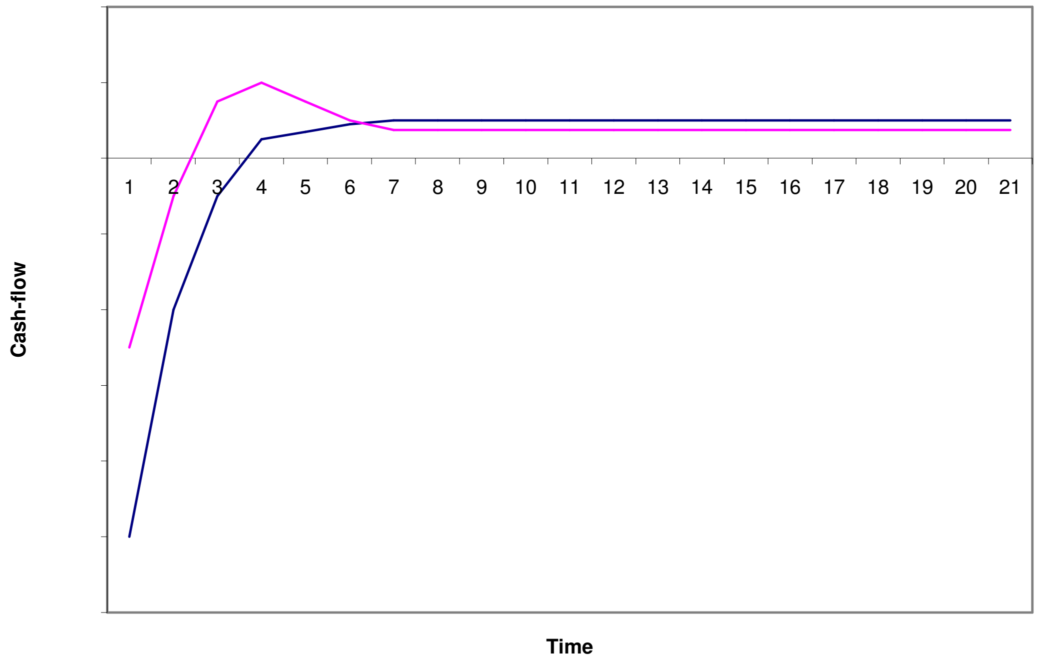

Figure 16.1: Cash flow with and without zillmerization

Negative cash flow during the term is not allowed – at least if this can be anticipated. If it is noticed, it must be eliminated, or reserves must be built beforehand to counterweight the negative cash flow.

Profit test is not simply a technique of premium calculation. It can be, and has to be performed not only during the development of the product, but also afterwards, when it is being sold (post-calculation).

The place of profit test within the life of the product is best illustrated by Goford's "Actuarial Control Cycle" (*Figure 16.2.*):

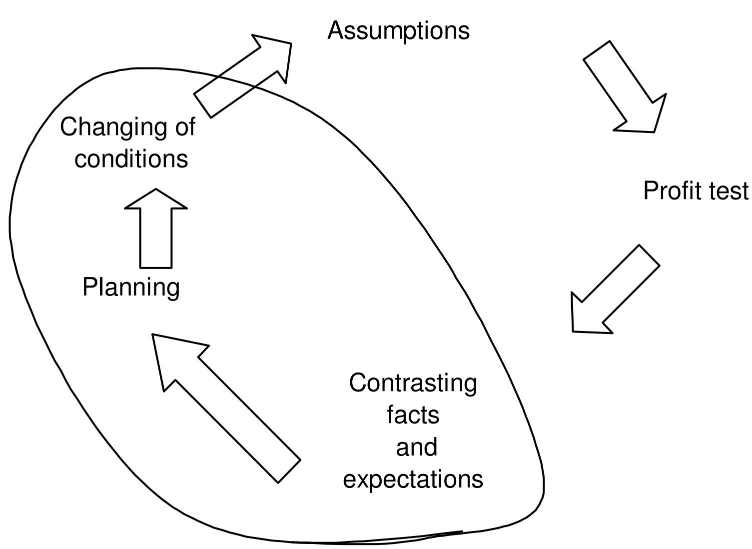

Figure 16.2: The Actuarial Control Cycle

Finally, the question arises what discount rate we should use during the calculation? Generally speaking, the answer is that this is usually determined by the owner himself, since this is the (minimally) expected yield of the owner. (He himself can determine this e.g. on the basis of CAPM)

16.2. Case Study: Calculation of the Expense Part of a Rider to Life Insurance Policies

As it is well known, European Union directives allow life insurance companies to offer accident- and sickness insurances as ''complementary risks'' along with life insurance policies. Thus, it is possible to sell life-, accident- and health insurances together, within one package. Usually this takes the form of constructing accident or sickness (sometimes life) insurance riders to the main policy.

Insurance riders still do not have so subtle methodology as the methods of main policies, and their models are often missing from the popular profit tests. This field is characterised on the one hand by the level premium independent of age and term[151] – mostly in case of accident insurance riders – and on the other hand by the lack of mathematical reserving. Also the calculation of expenses is often made on the basis of ''feelings''.

In the followings I try to present a model more subtle than the usual[152], which naturally also has its limitations. I take it for a given condition that premiums were calculated with the consideration of age, so I will deal with the “fair” and more or less sensitive calculation of the expenses. (The question of “fairness” arises basically because of the premiums depending on age – in case of level premiums this is not a problem.) Here I am concerned neither with the long term equalisation of premiums, nor with reserving. This model is suitable to carry out a sensibility test (profit test) on the effect of certain expense factors in an Excel model. But it is not my intention to discuss the Excel realisation of the model.

16.2.1. Should we use a level or an age-dependent premium?

Before introducing the model, let us summarize when level premium can be used, and in what cases it is not practical.

Premium calculation of an insurance company basically depends on three factors:

1. nature of risk
2. statistics available
3. habits of other competitors

Let us go through these factors!

Level premium can, or rather has to be applied if the risk in question is independent of age. In such a case, the (regular) premium will be independent of the term, too. However, most probably we will not find a risk that satisfies this prerequisite. Accidental death is usually considered to belong to this category, but it is well known that accidental mortality of men in their late teenage years and early twenties is significantly higher than the average.

Contrary to this, in certain situations level premium may be adequate. Basically in such cases when the risk contains no drastic trend depending on age, i.e. it has no tendency to rise, or even it significantly decreases with age.

In the following figures (“Basin”, “Random fluctuation” “Slight trend”) you can see cases when level premium is not only possible, but it is advisable.

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[151] Differences between the risks according to sex are not examined here, it is considered to be given that these risks are different, so different premiums should be calculated in the case of the two sexes – even if they are equal.

[152] Of course, this does not mean that it is impossible that a more refined approach than the one presented here may exist in the Hungarian market.

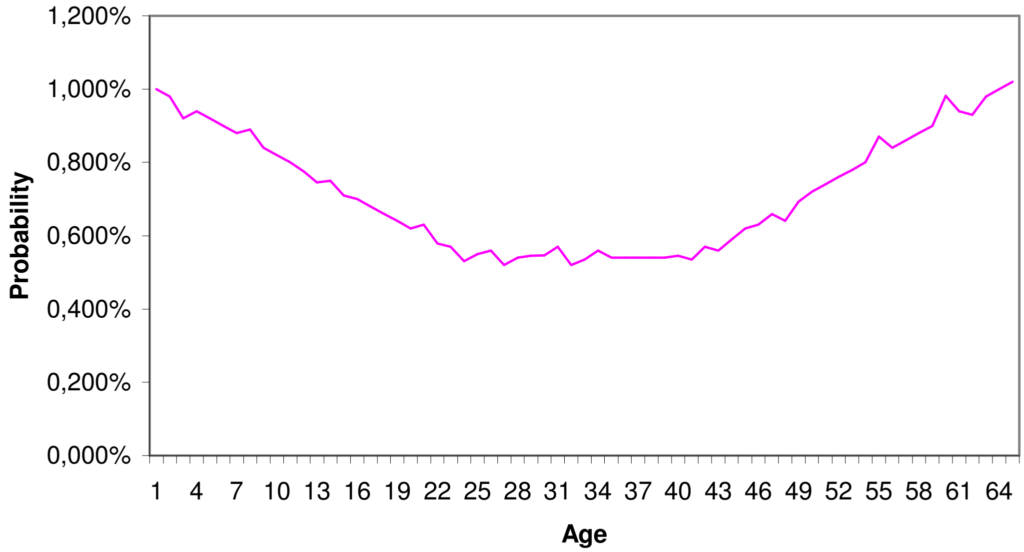

Figure 16.3: “Basin”

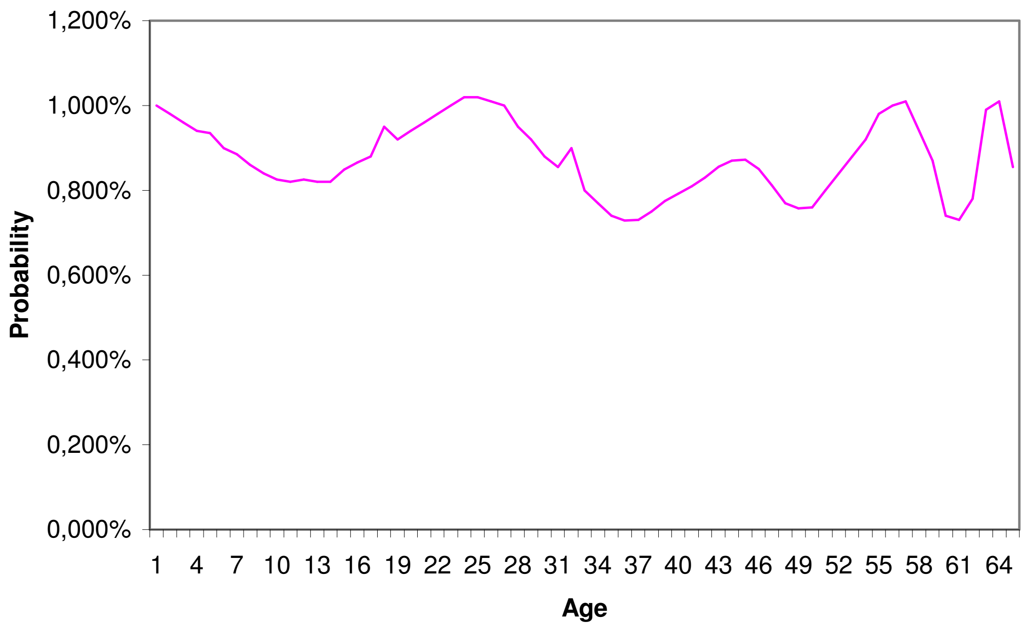

Figure 16.4. “Random fluctuation”

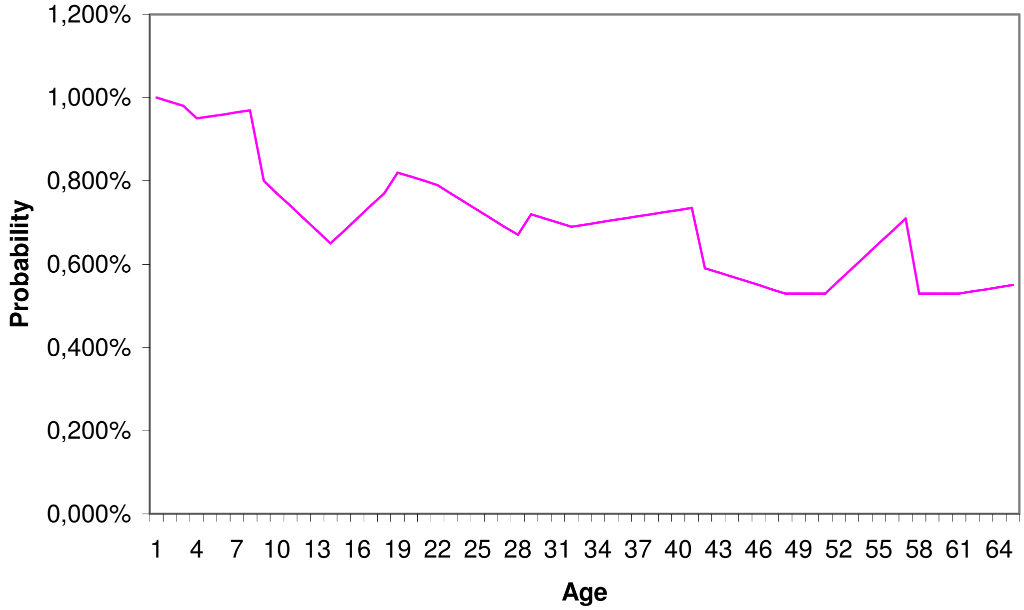

Figure 16.5: "Slight trend"

In the case illustrated by figure 16.6., level premium is still possible, but it already causes several problems.

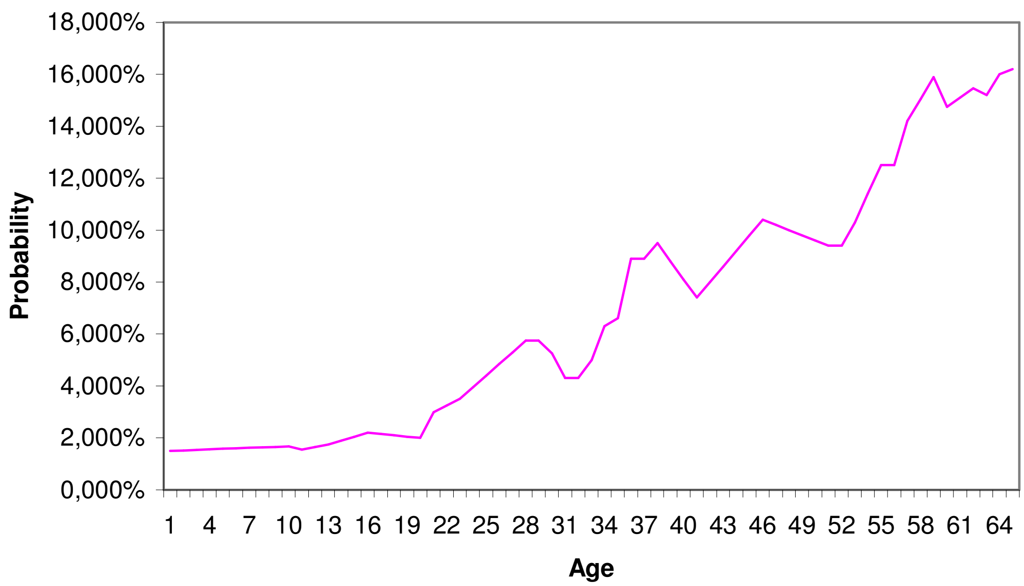

Figure 16.6.: "Drastic trend"

It is problematic, because if premiums are equalised, the portfolio will quite probably become auto-selected, which would mean that those who represent higher-than-average risk will consider the insurance to be exceptionally favourable, while those who represent lower-than-average risk will wait some time before taking out this type of policy. This will have the result that, all in all, expectations of the insurance company regarding the composition of risks within the portfolio will not be fulfilled. What is more, in such case it is worth for rival companies to enter the market with premiums differentiated by age.

If the insurance company uses age-dependent premiums, it is useful to consider the question of mathematical reserving.[153] But this also requires that the insurer keeps premiums of the individual policies fixed for a longer period of time (at least 3 years), i.e. there should be differentiation by age when entering the insurance, but aging during the term should not bring along changes in the premium paid.

There is currently a mixed practise regarding the equalisation of risk during the term and reserving. From a business aspect, a non-changing premium is more favourable, but accident and sickness insurance risks are not stable enough to enable the insurance company to engage itself for a longer period of time. Thus, from this perspective, this situation differs from the usual life insurance situations. As a consequence, insurance companies apply premiums that change yearly, or equalise premiums only for a shorter period of time (e.g. for five years), or they keep the possibility of changing the premiums, or they use a combination of all these possibilities.

The available statistics also influence the premium calculation of the insurance company. If there are no available statistics for the risks according to age for example, (up to now accidental risk has fallen in this category), the insurance company is prone to calculate a fix (level) premium based on the average of the risk community.

It is worth calculating premiums depending on age if there is a drastic trend according to age, even if there are no relevant statistics available. In such a case, the insurance company may use assumptions and analogies, or it may adapt existing data from abroad.

It often happens, that market ”practise” has also influence on whether the insurance company uses level premiums or premiums depending on age. In Hungary for instance, in the case of accident insurances, premiums independent of age and sex are most characteristic; there are examples of both ways in the case of disability waiver of premium, while in the case of hospitalisation daily allowance insurance tariffs usually depend on age.

16.2.2. The Problem

Now the goal of the case study is more apparent. Some questions have been answered, some are still open. It is decided that we plan some kind of life-, accident or health insurance rider. As for the risk, we find a strong trend increasing by age in the data. Net premiums have been calculated. These are:

- premiums depending on age and sex, changing yearly (generally[154] rising)
- there is no mathematical reserve and the technical term[155] of the insurance is one year.

The task is: to calculate an expense part on the net premiums.
Beside others, the following questions arise:

- How can expenses be spread in a “fair” way?
- How can loadings be determined? (The goal is to be able to carry out sensibility tests on loadings with a simple Excel model.)
- How can the effects of quota share reinsurance on the premium be planned?

The following sub-chapter deals with these questions.

16.2.3. Spreading of expenses

In the case study I use the generally known notations with the following differences, and I introduce the following special marks:

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[153] In case of single premium and long term, reserving is necessary even if premiums are independent of age!
[154] Despite the rising trend, temporary falls may occur.
[155] It is up to the insurer to define a product with formally a term of one year automatically renewed, or fix term. The automatically renewable policy enables the insurance company to correct the premium easily, or to leave the market if risks increase, while a fix term is more appealing to clients, because it makes planning easier.

nem: its possible values are: male, female

NP_x^{nem}: net premium of the insurance rider if the insured is of x years of age and of *nem* sex, in case of a sum assured of 1 Ft,

SA : the average sum assured of the rider at the commencement of the insurance (independent of age and sex)

P_x^{nem} : the size of the *sex* population of age x in Hungary

i_k : interest rate (discount factor) in the year number k. in the life of the product

fm: payback period of development expenses (year)

em: payback period of acquisition expenses (year)

tf_k : probability of surrender and termination (including termination due to death) in year number k. of the term

DB_k : number of new policies in year number k.

id_k : increase rate of the gross premium (and the average sum assured) in year number k.

In the model – which primarily serves for examining the effects of different expense factors – I assume an expense structure which is more subtle than the traditional expenses α, β, γ known from life insurances, but which is simpler than the structure used in big profit test models. The main idea is that I explicitly take into consideration the effect of the size of the portfolio and that of the average premium, versus the α, β, γ model; (although the effect of changes in the inner age structure of the portfolio and changes in time do not appear explicitly). I can do so, because I differentiate not only between expenses depending on the premium (and thus on the sum assured), but also between several other types of per policy expenses (that are independent of the level of premium). Of course there must be some hidden assumptions on the size of the portfolio and the level of premiums also in the α, β, γ models, but these are not explicit from the model's perspective.

The most important factor among those that I disregard here is the portfolio's inner structure (which is only represented by averages) and its development, and the ''transitionary'' types of expenses that are theoretically possible between the single and regular expenses.

The expenses I take into consideration have the following dimensions (**not** independent of each another):

- Single or regular expense
- Expense proportional to the gross premium or per policy expenses
- Expense of the whole portfolio, the portfolio of the given year, or the given policy

Since these dimensions are not independent of one-another, the following six combinations can be imagined (with the names that I gave them):

fk: development expense – arises only once – spread over the whole portfolio

ekfd: single expense occurring at the signature of every policy, in Forints/policy

eksz: single expense that occurs at every policy signature as percentage of the gross premium

foksz: regular expenses as percentage of the gross premium

foka: regular expenses in yearly absolute value (for the whole first year premium of all insurance riders)

fokfd: regular expenses in Forints/policy – for the first year premium of the given insurance rider

A profit test can be carried out easily if all possible expenses are listed on an Excel worksheet, put into one of the above mentioned categories, then the values are summarized by expense types. These sums will be the input data of the formulae. After that the value of different expense factors can be varied in the table, so the sensibility test of the premium can be carried out. Profit, allowances, provisions for adverse deviation, etc. should also be considered as expenses (e.g. as *foksz* expense), too.

Now, after the preparations, let's take a look at the possible problems that may arise in spreading the expenses. An evident solution is – so it is usually applied – that the *loading* is the same for all age groups, even if net premiums are different. But the solution developed for level premiums is not the best solution here.

Spreading expense factors proportionally on every Ft of the net premium would be a wrong method due to several of reasons:

1. Since the differences of net premiums by age are very high, certain age groups would bear a disproportionately great part of certain fix expenses, which is not fair.
2. Due to this unfairness, there would be an unnecessarily big difference between the gross premiums of different age groups.
3. But the most important thing that affects the insurance company is: the premium will not be ”self-financing”, i.e. if the age composition of the actual portfolio differs from the previous expectations, then the actual cover of expenses coming into the insurance company will also differ from the necessary amount. The reason for this is that expenses were not spread in the way as they arise.

In order to avoid these problems we create separate loadings for the different age groups and sexes, where our basic concept is to divide expenses between policies as precisely as possible where this division is unambiguous (disregarding differences occurring in the sums assured within the groups of the same sex, age and insurance type – that cannot be handled by a proportionate loading[156]).

This concept can be easily represented in relation to most of the expense factors, since they have originally been formulated this way (ekfd, eksz, foksz, fokfd). However it is a bit problematic in the case of the ”general” expenses like fk and foka. These first have to be spread over policies somehow. Now we choose the method of converting these two factors into one of the other four[157].

Evidently, these can be converted into the following expense factors:

- development expenses (fk): into single Forints/policy (ekfd) expenses
- yearly stock expenses (foka): into regular Forints/policy expenses.

The consideration that lies behind converting into per policy expenses is that general expenses should be shared equally per insured.

The conversion is done according to the following:

Converting development expenses: we assume that the payback period of development expenses is *fm* years, so it is loaded to the portfolio of the first *fm* years. Being a single expense, it is loaded to all riders taken out in this period of time, and for the sake of proportionality the loading is the same for all policies. We correct with a discount factor in the denominator of the formula, because the expense part with a longer payback period is carrying interest until it is refunded.

$$\frac{fk}{\sum_{j=1}^{fm} DB_j \cdot \prod_{k=1}^{j} \frac{1}{(1+i_k)}} \tag{16.1.}$$

Thus we raise the ekfd expenses per product according to formula 16.1.

Converting foka expenses: this is less complicated, since only one year is concerned. Similarly as above Fokfd expenses increase according to the following formula:

$$\frac{foka}{DB_1} \tag{16.2.}$$

Considering different factors:

fk and foka[158]: sharing them proportionally among all pieces.

[156] The step further is the solution already applied in the case of Unit Linked insurances, namely that certain expenses are completely separated from the premium and the sum assured, and are deducted monthly, in absolute value. Later this may be a possible way for insurance riders, too.

[157] In the case of Excel, this is only an intermediate conversion, it does not affect our assumption that the changes of these expense factors will also directly appear in the change of the premiums.

[158] This factor has to be recalculated every year.

ekfd (early Ft/piece expenses) – per product: here again we determine a payback period (*em* – it is practical to choose the same as *fm*, though their content is different), which means that these expenses have to be refunded on the portfolio alive during the payback period. These are spread in the ratio of the average gross premium. The extent of this factor is not influenced by the dynamics of portfolio development.

eksz (expenses expressed as percentage of the initial annual gross premium) – per product: nearly the same as the previous one, but here we do not have to care about the absolute value of the gross premium, but the payback period and probabilities of surrender. Dynamics of portfolio development is not a factor to count.

foksz (expenses expressed as percentage of the regular gross annual premium) – per product: simplest to deal with.

fokfd (regular Forints/policy/year expenses) – per product: divided by the average gross annual premium.

Considering the expense factors mentioned above and keeping our concept in mind, we get the following loading-formulae.

The loading expressed as percentage of the net premium can be calculated with the help of two kinds of expense factors:

1. expenses given in Forints/policy – these can be easily projected to the net premium. (ekfd, fokfd)
2. expenses given in the percentage of the gross premium (eksz, foksz), which have to be projected to the net premium.

If *b* represents expenses given in the ratio of the gross premium, and *n* represents the expenses given in the ratio of the net premium, then the loading can be calculated by the following formula:

$$\lambda = (1+\lambda)\cdot b + n \qquad (16.3.)$$

From which:

$$\lambda = \frac{n+b}{1-b} \qquad (16.4.)$$

$$\lambda_x^{nem} = \frac{ekfd}{SA\cdot NP_x^{nem}\cdot \sum_{l=1}^{em}\prod_{k=1}^{l}\frac{(1+id_k)\cdot(1-tf_k)}{(1+i_k)}} + \frac{fokfd}{SA\cdot NP_x^{nem}} + (1+\lambda_x^{nem})\cdot\left[\frac{eksz}{\sum_{l=1}^{em}\prod_{k=1}^{l}\frac{(1+id_k)\cdot(1-tf_k)}{(1+i_k)}} + foksz\right] \qquad (16.5.)$$

Expressing this explicitly (by using the relation deduced above):

$$\lambda_x^{nem} = \frac{\dfrac{ekfd}{SA \cdot NP_x^{nem} \cdot \sum\limits_{l=1}^{em} \prod\limits_{k=1}^{l} \dfrac{(1+id_k)\cdot(1-tf_k)}{(1+i_k)}} + \dfrac{fokfd}{SA \cdot NP_x^{nem}} + \left[\dfrac{eksz}{\sum\limits_{l=1}^{em} \prod\limits_{k=1}^{l} \dfrac{(1+id_k)\cdot(1-tf_k)}{(1+i_k)}} + foksz \right]}{1 - \left[\dfrac{eksz}{\sum\limits_{l=1}^{em} \prod\limits_{k=1}^{l} \dfrac{(1+id_k)\cdot(1-tf_k)}{(1+i_k)}} + foksz \right]} \tag{16.6.}$$

In order to simplify the formula 16.6., let us introduce the notation *kdf* (combined discount factor) as follows:

$$kdf = \sum_{l=1}^{em} \prod_{k=1}^{l} \frac{(1+id_k)\cdot(1-tf_k)}{(1+i_k)} \tag{16.7.}$$

Thus the formula above is simplified like this:

$$\lambda_x^{nem} = \frac{\dfrac{ekfd}{SA \cdot NP_x^{nem} \cdot kdf} + \dfrac{fokfd}{SA \cdot NP_x^{nem}} + \left[\dfrac{eksz}{kdf} + foksz \right]}{1 - \left[\dfrac{eksz}{kdf} + foksz \right]} = \frac{\dfrac{1}{SA \cdot NP_x^{nem}} \cdot \left[\dfrac{ekfd}{kdf} + fokfd \right] + \left[\dfrac{eksz}{kdf} + foksz \right]}{1 - \left[\dfrac{eksz}{kdf} + foksz \right]} \tag{16.8.}$$

16.2.4. The Effect of Reinsurance on the Premium

The loadings above were calculated without the assumption of reinsurance. But it is not sure at all that the reinsurer will undertake the reinsurance on net premium, so this effect has to be taken into consideration (of course, only if the insurance company reinsures the portfolio at all). Only the simplest type of reinsurance, the quota share reinsurance is discussed here. However, it is also the most widespread, and in relation to new businesses, it is given preference by reinsurance companies.

Let us suppose that the reinsurance contract is a quota share with r% retention, and the insurance company charges back a commission c% after the part handed over (for the gross premium). Since it is probable that c will not be equal to the expense part, the loading calculated without taking reinsurance into consideration will have to be corrected in a way, that the expense part, which is calculated on the basis of the new loading and which remains at the insurance company, should be equal to the expense part calculated with the former loading:

$$\lambda \cdot NP = r \cdot \lambda^{vb} \cdot NP + c \cdot (1-r) \cdot (1+\lambda^{vb}) \cdot NP \tag{16.9.}$$

Simplified:

$$\lambda = r \cdot \lambda^{vb} + c \cdot (1-r) \cdot (1+\lambda^{vb}) \qquad (16.10.)$$

Expressing λ^{vb}:

$$\lambda^{vb} = \frac{\lambda - c(1-r)}{r + c(1-r)} \qquad (16.11.)$$

where

λ: is the loading calculated without taking reinsurance into consideration

λ^{vb}: is the loading calculated with regards to reinsurance

NP: is the net premium.

In order to check the calculation, if the commission is completely equal to the loading, i.e. $c=\lambda/(1+\lambda)$, then:

$$\lambda^{vb} = \frac{\lambda - \frac{\lambda}{1+\lambda}(1-r)}{r + \frac{\lambda}{1+\lambda}(1-r)} = \frac{\lambda(1+\lambda) - \lambda(1-r)}{r(1+\lambda) + \lambda(1-r)} = \\ = \frac{\lambda + \lambda^2 - \lambda + \lambda r}{r + r\lambda + \lambda - \lambda r} = \lambda \qquad (16.12.)$$

If there is no reinsurance, i.e. if r=1, then the result is also:

$$\lambda^{vb} = \frac{\lambda - c(1-1)}{1 + c(1-1)} = \lambda \qquad (16.13.)$$

IV. QUESTIONS REGARDING THE LIFE INSURANCE INDUSTRY

17. SOME PROBLEMS OF THE LIFE INSURANCE INDUSTRY

KEY WORDS

Insurance application	Administrator
Broker	Mortality profit
Investment profit	Sources of profit
Medical statement	Medical examination
Sales channels	Hidden profit sources
Network	Temporary decline
Commission system	Renewal commission
Commission regulation	Acquisition commission
Calculated profit	Solvency
Claims handling	Area director
Classical branch offices	Product development
Underwriting	Recruiting
Expense profit	Surrender/lapse profit
Policy	Agent
Policy issue	Director of sales
Policy administration	Waiting period

In the following we try to deal with some of the problems regarding life insurance, though our list might be incomplete, we decided to create a kind of logical order. The reasons are mainly practical, as during his own business the author was mostly faced with these problems and has good reason to suppose that others are or will be confronted with them as well.

17.1. SOME PROBLEMS OF FOUNDING A LIFE INSURANCE COMPANY

It was mentioned previously that a typical life insurance is different from the other insurance types in two aspects:

1. Claims can be calculated with high exactness,
2. The insurance contracts are usually signed for decades, and the premium paid by the policyholders creates the funds for claims and expenses gradually.

These special features are reflected in the profitability of the newly launched insurance companies. It is natural, that every company, so as every insurance company, whether its main products are life or other kinds of insurance, show a deficit in the first years of its existence. The main reason for that is that the expenses of founding such a company (buildings, rental fares, salaries, devices, such as personal computers and software) has not been compensated yet by sufficient premium income. However, compared with property insurance companies, the life insurance companies have a specific initial source of loss. This specific source of loss can be connected to the problem mentioned at the topic of zillmerization, namely that (in the case of insurance with typical, i.e. regular premium payment) the expenses of the insurance companies resulting from the life insurance (commission, medical examination, policy administration) incur at the beginning of the term, while the cover of these expenses from premium loading are arriving in a relatively slow rate. One possible solution to this problem is zillmerization, which means that the insurance company borrows money from the premium reserve of the client. This sum can be that part of the risk premium in the first year or years, which is not essential for paying up the possible death in that year. If the insurance company determines the commission level to such a degree, that the sum borrowed by zillmerization meets the expenses of signing the insurance policy, this problem ceases to exist, which means that the newly founded life insurance company cannot be differentiated from the newly founded property insurance companies in the terms of initial losses. However, if the fights for the best agents in the business force the companies to decide on a higher commission rate, then this results in the above mentioned additional loss factor.

In these cases the insurance companies appropriate larger sums for the signing of an insurance policy than the premium income of the given policy in the first year. This means that the better the launching of the insurance company is and the faster it gets new insurance policies, the higher losses it has in the first few years, or until the new policies outnumber the old ones that managed to recover their initial losses. This period producing losses can last up to 5-10 years. This is usually longer than the loss-producing period of the newly founded property insurance companies, as in the case of (usually one year long) property insurance policies the initial loss factor caused by the commissions is missing. That is why life insurance companies are usually founded by firms with high capital investments that have the time to wait out this 5-10 years period.

In spite of the initial losses it is worth to establish a life insurance company, because the business itself is much safer compared to property insurance companies, due to the predictable feature of the claims. On the other hand, by means of the premium reserves bound by the policies for several years, the life insurance companies obtain sources that can be invested in long terms. In the case of property insurance, these incomes are missing or insignificant.

It is natural that even the experienced and calm owners would like to shorten the initial loss-producing period. That is why the pressure set by owners is particularly high to increase the premium loading in the case of newly founded life insurance companies.

17.2. Some Problems Arising in the Course of Company Operation

17.2.1. Product Development, New Policies

Product development is a strategically important area of every life insurance company, as it is the process of product development that determines the range of products defining the face of the company.

Most of their products can be traced back to the basic types of elemental life insurances mentioned earlier, however some details of the particular conditions (the possible entering age of the insured person, the possible duration of the policy, the age and sum limits of the medical examination, exclusions etc.) are usually different, characterizing a given company. The task of the product development process is to “monitor well” the needs of potential customers and to create a the “mix” that satisfy them perfectly from the available raw material. From time to time significant life insurance innovations emerge from these combinations.

During the process of development it is not only the marketing, but also the insurance technical aspect that must be considered. Let’s see an example: if the insurance regulations provide special tax reductions in case of term insurance policies but not in the case of endowment ones, then the insurance companies might be incited to sell their endowment insurance policies not as an endowment one, but divided into independent term and pure endowment insurance. However with this solution the range of possible premium increasing and profit sharing methods will be technically narrowed down. It is only the technique of revalorization that goes well with this solution, as this is the only technique where the insurance premium of both the term and the pure endowment insurance increases in the same degree and the initial proportion between the two sum assureds – that can be e.g. one to one – can last for the whole insurance term. If the company chooses a system for increasing the insurance premium that depends on the inflation rate, then the increase of the sum insured in case of term insurance and pure endowment insurance will be different already in the first year, as in this system it is only the amount of premium increase that is given, but the increase of the sum assured depends on the proportion of premiums in accordance with the tariffs, and in the case of the above mentioned two different policies it can be the same only by chance.

There are certain cases when some information is missing during the process of product development and this has to be surmounted by technical solutions. For the most of Hungarian insurance companies such missing information can be the product type selection (mortality) table that shows the rate of mortality among those who choose different policies (such as term life insurance or life annuity insurance), since the auto-selection works in the way that among people choosing term life insurance the rate of mortality is higher from the average, while among people choosing annuity insurance it is lower. In the case of term life insurance the insurance company has the right to apply premium correction factors during the process of underwriting, which means that the selection table is not an urgent need. However, in the case of annuity policies there is no underwriting, so there is no chance for correction, i.e. the effect of auto-selection must be calculated into the insurance premium beforehand. Without the selection table insurers usually use the method of age decrease, which means that during the calculation they consider every insured to be five years younger than their real age, which results in a higher premium.

The method of age decrease makes the calculations more simple by the means of saving special mortality tables and even good from the aspect of marketing. Some insurers do not use a special mortality table compiled for women to calculate their insurance premium, but use the men's mortality table considering the female customers to be five years younger than their real age.[159]

17.2.2. Computer Technological Background

Nowadays it is essential for a modern life insurance company to have a serious computer technological background. In all of the developed countries, and in Hungary as well, the life insurance companies administrate policies almost exclusively centralised, on computers. Policy administration (the supervision and underwriting of insurance applications collected by the agents) is also centralised, as well as policy issue, and correspondence with the clients (because of premium increase, profit sharing etc.). Due to the centralised administration system, the managers always have up-to-date information on the composition of the total portfolio, the proportion of lapse, the amount of premium reserve, the performance of agents and agencies etc. Thanks to the up-to-date information, it will be possible to intervene quickly and accurately where it is necessary.

17.2.3. The Safety of the Insurance Company

If we want to formulate generally what the product sold by an insurance company is, we could say it is safety itself. That is why it is extremely important for the company to be stable, creating safety for itself as well. This effort for safety can be traced back in all momentums of the functioning of an insurance company. Through the examination of the "life-cycle" of a concrete life insurance policy, let's have a look at the safety solutions that the insurer uses.

Everything starts with product development and the selection of the proper product. The product sold later must fit into the risk managing system of the insurance company (and vice versa: this system must be worked out in accordance with the product). That is why it is important to know the specialities of the risks embedded in the product itself. With a short bypass let's have a look at the traditional life insurance types that are important from this point of view. Where the risk of the company is the death of the insured person, the most considerable ones are the following: the term, the endowment and the term fix life insurance. If we look at the risks of the company, the order increasing by risk is term fix, endowment and term insurance. It is obvious that the term insurance carries larger risk than the endowment insurance, since in case of endowment insurance the pure endowment part decreases the effect of the risk part. The reason for the term fix to carry smaller risk than the term one is that the insurance company pays in both cases, but in case of the term fix benefit payment is always at the end of the insurance term, while the endowment insurance pays in case of the earlier death of the insured before the end of the term. The lower risk is indicated by the lower fluctuation of claims. If the insurance company does not want to deal

[159] Since they usually live longer.

with underwriting too much, then the best choice is to sell term fix insurance, as the Austrian companies entering the Hungarian insurance market and selling life insurances illegally have done in the early 1990s.

After the insurance application, but before signing the policy the insurance company carries out a very precise underwriting procedure (discussed in more detail under the chapter 17.5.3) in order to filter out the anti-selection.

If the risk is death type (and not pure endowment type as in the case of annuity insurance) and the sum assured is more than the risk bearing capacity of the insurer, then the insurer reinsures the policy.

The risk bearing capacity is connected to the variance of claims, which is connected to the level of the sum assured. The maximum level of variance can be such that the insurance company can pay incurred claims with high probability – using its capital resources and reserves.[160]

Reinsurers usually sign short term (mainly one year) contracts only for the death sum assured (to be more precise, for that part of the sum assured that is above the premium reserve) with the life insurance companies, which is why they do not create premium reserves of the undertaken risks. The net premium of the reinsurance is usually calculated with the help of a mutually accepted mortality probability sequence. After the signature and the possible reinsurance of the policy, the insurance company starts to collect the insurance premiums and build up the premium reserve. To make sure that the premium reserve can cover the costs of benefit payments, some insurance companies use the method of conservative premium reserving, which means that in the premium reserve calculation – leaving everything else the same – the company considers the insured person to be one year older than his real age, thus resulting in a higher premium reserve all together (because this method doesn't have the same effect on every single policy) than without the age increase. As we have mentioned before, this method is called “conservative” premium reserving.

The created premium reserve must be invested by the insurance company and in the case of traditional life insurance policies it has to reach the guaranteed yield in accordance with the technical interest rate. In this investment safety is essential for insurer. The basic principle is that if the company has to choose between the secure but low yield and the insecure but high yield investment form, then they choose the first option in every case. That is why the clients of an insurance company do not expect especially high, but rather safe yields from their insurers. The safety of the investment is secured by the method of diversification, i.e. the insurance companies themselves use the strategy of self-insurance. The diverse composition of the investment portfolio is, by the way specified by the “Act on Insurers and Insurance Activities”.

And finally before benefit payment is due (in case of death, maturity, paying the annuity) the insurance companies investigate the justness of the claim carefully to avoid insurance frauds.

17.3. The Sale of Life Insurance, Sales Channels

The most important segment of a life insurance company from the point of profit (if there is a “most” important segment at all) is the sales network. As the life insurance policies are typically long term contracts, it is extremely important to make sure that the clients choose this type of insurance not because of a sudden decision but after a thorough consideration, which means that they consider the premium payment to be advantageous after several years as well. Since the everyday people in Hungary do not possess even minimum level knowledge of life insurance and insurance at all, it can be presumed that they will receive the greater part of their knowledge on the subject from the representatives of the insurance company[161]. That is why it is especially important to have well educated representatives,

[160] Which are controlled by the minimum requirements, the so-called solvency regulations of the European Union.

[161] From its agents, and in this respect brokers are also representatives of the insurance company.

whose interest is to offer real and not false information to the clients. This is the main reason why "passing" agents (who are not professionals, only try to sell a few occasional policies) are not suitable for the sales of life insurance. The best thing for an insurance company is to work with the same, stable educated team of agents, who would not sacrifice their "professional good name" for the sake of some short term advantages (insurance policies based on manipulated data, that are this way unstable). Unfortunately in Hungary it happens quite often that the agents are bunglers not doing a satisfying job. (The scale of bunglers is very extensive. It starts with some basically honest agents whose only reason for selling a life insurance policy is "why not pay life insurance premiums instead of paying for 20 packets of cigarettes each month" and it ends with the type – acting in bad faith – who offer a very high and long term yield (exceeding the interest of the best bank deposit) after the premium reserve.)

Bunglers can cause great damage not only to the clients, but also to the insurance company whose policies they sell. It is not just about the dissatisfied clients who can deter everyone in their environment from the given company and even from signing a life insurance policy, but about the possibility that the level of undertaken risk is not clear to the insurer due to false or unsatisfactory data provided by the bungler agents, and so the insurance becomes effective with a premium lower than necessary.

There are several sales channels for selling life insurance policies. The traditional way is the sales through insurance intermediaries (agents or brokers).

Apart from some exceptions the sales through **brokers** is not so wide spread in Hungary. The main reason for this is that the brokers are usually specialised for the sales of the valuable property insurance policies and not for the sales of life insurance, that can be considered as a mass insurance type from this point of view. However, there are certain signs that in building their own network, insurance companies are using elements characteristic of broker firms (without transforming their own network into a broker firm). Some of the insurance companies start to build up networks with units owned by the manager or managers, that are in contractual relation with the insurance company and are selling exclusively their policies (so they are not brokers), and these managers employ the agents, who are usually enterpreneurs. Regarding the settlement of accounts, the process is the same as in the case of broker companies. There is an opposite tendency as well, when certain brokers organise an independent agent network for the sales of life insurance policies (who often don't stay formally brokers, but become agents of several companies at once).

In Hungary life insurance companies sell their insurance policies primarily the traditional way, usually through their own agent network. Since it is the most important and wide spread method in Hungary, we will discuss it later under subchapter 17.4.

The modern life insurance sales channels can be alternatives of the insurance company's own agent network:

- branch offices of banks ("bank insurance")
- direct mail,
- call-centres,
- the Internet
- others.

Sales through the **branch offices of banks** is spreading in Hungary, and many insurance companies were founded by banks, where the only sales channel is the branch office. The success of bank insurance depends strongly on the traditions of the given country. In some of the Western countries (e.g. Portugal, Spain and Belgium) this is the most important sales channel, but in other countries it has only marginal role. It cannot be predicted what the role of bank insurance will be in Hungary, strong boost or run-down can both be imagined. However, it is important to mention that it is only the very simple life insurance products that is suitable for sale in bank offices, which means that the structure of products offered by banks is very different from that offered by agent networks.

The method of **Direct Mail** (when the companies send letters to clients offering their life insurance products) is not present on the Hungarian market today. Although there were several attempts to introduce this method, it is not probable that it will be successful in Hungary.

The **Call-Centres** are in the same position as the direct mail method, but there were more attempts made in this area.

The Call-Centre as a sales channel means selling through phone, and it has two different types: the active and passive type.

The **Active Call-Centre** is when the insurance company operates with outgoing calls, i.e. it is the call-centre that turns to the potential clients offering them the services of the insurance company. In some countries (e.g. in Great Britain) these active call-centres are really successful in the sales of certain community insurance types (mainly in the case of house- and third-party insurance policies), but (until now) failed almost everywhere in the case of life insurance[162].

The **Passive Call-Centres** are much more common. The main point of this call-centre is that the insurance company advertises in the media the number of the call-centre, which is usually free and can be reached night or day, and the products that are sold by calling this number. There are many passive call-centres in Hungary, but the records show that the clients usually dial these numbers not with the intention of purchasing but to make complaints and report claims. That is why their importance is shifting from sales to handling and managing existing policies.

Nowadays the sales by the means of the **Internet** is in an experimental state in Hungary.

From time to time insurance companies launch **other** sales channels as well. Some examples: they have tried to sell life insurance policies in post offices and through trade union members.

Lets have a detailed look at the most important sales channel: the insurer’s own agent network.

17.4. Sales Through the Insurer’s Own Agent Network

17.4.1. Network Organisation and Management

The agent network can be independent or be the part of a “general” network. In accordance with this there are two major types of networks in Hungary:

- classical branch offices, and
- networks dealing with sales only.

It is usually the non-life or composite insurance companies that have classical branch offices, because it is the property insurance where it is important to hold policy administration and claims handling in a branch close to the insured property. The main point of the classical branch office network is that the branch not only deals with sales, but also with risk underwriting, policy administration and claims handling. The sales itself is only one function of the network unit, and the leader of the sales team is not the same person as the manager of the branch but one of the substitutes.

In the case of specialised life insurance companies it is not reasonable to have a classical branch office network, as the customer service and claims handling can be carried out centrally as well. Here it is enough if the network deals with sales only. That is why some of the composite insurance companies have tried to establish an independent network which is different from the networks dealing with property insurance and deals only with life insurance sales. As we have mentioned before some insurance companies have tried to copy the Western example and give the units of the network to its managers. (The units of the sales network built from units with independent owners are not to be mistaken for brokers, since brokers are not committed to only one insurance company.)

[162] There were two unsuccessful and terminated attempts to introduce the active call-centres for the sales of life insurance in Hungary at the end of the 1990s.

All of the networks (apart from the small insurance companies) have different levels. The leaders and the staff of the different levels of sales networks have different names. Some of the companies split the country to regions and the leader of these branches are called regional directors. This is common in the case of larger insurance companies. In most insurance companies the biggest unit of the network is usually an “area” (e.g. a county) defined in different ways at different companies, with a leader called “area director” at the top. In the regional system there are several ’territorial “area directors” under the leadership of a regional director. The “area director” directs more than one team of agents. At the top of each team of agents (one team usually consist of 5-20 agents) is the so-called “director of sales” (in some places the “director of the branch” is also used, but this name can be misleading as it refers to the classical branch network). The official name of the sales person under the leadership of the director of sales is the agent, though some people use it unwillingly. They are usually called mediators, representatives or advisor (professional, bank insurance, etc… advisor).

The agents get commission for their work done. This is the most important factor for them, so the insurer can encourage agents for better achievement by means of the commission system. The possible types and scale of commission is laid down by the commission regulation. It is basically the commission regulation that transfers the will of the centre to the network. It can be said with a slight exaggeration that the only communication channel between the centre and the network is the commission regulation. Because of its importance it is discussed in a separate subchapter.

17.4.2. Commission System, Commission Regulation

Regarding to the commission regulation there are more than one requirements. The features of a satisfactory commission regulation are the following:

It is **comprehensive** that is, it covers all aspects. Comprehensiveness means completeness from more than one point of view.

On the one hand the commission regulation must include all of the expenses incurred in connection with sales. These expenses can be the money given to the directors and the agents to cover their costs of clothing, the costs of company cars, the fix salary of the directors of sales. These expenses only incur in connection with sales though they are not considered to be part of the commission system. From the point of view of expenses comprehensiveness is essential so that the commission regulation can provide a satisfactory basis for profitability calculations. In other words it means that the commission regulation must consider the sales network to be an independent profit centre, so it must include all of the expenses and there should be no costs in connection with sales that are not part of the commission regulation.

On the other hand the commission regulation must be complete from the point of view of agent hierarchy as well, since it has to include all of the different posts that are in any relation to sales. E.g. instructors supervising the network, brokers, possibly the officers of the bank who sell bank insurance, etc. must have their place in this hierarchy.

Thirdly the commission regulation must include the detailed description of all instruments used during the work of the agents such as the method of planning, supervising and examining the work-plan of agents etc.

Fourthly it must include every encouraging instruments that is different from the commission such as the system of the prizes of different competitions, the system of “encouraging” travels, career opportunities for the leaders (and leaders-to-be), that means not only advances in the hierarchy, but gaining some of the status symbols connected to the increase in the number of policies sold (e.g. company mobile and car, etc. regulating exactly the connection between the benefits and the acquired portfolio).

Fifthly it must include the benefits of that team leader who establishes a new team of agents on his own that can become independent in the future. These benefits are essential in making the team leaders interested in encouraging their best agents to be independent and widen the network of sales.

The sixth feature of the commission regulation is that the regulation must exactly regulate all commission types that are due according to the retaining level of the portfolio of an agent or their leaders, so that keeping the portfolio becomes an automatic task instead of a question of occasional actions.

The good commission regulation is **stable** in two ways: on the one hand the system itself has to be unchanged for a long time, on the other hand the commission rates have to be constant as well. Every effort should be made to have less and less people possess the power to change the rates, as rarely as possible. (The best solution would be if only one person, the director in charge of the network could change the commission rates). The reasons for the change must be controlled precisely.

The commission regulation must be **multi-level,** which means that the salary of all the agent leaders of all levels must derive directly from the performance of the agents in their team exclusively (or at least primarily). The fix salary of the director of sales must be set to minimum and should be paid for a temporary period, which should be shorter and shorter with the development of the company!

It must be kept in sight that for an insurance company it is better to have no director of sales at all in one region than to have one with a very high fix salary and doing nothing for it.

Maximally proportional to performance: that partly means the multi-level feature in the above sense, and partly that the system must not contain any commission title (e.g. interventional and procedural commission) that doesn't increase strictly proportional to the volume of business, but exactly contrary to it. (The interventional commission is highest when the paying morality of the clients is worst, i.e. when the insurance company has financial difficulties. Moreover these cannot be calculated beforehand, and their use is not in line with the requirement of predictability, without which profitability cannot be measured well.)

Of course the interventional and procedural commission gave answers to real problems so if they are eliminated, new solutions must be found taking over their functions. The new commission used is the internationally well known **renewal commission**.

Making the renewal commission general means at the same time the spreading of **service agents**. This means that all policies have a service agent at every moment whose task is to do every "agent type" job in connection with the policy (e.g. intervention, handing out letters of premium increase, providing information, etc.). (The system of service agents is a question of company philosophy. Its application means that company and the agent do not only deal with the client until the policy is signed, but the agent serves as a kind of personal financial advisor.)

It is the service agent who receives the renewal commission. At first the service agent is the agent who managed to sign the policy with the client, but if he leaves the company or the region then another agent receives the policy. This means that the system of the renewal commission is general, because in case of a policy the title to renewal commission never ceases to exist.

A performance proportional commission system means that the commission is always proportional to the premium income. If the acquisition commission is determined as a percentage of the first year's premium then the agent will receive it immediately when the client pays the annual premium, but will receive it in 12 instalments if the client chooses to pay the insurance premium monthly. This is called "earned as paid" commission in insurance terminology. It means that the insurance company can avoid the problem of commission chargeback.

The agents are encouraged to convince the clients to choose the one year payment which is the most secure method of payment from the company's point of view. It can happen that the acquisition commission does not depend on the frequency of payment but in these cases the rules of commission chargeback must be laid down precisely. The renewal commission is always due from the **beginning** of the second year in the same frequency as the client pays the insurance premium so it is similar to the acquisition commission from this aspect. The renewal commission encourages the agent to intervention automatically as without the premiums paid by the client the agent won't receive the commission. Of course the intervention must be backed up with an interventional list by the informatics and the leader of the agents has to supervise its execution. The authority of the leader of the agents has to be defined properly for taking away the service agent function from those agents who do not intervene satisfactorily.

Controlled in detail: nothing can be left to the invention of the person using the regulation. It would carry the risk of creating several different "mutations", or uses of the regulation in opposite to the original intention due to "legal gaps".

The computer record of agents, insurance policies and commissions must be controlled as well. It is a basic requirement to keep a record of the total commission history per policy and per agent. The system must allow service agents to move between policies and teams.

Flexible: it is important for the company to adapt itself to the changing circumstances without changing the structure and principles. Flexibility is a kind of principle that must be considered all the time during the development of the concrete commission system. Flexibility can be reached by some special solutions such as leaving blank positions in the hierarchy in case of introducing new levels (as yet of unknown purpose) later on.

For the sake of flexibility the insurance company may split the different sales channels without integrating them into **one** hierarchy, and plans more than one channels strictly divided and operating parallel to each other. One example of that is when the elite policies are sold separately from the common insurance policies by an independent team, having their own leaders and with a different commission system, etc.

Well-edited: The good commission regulation is well-edited, making it understandable for its users. This can be achieved by commentaries connected to the regulations. These commentaries are not part of regulation, but give useful help and hints to the user.

17.4.3. Recruiting

Organising the agent network is a continuous task due to the natural fluctuation and the development of the company. The official term for winning new agents over to the company is **recruiting**.

Recruiting is usually the duty of the unit leaders as they are the ones responsible for the satisfactory sales performance that cannot be reached without a team of adequate size and quality. During recruitment it is important for unit leaders to know what kind of agents they are looking for.

From all of the insurance types selling life insurance is the most difficult task. That is why it is fortunate – though not a basic requirement – if the agents selling life insurance policies have a college/university degree.

As the life insurance is a long-term financial commitment towards the clients, it is essential to have clients who trust the company. It is the agent who is the representative of the insurance company, so the agent has to be able to win the clients' confidence.

Life insurance is an instrument for making our lives predictable, therefore those people become clients who have a need (or this need has been aroused in them) for a consolidated life. The agent has to radiate this consolidation as well, this is why elderly or middle aged agents are more suitable for selling life insurance policies than younger ones. This is also related to the fact that the typical client is usually middle-aged.

The training of new agents is a very important part of the recruitment process. The agents must be educated and trained well enough to be able to give thorough information. Although training is most emphasized at recruitment and directly afterwards, it really is a continuous requirement as the knowledge of agents must always be adapted to the changing circumstances.

It is important that recruiting should aim at new agents instead of winning over well trained agents from other rivals. On the one hand it is a question of ethics, on the other hand those agents who are won over easily can also leave the new company any time.

17.5. Technical Duties Regarding the Signing and the Administration of a Life Insurance Policy

It is useful to go through the history of a life insurance from its signature to its termination to know the different duties that might have to be done in relation to it.

17.5.1. The Insurance Application

The agent fills out the insurance application (or proposal) form of the policyholder and the insured person and collects the insurance premium of the first insurance period. The money is transferred in 1-4 days – according to the business regulations – to the unit handling the contract (the centre or a given branch office). Simultaneously with the signature of the application, the medical statement form is also filled out about the health conditions of the insured person and/or (depending on the age of the insured, the type of insurance and the sum insured) makes an appointment with the M.D. of the insurance company who examines the insured person thoroughly.

The word **insurance application** has two meanings here. In spite of the fact that the active party is usually the insurance company (the agent representing the company), when a life insurance policy is signed it is the client (the policyholder) who takes the initiative formally by **proposing** the company to sign a life insurance contract. The contract comes into existence when the insurance company accepts the application of the insured person. It is a practice to use the term **insurance application** to the form which is filled out by the policyholder when proposing to the insurance company.

The **medical statement**, is an official form as well, where there are questions asked by the insurance company regarding the medical condition of the insured person. The medical statement must be filled by the insured person as part of the **reporting liability** of the insured and consciously incorrect statements might involve punitive sanctions and finally the refusal of the payment of the sum insured.

The (first) insurance period that the premium collected by the agent from the policyholder when signing the application depends on the frequency of the premium payment. In the case of a single premium insurance the insurance period is the whole insurance term, in case of monthly premium payment the insurance period is one month, in case of quarterly payment the period is 3 months, which means that the insurance period is the term covered by the premium.

For the sake of informing the client the application is usually filled out in two copies, and the agent leaves one of copy by the policyholder. Because of the same reasons the agent also hands over the conditions of the given insurance. These conditions have usually two levels: the **General Life Insurance Regulation** (or the General Conditions of Life Insurance) contains the general rules of life insurance policies of the given company including the most important life insurance regulations of the civil code. The **Policy Conditions** contain the conditions of the given life insurance product which might be different from the regulations laid down by the General Life Insurance Regulation. If the insurance contract is accompanied by different insurance riders then these might each be regulated by different special conditions.

17.5.2. Policy Administration

The first task in connection with the insurance application is **policy administration,** that is a formal and technical check of the data supplied by the client and the completeness of the application. The person doing the policy administration is called the **administrator**.

The objects of the supervision are:

- First of all whether all of the required columns are filled out by the client. If something important is missing then the administrator gives the insurance application back to the agent who has to visit the client (or clients) again.
- The correctness of calculations, i.e. whether the agent calculated correctly the following:

- the **age of the insured person** at the time of the signature of the contract. The Hungarian practice (which is not general internationally) is that the age of the insured person is calculated by subtracting the client's birth year from the effective year of the policy.
- the insurance premium paid after the **initial sum insured**. The initial sum insured is the sum laid down in the application and which serves as the base to the insurer for calculating the initial premium. The initial sum insured can be different from the current sum insured – due to inflation handling techniques, premium increase and investment profit sharing.
- **the authenticity of the signatures**. (This is important for avoiding the possibility of agents signing the application instead of the clients to avoid the difficulties of handling incomplete information.)

If the application and the medical statement is complete and correct officially, than the policy is ready for underwriting.

17.5.3. Underwriting

To avoid anti-selection the insurer must use the method of **underwriting** before almost all policy issues. What are the risks of the insurer in the case of the different types of products?

In the case of **Term Insurance** the risk of the company is that the client will pass away "early" (before the maturity of the insurance). From the point of the insurance company those clients whose expected life span is shorter than average signify bad risk. Underwriting is to avoid and manage the bad risks of a company.

In the case of pure endowment and annuity insurance policies the risk of the company is that the client will be alive at the time of maturity or that the client will live longer than average. It can be said that at this form of insurance it is the client with a bad health status who signifies a good risk. It would be logical to adopt an underwriting method similar to those used in the case of term insurance (medical examination for example) and if it turns out that the client is in perfect health condition the company would take the risk with increased insurance premium. However there is no underwriting in the case of pure endowment insurance at all. One of the reasons for that is that this kind of insurance is usually not sold separately (because it is difficult to have the client accept the fact that in the event of death all the results of saving will be granted to the insurance company, or the risk community). The other reason is that from the client's part – as the probability of surviving is more likely than the probability of death – survival carries less chance of manipulation in itself, so the possible profit for the client is much smaller, which does not cause anti-selection on a big scale.

In case of annuity policies the insurance companies compensate the auto-selection occurring necessarily by the method of age decrease built in the policy, or by using some kind of projected mortality table. The insurer supposes that only those will sign an annuity policy whose chances of survival are the best, this way insurance premiums are higher, but there is no underwriting. This is why it is not worth signing an annuity insurance for those clients whose chances of survival are worse, which means that the insurance companies leave a significant potential segment of the market uncovered. This is why in some countries there are trials for a kind of reversed underwriting in case of annuity insurance policies, which means that if someone can prove that they are ill then they can get reduction from the insurance premium. This kind of practice has not appeared in Hungary yet.

Underwriting can be avoided with certain methods in certain cases. These methods are usually used in the case of cheaper mass insurance types where the expenses do not contain the cover of underwriting or where the nature of the sales channel is not suitable for it (e.g. in banks or in case of commercial credit life insurance). The most common of these methods are:

- group selling – not for selected groups
- applying exclusions of wide range

- gradual increase of the of term sum assured (i.e. the total sum will be valid in 3-5 years, by the time the effect of underwriting would be over. Until then the death benefit increases gradually starting from a low amount.)
- declaring a waiting period (this can be currently maximum 6 months in Hungary)

Sometimes it is compulsory to have a **waiting period** in case of underwriting as well. When there is a waiting period and the insured event happens during this period, the insurer (apart from cases such as accidents) pays not the sum insured, but only the premium reserve to the beneficiary. If there is a medical examination before signing the insurance contract the insurance companies usually disregard the waiting period.

The underwriting is the process of examining whether the application carries a normal scale risk or it should apply higher premiums because of the higher health or occupational risk factors (sport, hobby). There are usually three categories of the risks:

1. Health
2. Occupation, sport, hobby
3. Financial

In accordance with this there are three types of underwriting:

1. Medical
2. Non-medical
3. Financial

The documents of the medical underwriting could be the medical statement and in certain cases the statement from the M.D. (that can have different degrees).

The inputs of the non-medical underwriting are usually the appropriate data of the application form and filling out a special questionnaire in some cases.

The financial underwriting is not so common, it is usual only in cases with very high sum insured. The task of financial underwriting is to examine whether the client's purpose to sign the policy is well-established enough on a financial basis. The insurance company must be sure that the client can pay the insurance premium (so that the energy invested in the acquisition of the policy was not wasted) and that there is no purpose of insurance fraud.

In the course of underwriting it is essential to clarify whether the documents supplied (application, medical statement, the records of the medical examination) contain enough information to accept the application.

If some additional information is needed, then the underwriters get it from the agents, by mail or sending the client to a new medical examination. There are 15 days provided for the underwriting. If it is not enough then the insurance company has to refuse the application in accordance with the civil code or the policy will come into force automatically.

The results of underwriting can be different:

- in most cases the insurance application is accepted,
- in some cases, when the insured carries a higher risk (has a bad health or because of other reasons, e.g. has a dangerous job or pursue dangerous sports) the insurer can increase the premium or declare exclusions. Of course the premium increase is an alteration of the insurance application, and it has to be accepted by the client.
- in some cases the application can be refused.

One of the possible methods of increasing the premium is to raise the age of the insured (that finally results in premium increase). Nowadays it is more common to increase the premium directly with the starting point of the so called extra-mortality. The rate of premium increase is usually calculated from tables (generally compared by reinsurance companies). The increase can be expressed in the percentage of the premium, in per thousand of the sum insured or as a fix amount.

In addition or instead of premium increase it can happen that some events are excluded from the insurance.

It can happen that the insurance company does not accept the risk and refuses the application. The refusal can be permanent or temporary, when it is called temporary decline. The permanent refusal happens only in the case of extremely high risks such as a very serious fatal illness that was unknown by the client as well. Temporary decline is usual in case of pregnant women.

If the insurance company accepts the application, or the exclusion or premium increase is accepted by the client, the next step is the policy issue.

17.5.4. Policy Issue

The **policy** is a document issued by the insurance company containing the fundamental conditions of the contract. By issuing the policy the insurance company accepts the application of the policyholder so the contract comes into force at this time.

The life insurance contract has two parts, the insurance application signed by the client or clients and the policy signed by the insurer.

17.5.5. Indexation, Indexation Letter

There are certain repeated tasks with every insurance during its term. These are usually connected to the **policy anniversary**. The policy anniversary is usually that day of every year when the policy came into force. There are some Hungarian insurance companies that settles the date of the anniversary to be the first day of the month following the signature of the policy, but this practise is less and less common. The commencement of the policy adjusted this way is called the "**technical commencement**".

In the case of traditional insurances it is usually the policy anniversary (or a given day of the year) when the investment profit originated from the investment of premium reserve is credited to the client who has the option of **premium increase** at this time (this is true for modern life insurance policies as well). Making use of premium increase options technically means the purchase of new policies (even in case of the revalorisation technique, though it is not so obvious) that have a term equal to the remaining term of the original policy, and the entering age of the insured equal to the current age of the insured person. In case of making use of premium increase options the premium reserve is the sum of the premium reserves of the initial sum insured and the increased sum insured.

Due to investment profit sharing and premium increase the **current sum insured** becomes higher. The current sum insured is the sum insured determined in the policy (the initial sum insured) raised with the sum insured of new policies chosen by the client as an increase option and/or with the sum insured in those single premium insurances that are obtained by the client through investment profit sharing. If the client did not make use of premium increase option or the policy doesn't receive investment profit sharing (or it is administered on a separate account), the current sum insured can be the same as the initial sum insured.

With the increase of the current sum insured the current premium reserve of the traditional insurances will be higher as well.

The premium reserve always consists of the following parts:

- the premium reserve of the main policy,
- the reserves of the investment profit sharing
- and the premium reserves of the new insurance policies purchased by the client through premium increase options.

17.5.6. Claims Handling – Making Use of Non-forfeiture Options, Insured Event, Maturity, Benefit Payment

Making use of non-forfeiture options and the occurrence of the insured event results in the insurer's liability of benefit payment. In case of benefit payment the contract usually ceases to exist, but not in all cases. In case of paying up the policy, partial surrender or if in unit linked life insurances money is withdrawn from the funds, the policy is continued. In the case of term fix insurance death is an insured event but the policy is not terminated. In case of some insurance types the maturity of the policy is an insured event as well (term fix, endowment insurance) but in other cases it is not (term insurance).

The sum paid is corrected with the sum originated from the differences of the frequency of premium payment at the time of calculating the benefit. If the insurance company supposes annual premium payment in its calculations and conditions but gives the allowance of monthly, quarterly or semi-annual payment, then at the time of making use of non-forfeiture options and when the insured event occurs, the part of the annual premium not paid in yet can be subtracted from the amount of benefit.

In case of supposing monthly payment as a basic principle the case is just the opposite. If the clients chooses not monthly but less frequent payment, a so called **unearned premium** is formed, that the insurer pays back to the policyholder together when paying the benefits. The unearned premium is the premium for the whole months remaining from the given insurance period, as it has not been “earned” by the insurer yet.

17.6. The Profit of the Life Insurance Company

The business insurance companies work for profit. At the end of discussing the subject of life insurance let’s have a look at the different parts of the final goal of the operation of life insurance companies. Basically there are five different types of profit:

- calculated profit,
- expense profit,
- premium reserve (and capital) investment profit,
- mortality profit, given from the difference between the calculated and real mortality rates
- hidden profit sources.

Of course all of these profit types can be negative as well, which means that the insurance company has loss. Let’s have a look at these profit types one after the other:

1. The ***calculated profit*** is planned to be included in the premium loading and in the expenses of modern insurance types as one of the expense elements.

2. The ***expense profit*** is the result of the fact that the insurance company during its operation does not use the total premium loading collected to pay for its expenses.

3. As we have discussed before, the ***investment profit*** is the interest earned by the insurer on the premium reserve and the yield of the asset funds of unit linked life insurances. It is important to know that the profit of the insurance company and the yield of the premium reserve and asset funds are different, which means that the insurer can show a loss and still pay profit share to its clients after the yield of the premium reserves and also the yield of the asset funds can be favourable. Naturally the insurer doesn’t have to share the yield of investing its own capital, it increases or decreases its profit and not the clients’.

4. The ***mortality profit*** or loss results from the difference between the calculated and the real mortality rate. If the real mortality is higher than the calculated one that means mortality loss in the case of term insurances but in the case of pure endowment (and annuity) insurances it means mortality profit and vice versa. If an insurance company is not specialised in annuities, then its portfolio is dominated by term insurance types (where the real risk of the insurer is death, as in case of term, endowment, term fix and unit linked life insurances), which means that the real risk is the higher-than-average mortality. If the insurer calculated correctly, there is no general increase in the rate of mortality among the insured population due to economic recession or similar reasons and the underwriting was correct, then the insurer has to realise mortality profit. The reason for this is that from the point of mortality the clients of an insurance company are in a better position than the national average which is the base of the calculation. Since people who sign a life insurance policy and can regularly pay its premium presumably live in good financial conditions and

have long-term plans, which means that they care for themselves etc. In the process of underwriting the insurance company makes a selection among the population and this creates profit as well.

The mortality profit is not absolutely due to the insurance company under all conditions. In this regard the practice is different in Hungary. Most of the insurance companies keep the whole mortality profit for themselves, but some of them share it with the client according to the rate of the investment profit sharing.

In the developed countries life expectancy has significantly and continuously increased in the past decades, and this tendency is expected to continue, this way in case of annuities the danger of the mortality loss is high, one can say that it is the mortality loss and not the profit that can be regarded as “default”. However there are some mortality tables taking the annuitant auto-selection into account and projecting the increase of life expectancy[163] that can be a useful in fighting against mortality loss.

5. The ***hidden profit sources*** are not so much “in view” as the “open” profit sources discussed so-far, but they are just as important for insurers. We try to introduce – without discussing all types – three different hidden profit sources. These hidden profit sources are mostly typical in case of the classically calculated traditional insurances. The transparent structure of unit linked life insurances and the profit-test techniques used for their calculation make these profit types more open and realised in the above profit source categories.

The first hidden source is the **surrender profit**. On the one hand it comes from the fact that in case of surrender or if premium payment fails than the insurer doesn’t pay all the premium reserve back to the client. On the other hand the source of this profit is that the insurance company takes back from the agent the acquisition commission paid in case of surrender before the end of a given period (for example 2 years). It is important to know that this kind of profit is not favourable for the insurance companies, because surrendering the policy means the reduction of the portfolio. On the other hand we have already mentioned that one purpose of holding back part of the premium reserve in case of surrender is to balance the expense-increasing effect of anti-selection. Also there are some cases when commission chargeback from the agent is not possible, if at the time of surrender the agent does not work for the insurance company any more.

The second hidden source is that the “**new” insurance created by premium increase** is usually given to the client according to normal tariffs and the commission is embedded into the premium loading part of the normal tariff. However after premium increase the agent does not receive acquisition commission, which means that this remains at the insurer as a source of profit. In reality this hidden source is a type of expense profit.

The third hidden source comes from **zillmerization**. As we have already discussed, zillmerization means that the insurance company borrows part of the premium reserve of the client which is paid back gradually with all the interests during the insurance term. The interest paid to the client by the insurance company is the same as the technical interest rate, which – mainly at inflationary times – falls behind the market loan interest rates. This means that there is some interest profit deriving from zillmerization.

It is important to note that the existence of the hidden profit sources does not mean that the insurance company cheats on the clients as these hidden sources are the part of the normal profit similar to that of other companies in different sectors. Making the hidden profit sources visible and making proper accounts to the clients would probably not be worth the effort, i.e. the expenses of the accounting would be more than the profit, and in the most cases making these sources visible is not possible (e.g. in case of surrender profit the monitoring of anti-selection due to surrender is problematic).

[163] In Hungary projections like this are made by László Hablicsek.

18. TECHNICAL INCOME STATEMENT

KEY WORDS

Premium due date	Commission chargeback
Direct debit	Mathematical reserve
Written premium	Unearned premium
Embedded value	Technical income statement
Profit dependent premium refund reserve	Individual outstanding claims reserve
Reserve for claims incurred but not reported	Surrender reserve

18.1. THE TECHNICAL INCOME STATEMENT IN GENERAL

Every economical organisations has to know the processes and factors that influence its results. For Some companies, such as insurance companies the demonstration of these is officially regulated. A document of this kind is called technical income statement. In the case of life insurance companies – because of the special features of the business – calculating the profit is a very complicated process requiring many special calculations.

In the following we'll examine the technical income statement of the traditional life insurances, because in case of Unit Linked insurances – due to their transparent structure – it is far more simple to create such a statement. This means that by examining the traditional type, we deal with the more difficult task[164].

The technical investment statement is the appendix of the yearly balance sheet. The balance sheet tells us the value of reserves and profit on the turning day, but the processes resulting in these figures are not visible. The purpose of the technical investment statement (as usually the purpose of income- and result statements connected to the balance sheet as an appendix) is to show: from the values of the previous turning day what processes resulted during the elapsed time in the reserve- and profit figures stated in the balance sheet. The question is that during the accounting period what happened to the initial assets of money and to the new ones arriving in the meantime (mainly premium income) until they reach the final figures of the balance sheet?

The technical income statement can be closely connected to the **profit test,** but is very different from in some aspects. The main difference is that the profit test is a preliminary test for an imaginary portfolio, but the technical income statement explains the real processes on the real portfolio afterwards. This means that when making an income statement we have to use the same model as in the case of making a profit test. The difference is that we take the real figures as a starting point and considers the real happenings of the period in question.

Planning and **controlling** mainly differs from the technical income statement in that these give an acting line into the hands of the management and a report of the momentary status of its realization. Planning and controlling doesn't need to be comprehensive, precise, dealing with all the details. In the case of the technical income statement the most important things are comprehensiveness and the precise[165] equality of the appropriate data.

The technical income statement shows the processes resulting in the figures at the end of the year and thus it is able to split these processes into different parts.

In case of the technical income statement we have to give account of two things:

1. What happens to the money collected during the year? (Mainly to premium income, but there are other income types as well, e.g. increase of share capital.)

[164] However the Unit Linked insurance stands on a totally different "product platform" than the traditional insurance, so the technical income statement of these insurance types must be examined in a different way.
[165] It is essential to know that in the practice we have to make compromises and sometimes apply approximation solutions instead of calculating the precise value.

2. What happens to the money already reserved at the beginning of the year during the next year? (insurance technical reserves, share capital)

The profit consists of five different parts:

1. Mortality profit
2. Surrender profit
3. Investment profit
4. Profit correction factor
5. Expense profit

18.2. THE PATH OF THE MONEY COLLECTED DURING THE YEAR

First we look at figure 18.1. that shows the schematic path of the collected premium within the insurance company. This figure can be of our help later on as well, though not all of its branches (e.g. the change of the external sources and the share capital) will be dealt with and some of the branches (e.g. the risk profit and claims reserve) will be discussed in more detail.[166]

[166] It was my intention to show a figure that is general and not applied for a concrete life insurance company dealing with traditional life insurance policies.

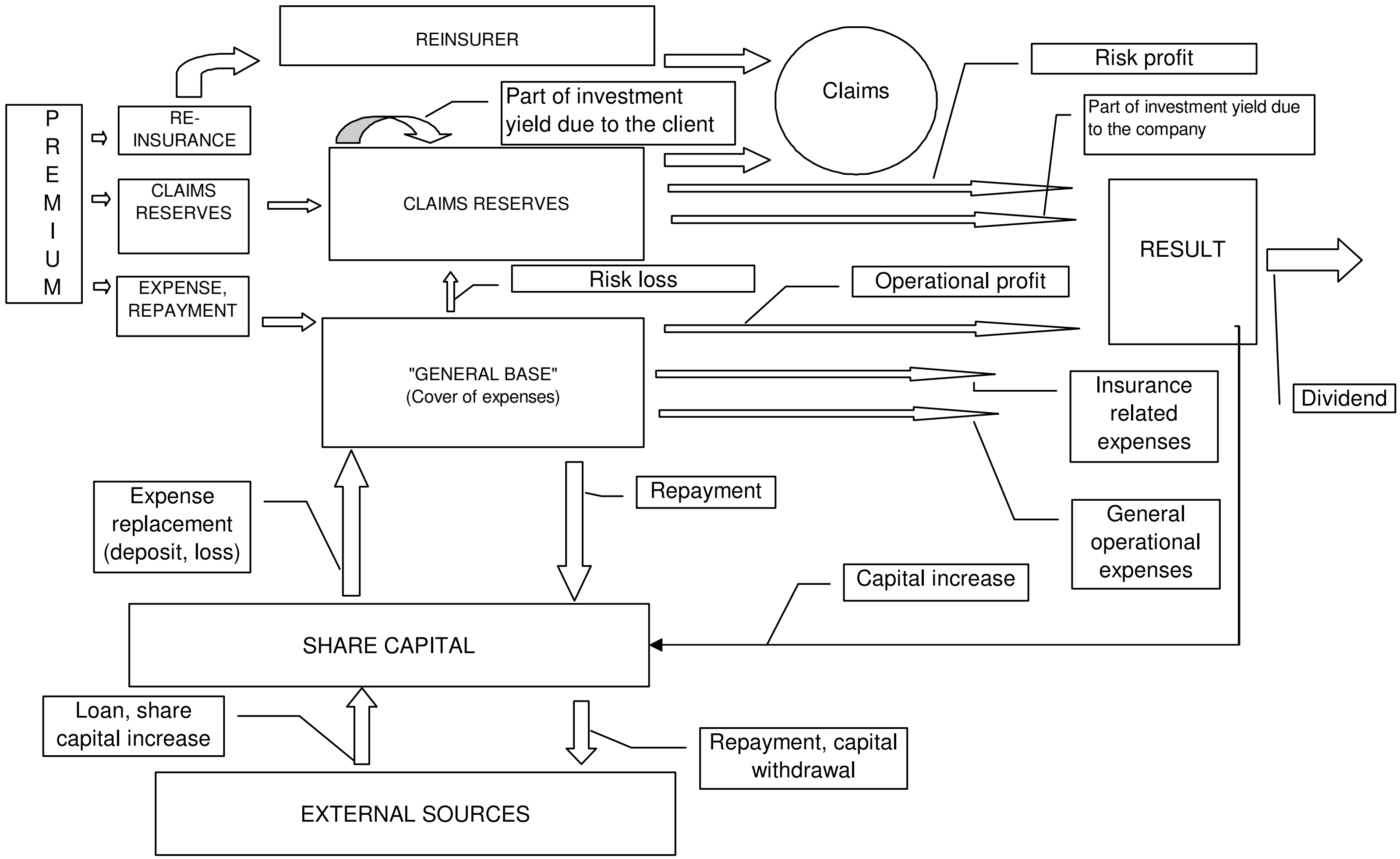

Figure 18.1.: The path of the premium within the insurance company

At first sight the premium income (at this point it is the **written** premium and not the actually collected **premium income**) is divided into two parts when arriving to the insurer:

- the net premium filling the reserve
- the premium loading covering expenses.

At second sight we have to make a correction, namely that the premium (in case of regular payment policies) is typically not divided between the net premium and the premium loading. Because of the gross premium reserving technique, i.e. zillmerization the division is changing in time. From the first premiums the sum covering the expenses is higher than the premium loading and much less or nothing fills the reserve, but in case of the later premiums one part of the premium loading also goes to the reserve.

This is problematic, because all premiums must be individually divided into reserve filling and expense covering parts. The parameters of the division are:

- the type of insurance[167]
- the nature of premium payment (single premium, regular premium or top-up payment) and the frequency of the payment (annual, semi-annual, quarterly, monthly)[168]
- the insurance term
- the entering age of the insured person
- the sex of the insured person
- the age increase applied by underwriting
- the sum of the premium[169]
- the number of the premium payment (this is important because the effect of zillmerization[170] is changing in time)

The current premium depends on whether there was a premium increase before or the client has accepted it or not. Fortunately this does not affect the internal ratios of the premium.

So in the end the premium income is used in two ways: it either goes to cover expenses or to build up reserves. The difference between the real expense and the sum covering the expenses is the expense profit or loss. The loss can be covered from the share capital. The reserves are used to cover claims and surrender benefit payments (and the transforming of policies, mainly and almost exclusively paying up the policy). The difference of the expected and real values of these are the sum of two factors: the mortality and surrender profit. The purpose of differentiating between the two is to monitor the difference between the "proper" termination of policies (which is the occurrence of the insured event) and the "non-proper" termination (surrender or transforming the policy).

In the above we have calculated based on written premiums because this is the logic of the traditional insurance and generally the computer system serves this logic as well. On the other hand the written premium can be paid earlier or later than the premium due date. The main reasons of the deviation are:

1. In case of Direct Debit, when the insurer withdraws the premium from the account of the client:
    a. The insurer withdraws the money not at the proper due-date but on a given date of every month, that can be in certain cases earlier or later than the due-date. If it is withdrawn earlier than the due-date, the money is put on a so-

[167] And – naturally – the structure of the concrete policy as well (main policy – insurance riders) but that is not discussed at this point.
[168] For the sake of simplicity we deal with regular, annual premium payment policies.
[169] It is important because of volume-depending reductions, but this is not discussed here.
[170] The situation is not that bad: zillmerisation not decreases but increases the net premium, so the ratio of the net premium and premium loading remains constant.

called parking account and entered into the books as premium income only on the due-date.
   b. There is no enough money on the account of the client so the insurer can only withdraw the money later than the due-day or never at all.
2. In case of paying by postal cheque or individual money transfer:
   a. The client forgets about paying the insurance premium or doesn't have enough money, the premium will be paid later or never at all.
   b. It happens rarely that the client pays the postal cheque earlier than the due-date.

In case of paying later the insurance company can ask for interest on overdue payments, but this is not practiced by Hungarian companies so there is no need to count with this correction factor (it would be part of the profit correction factor discussed later).

The insurance company gets a kind of latent interest profit after the premiums paid earlier but the premiums paid later mean latent interest loss. (Latent, because the interest of reserves and investment profit sharing is paid according to the written premium – and in the end this should be compensated by the interest on overdue payments). These latent profits and losses are not discussed later because:

- most of the premiums are paid by *direct debit*, which is quite trustworthy.
- in the case of *direct debit* the interests of premiums paid earlier and later equalize each other.
- the policies of considerable overdue payment usually become surrendered and the insurance company assert the interest loss at that time.
- Benefit payments could also be late, and the latent profit of overdue benefit payment compensates the latent loss due to overdue premium payment, and this is also not calculated.[171]

Altogether the results must be corrected at the end of the year by the following:

(premiums paid in, but not due in the given year) – (premiums not paid in yet but accounted).

I call the value calculated this way profit correction factor.

All **other income** must be examined properly according to the accounts. The most important of these is the commission chargeback (that was paid in effectively), the others are not significant factors.

If there are other expenses (not connected to the insurance activity), they have to be compared to other income, and the balance of the two has to be written to the expense profit.

18.3. The Path of Money Already at the Insurer at the Beginning of the Year

There are two different types of these:

1. The reserves
2. The share capital of the insurance company

The **reserves** are divided into two main groups:

1. mathematical reserves
2. other reserves such as:
   a. unearned premium reserve
   b. surrender reserve
   c. outstanding claims reserve (individual+ IBNR[172])

[171] Of course the correct thing would be to determine these latent factors as well, but it would be an enormous work to do so.
[172] Incurred but not reported, international abbreviation.

d. profit dependent premium refund reserve (not every insurance company has to use it)

Since they behave differently (although their behaviour is connected to each other), they are discussed separately.

The **mathematical reserve** changes due to the following :

1. premiums filling the reserve
2. benefit payments (death, maturity[173])
3. distributing of the premium reserves of the deceased among those still alive.
4. the share of the clients from investment yields (temporarily it can be part of the profit dependent premium refund reserve – but it is not necessary)

The insurance companies generally use computer systems that calculate automatically the changing of the above mentioned first three factors with the help of the reserve functions built in the system, and it is not possible to show these factors separately.

The value of the 4th factor can be clearly determined when crediting the profit share.

The components of the other income elements can change because of different reasons.

In case of annual premium payment the **unearned premium reserve** – as the premium immediately becomes part of the reserve – is not a meaningful category, so it is not discussed here. The difference between the decrease and the increase would be an income modifying factor (expense profit or mortality profit). The value of the unearned premium reserve is usually unambiguous and can be easily taken from the computer system.

The value of the **surrender reserve** is unambiguous as well. The difference between the value at the beginning and the end of the year can be accounted for in the expense profit (or maybe in the correction factor).

The mortality profit must be decreased by the difference between the value of the **outstanding claims reserve** at the beginning and the end of the year, because logically it belongs there.

The insurance company doesn't necessarily have **profit dependent premium refund reserve**. It is formed when the investment profit share is not credited continuously to the premium reserves. As we are supposing continuous crediting here, we don't use it. If it existed, then its value at the beginning of the year would be added to the premium reserves formally during the year.

The main factors effecting the **share capital**:

- using up the share capital, if the result is negative and not deferred, and a loan hasn't been taken out
- the yield of investing the share capital
- decision on increasing or decreasing the share capital (but this is of course not an income factor)

18.4. CALCULATING THE FACTORS OF PROFIT

18.4.1. Expense Profit, Income Correction

This profit factor is calculated as a remainder, so it is defined following way:

Expense profit = total profit – mortality, surrender and investment profit

[173] Of which a special case is the annuity.

On the other hand its name is justified because the main factor of this income is the expense saving (or exceed the expenses if the value is negative).

The profit correction is an important part of the income (according to a decision it can be accounted independently or can be considered part of the expense profit). This is formed because we calculate mortality- and investment profit to "theoretical" values, based on the written and not the actual premium income. With the help of the profit correction we adjust it to the actual premium income.

According to the above the expense profit is roughly: all actual expenses must be subtracted from that part of the premiums, written for the given year that is saved for covering expenses (this figure is corrected by the difference between the written and actual premium income). It is important to note that those expenses that were accounted somewhere else are not part of the total expenses incurred (primarily the fund management fee can be mentioned here).

The expense profit contains also the difference of other income and other expenses.

One part of the expenses is the interest paid for the capital loan. (The loan itself appears between the sources as a kind of external source, which equalizes the increase of assets. The repayment of the capital loan decreases the assets and sources on the same level, that is why we have to deal with the interest only regarding the profit.)

A separate part of the acquisition expenses should be isolated within the expense income. Namely: the sum paid for acquisition and that part of the zillmer premium that is saved for z must be matched. They should be corrected continuously with the decrease of deferred acquisition costs due to surrender, with the successful commission chargeback and with the unsuccessful commission chargeback accounted as loss.

18.4.2. Mortality (risk)[174] profit

The calculation of the mortality profit is discussed in detail in the following. For an accurate calculation we have to have the components with premium reserve in detailed figures.

The insurance portfolio is divided into the following groups by policies[175]:

1. From those components that were valid at the beginning of the year those that:
   a. are still valid at the end of the year or came to an end during the year when the insured event has occurred;
   b. came to an end with lapse or surrender during the year;
   c. were the policy was transformed or paid up.[176]
2. From the contracts signed during the given year:
   a. are still valid at the end of the year or came to an end during the year when the insured event has occurred.
   b. came to an end with surrender during the year.
   c. were the policy was transformed or paid up.[177]

Lets see how the mortality risk is calculated in each group:

In group 1.a. the calculation does not depend on the type of the insurance, it can be pure endowment, term or other kind as well and it also doesn't depend on whether the policy is single- or regular premium. Calculating the net premiums paid is not a simple task.

In groups 1.b. and 2.b. the mortality profit is not calculated (even if it could be), but accounted as part of the surrender profit.

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[174] The term "risk" profit is more adequate if we also have risks such as accident, that doesn't originate from mortality differences.

[175] If the client has the option of adding and leaving components (insurance riders) from the main policy during the year, then here we have to use the term "components" instead of policies.

[176] Paying up the policy is in reality a kind of policy transformation, but – due to the usual Hungarian company policy – this at the same time the only type of policy transformation.

[177] This will be an empty set with great probability, group because it is a common policy not allow the policy being paid up in the first year, and clients are usually don't want to transform single premium policies.

In the case of 2.a. the formula of the mortality profit becomes simpler, since there is no reserve at the beginning of the year (i.e. it is 0) and there is no bonus paid during the year (because this is due only at the end of the first year) except the extra bonus in case of death.

In group 1.c. the policy transformation is considered as a surrender and a new policy at the same time. The difference between the premium reserve released and the single premium of the new policy (initial premium reserve) is accounted as surrender profit. From that point on the transformed policy is regarded as a new policy.

In group 2.c the procedure is almost the same as in group 1.c. There is only one difference, that here we don't have to register two new policies, it is enough to register only the second one after the policy transformation. (Though it is almost certain that this group is will be empty.)

As we do not deal with components without premium reserve (e.g. accident riders), we only have to mention that the process is simpler there, because there is no premium reserve, no bonus and no interests.

The mortality profit for the whole portfolio is the sum of the above terms decreased with the balance of the outstanding claims reserve at the beginning and the end of the year.

18.4.3. Surrender Profit

In groups 1.b. and 2.b. of the partition of the previous section we simply take the premium reserves of the surrendered policies, (including earned but not paid bonuses) and subtract all surrender benefits.

The premium reserve released at the time of policy transformation must be added to this figure (i.e. we add the premium reserve before policy transformation – including the earned bonus – and subtract the premium reserve after the policy transformation).

Control possibility: calculating the mortality and surrender profit at the same time: if we do not differentiate between the individual components of the portfolio in the general formula of calculating the mortality profit then we get the mortality and surrender profit as one figure.

18.4.4. Investment Profit

There are two parts of the investment profit:

- the yield of the invested share capital (the share capital reduced by deferred loss).
- the yield of the premium reserve in the year (the bonus or technical interest paid to the client during the year – that can also be negative – must be subtracted from this figure).

From the above figures and from the results of long calculations we get a detailed picture of what happens to the money of the clients and the insurance company during the year.

18.5. CALCULATING THE MORTALITY PROFIT

Calculating the mortality profit is very important, so let's look at it in detail.

18.5.1. Mortality Profit of Insurances With Single Premium

The recursive premium reserve formulae suggests that there is a mortality profit if the premium reserve of the deceased person not only covers the death benefits and the premium reserve credited to those still living, but reserve is released even above these. There is mortality loss when things happen in the opposite way (e.g. when none of the insured of pure endowment insurance die in the given year.)

According to the above mentioned, if we suppose that:

- the insurance year and the calendar year coincide,
- there is no profit sharing,
- there is no surrender and no new policies are signed,

then the mortality profit can be calculated the following way:

(all premium reserves at the beginning of the year compounded to the end of the year) – (the year end premium reserve of those still living) – (the death benefits compounded).[178]

(In the case of pure endowment naturally the value of death benefits is 0.)

In the end the coincidence of the insured- and the calendar year is not important, because the result will be the same if we left this supposition.

It is relatively easy to correct the figures with the help of profit sharing: the value paid during the year is added to the formula (compounded) and the effect of profit sharing is included in the year end premium reserve (as it usually is).

The arriving premium of the new policy must be accounted as premium income (and compounded to the year end). All the new policies of the year must naturally be included in the year-end portfolio (if the insured is still living).

But it has to be corrected by the surrenders, because the result of surrender is not entirely mortality profit – it is mainly surrender profit. The correction is the following: the initial reserve of those who surrender their insurance during the year is subtracted from the premium reserve at the beginning of the year and w perform a separate calculation on this portfolio. Naturally they are not included in the year end portfolio, which practically means group of "living" policies from that time.

According to the above mentioned, the definition of the corrected mortality profit for the not surrendered portfolio is the following:

(all premium reserves at the beginning of the year of those not surrendered during the year compounded to the year end) + (the initial premium reserve of those new policies signed during the year compounded to the year end) + (the profit share of policies terminated but not due to surrender compounded to the year end) – (the premium reserve of those still living at the end of the year and the compounded death benefits)[179]

In the case of surrendered/lapsed policies the procedure is the following: the premium reserves at the beginning of the year compounded to the time of surrender can be divided into three parts:

1. the part given back to the client (in case of surrender)
2. the mortality profit/loss for the last year fragment
3. profit/loss of surrender.

We can also say that the profit of surrender is what remains from the premium reserve compounded to the time of surrender after subtracting the first two parts (and this way we have defined its method of calculation).

Calculating the mortality profit is very simple:

(the premium reserves of the surrendered policies at the beginning of the year compounded to the time of surrender) + (the profit share given at policy anniversary compounded to the time of surrender (if there was a policy anniversary until surrender)) + (the value of the profit share not yet distributed at the time of surrender) – (the estimated profit share at the beginning of the year compounded[180]**) – (premium reserve at the time of surrender (including the effect of the possible indexation)) – (all benefit payments in the given year compounded to the time of surrender)**

[178] Thus not the so called mortality services reduced with the premium reserves.

[179] If we want to make this formula valid for annuities as well, we have to be more general – the sum insured already paid or the claims already compounded to the year end.

[180] Since the estimated value of the profit share not yet distributed is considered to be the part of premium reserve at the time of policy anniversary, that would be counted twice after distribution. Moreover the distributed value is a precise value, while at policy anniversary we only have an estimate.

Naturally if the surrendered (lapsed) contract was signed during the year we have to calculate with the initial- and not the beginning-of-year premium reserve.

It is relatively hard to make a clear situation regarding the bonuses not yet distributed that should be paid after the surrendered policies in the given year. One part of these will be accounted as investment profit, another part as surrender profit. I advise the following (the above relation already reflects this):

- we should estimate the bonus due to the client at the time of surrender (even if we don't want to pay it out in this case),
- if there was no policy anniversary in the meantime, we have to subtract the estimated bonus credited at the beginning of the year (if there has been a policy anniversary we have already done this correction),
- this figure should mean the premium reserve at the time of surrender.

The mortality profit is the sum of mortality profits calculated for the surrendered and not surrendered contracts.

As a control we should examine whether the above mentioned formula is supported by the recursive formulae in case of term insurances or not.

The recursive reserve formula of the single premium term insurance is:

$$V_{t+1} = V_t \cdot (1+i) - \frac{d_{x+t}}{l_{x+t+1}} \cdot \left(1 - V_t \cdot (1+i)^{\frac{1}{2}}\right) \cdot (1+i)^{\frac{1}{2}} \tag{18.1}$$

From this:

$$\frac{d_{x+t}}{l_{x+t+1}} \cdot \left(1 - V_t \cdot (1+i)^{\frac{1}{2}}\right) \cdot (1+i)^{\frac{1}{2}} = V_t \cdot (1+i) - V_{t+1} \tag{18.2}$$

This can be interpreted easily. If there is no death, then the second (subtracted) term on the right side of equation (18.1) or the sum of this for all policies will be the mortality profit, that is:

$$\sum \frac{d_{x+t}}{l_{x+t+1}} \cdot \left(1 - V_t \cdot (1+i)^{\frac{1}{2}}\right) \cdot (1+i)^{\frac{1}{2}} ,$$

since at this time we have the premium reserve compounded to the end of the year for all policies, but these subtracted terms are not needed for the further benefits.

Common misunderstanding: A general "everyday" definition of the mortality profit is the sum of the following for all policies:

= sums insured × death rate compounded to the end of the year – death benefit payments compounded to the end of the year

This formula is a more or less correct – though not precise – estimation of term insurance, that we see from the following formula:

$$\frac{d_{x+t}}{l_{x+t+1}} \cdot \left(1 - V_t \cdot (1+i)^{\frac{1}{2}}\right) \cdot (1+i)^{\frac{1}{2}} = q_{x+t} \cdot \frac{l_{x+t}}{l_{x+t+1}} \cdot \left(1 - V_t \cdot (1+i)^{\frac{1}{2}}\right) \cdot (1+i)^{\frac{1}{2}}$$

The $\frac{l_{x+t}}{l_{x+t+1}} \cdot \left(1 - V_t \cdot (1+i)^{\frac{1}{2}}\right)$ is an estimation of it. This must be exactly one here.

$\frac{l_{x+t}}{l_{x+t+1}}$ is a number greater than one, $\left(1-V_t \cdot (1+i)^{\frac{1}{2}}\right)$ less than one, their product is somewhere around one, but not exactly.

18.5.2. Mortality Profit of Insurances With Regular Premium Payment

According to the relations concerning single premium insurances the reserve of insurances with regular premium payment (**if we do not use the zillmerization**) is different from the single premium insurances in only one aspect, the net premiums paid at the beginning of the year.[181] That is why in the definition of mortality profit this is the only factor that has to be modified. The mortality profit of insurances with regular premium consists of the following:

(All premium reserve at the beginning of the year of those who did not surrender or pay up[182] their policy during the year, compounded to the end of the year) + (The initial premium reserve (before premium payment) of those who took out new policies during the year, compounded to the end of the year) + (The profit share of those contracts that terminated but not because of surrender or paying up, compounded to the end of the year) + (The net premiums paid, compounded to the end of the year) – (The premium reserve of those still living at the end of the year) – (the compounded value of benefits paid[183])

It is a very important supposition in the above that premiums are paid in regularly, which means that the written premium and the actual premium payment is the same.
For the paid up and surrendered policies the following formula is used:

(The premium reserve at the beginning of the year of the paid up and surrendered policies compounded until surrender/pay-up) + (All net premiums paid, compounded to the same date) + (The profit share paid at policy anniversary compounded until surrender (if there was a policy anniversary until surrender)) + (The value of profit share not yet distributed at the time of surrender) – (The compounded profit share estimated at the beginning of the year) – (the premium reserve at the time of surrender (including the effect of the possible indexation)) – (Benefits paid in the given year, compounded until surrender)

We can regard paid up policies (by product type) as a separate group of single premium policies, that commence at the time of paying up, and for which we can calculate the mortality (or possibly surrender) profit according to the general rules. On the other hand the mortality profit relevant to the year fragment until paying up and surrender (both simply referred to as “surrender” from now on) increases the mortality profit of the contracts existing in the given year.

The **zillmerized** case differs from the above only in the aspect that the interpretation of “net premiums paid” must be modified to “that part of the zillmer premium which fills the reserve”, as it has been discussed earlier. In the classical case this is P_1, in the non-classical case this is the sequence of p_i-s in the first k years, and $A_{x+k:n-k} - PZ \cdot \ddot{a}_{x+k:n-k} + PZ > p_{k+1}$ in year (k+1), after that it is PZ.

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[181] In case of more frequent premium payment (not annual) the sum of the reserve can change during the year.

[182] I.e. we want to account the effect of paid up policies in the surrender profit! In this case – above this – we also have to suppose a new, different type, single premium insurance.

[183] If we want to use this formula for annuity as well, we have to be more general – all paid sum insured compounded to the end of the year.

18.5.3. Deviation from the Standard Mortality Table

When calculating premiums, many of the companies suppose that due to the effect of underwriting the expected mortality rate will be less than the population-mortality in the first years, and after that gradually coming closer the expected mortality of the insured portfolio (used in the calculations) will reach population mortality in some years. The premiums calculated with this method are naturally lower than the premiums calculated all the way with the population mortality table.

In this case the premium calculation is typically (but not in every case) done with the method of profit testing, therefore there is no individual net premium (if there would be, the net premium would be lower than the net premium calculated the usual way). The net premium is also used in premium reserve calculations in the case of the profit test method. However this net premium is calculated the usual way – i.e. it is calculated exclusively according to the population mortality table – and not with the mortality rates decreased at the beginning of the term.

This method of calculating the premium reserve supposes that a larger part of the premium must be set aside for benefits than what was taken into consideration when calculating the premium, which means that the practice doesn't use the same principles in every area. As the practice is not derived from strictly uniform principles, we have to choose between the two net premium (and also mortality benefit) calculation methods in case of calculating the mortality profit. The result won't be totally correct in either cases, so it is suggested to choose the simpler solution.

18.5.4. A Detour: What is the Benefit of the Term Fix Insurance?

When calculating the mortality profit (and generally: when accounting benefits) we have to face the problem: what exactly is the benefit of the (regular payment) term fix insurance? In the section below examine this question in detail.

The answer is not totally unambiguous, because the too obvious answers usually contain contradictions. Such obvious, but not satisfying answers are:

- The benefit is specified in policy conditions, that is the maturity sum paid at the time of maturity. The basic problem with this solution is that it supposes that it doesn't matter whether the insured dies during the insurance term or not, while we know that in this case the premium reserve is filled up. From the point of the risk community everything is right, since the premium paid by the clients still living is enough on the level of the risk community to cover all benefit payments at the end of the insurance term. However on the individual level the accounting does not follow the real happenings.
- In case of death it would be logical to account the filling up of the reserve as a benefit. The problem with this is that in this case (one part of) the benefit paid at the end of the insurance term would be accounted twice.
- If the filling up of the reserve is considered to be a benefit, but the maturity sum paid after the policies of deceased is not (to avoid duplicated accounting), then we find ourselves in an obvious contradiction with the actual cash flow of the insured, as these benefits are not used for the filling up of reserves.

The solution to the problem is to **define** the **benefit of the policy** in a **different**, but equivalent way to make the complexity of the background more visible.

The traditional term fix insurance is nothing else but a special endowment insurance. Special, because its sum paid in case of death is the value of the maturity sum insured discounted to the time of death[184]. According to this:

[184] The proof is the following: the annual net premium of the term fix insurance:

- If the insured person lives at the time of maturity then the insurer pays the maturity sum.
- If the insured person dies during the insurance term then the pure endowment insurance ends immediately, but the insurer pays the discounted value of the pure endowment part to the beneficiaries. However the insurance contract implicitly contains that the beneficiary does not receive this money but purchases a special policy without any death risk (in fact a single premium term fix insurance), so he pays that money to the insurer as a premium and receives the benefit of the new policy at the time of the original maturity. By that time this sum is reaches exactly the original maturity sum due to the interests.

So what should be accounted as benefit?
If the insured person lives until maturity: only the payment of the maturity sum.
If the insured person dies during the insurance term, then the benefit can be divided into two parts:

- the discounted value of the maturity at the time of the death of the insured (the policy is terminated at this time but a new one is formed implicitly). But at the same time it must be accounted as a premium income of the single premium term fix. In fact this implicit premium income will be the base of the further benefits, and this helps to avoid the duplicate accounting of the same benefit, but at the same time we follow the insurer's outgoing cash flow.
- at the end of the term the single premium term fix insurance is terminated and its benefit accounted.

The question might arise why is this process of implicit cash flows is necessary in the case of death. First of all because the insurer has to react to the important happenings such as the significantly different reserve after death and the lack of further premium payment. On the other hand the beneficiary might think that the insurance company does not provide any benefit at that time, but later or continuously, that is why this payment has to be corrected by considering it to be premium income (which is the opposite of benefit payment).

$$P_{x.\overline{n}|} = \frac{v^n}{\ddot{a}_{x:\overline{n}|}}$$

If we consider this to be a term insurance with varying sum insured and a pure endowment insurance, then the premium of the two parts will be the following:
Equivalence equation for the term part:

$$l_x \cdot \ddot{a}_{x:\overline{n}|} \cdot P^k = d_x \cdot v \cdot v^{n-1} + d_{x+1} \cdot v^2 \cdot v^{n-2} + \ldots + d_{x+n-1} \cdot v^n \cdot v^{n-n} =$$
$$= v^n \cdot \left(d_x + d_{x+1} + \ldots + d_{x+n-1}\right) = v^n \cdot \left(l_x - l_{x+n}\right)$$

The same for the pure endowment part:

$$l_x \cdot \ddot{a}_{x:\overline{n}|} \cdot P^e = v^n \cdot l_{x+n}$$

The premium from the sum of the two parts:

$$P^k + P^e = \frac{v^n \cdot \left(l_x - l_{x+n}\right) + v^n \cdot l_{x+n}}{l_x \cdot \ddot{a}_{x:\overline{n}|}} = \frac{v^n}{\ddot{a}_{x:\overline{n}|}}$$

We can see that this is really the same as the original term fix premium.

The term fix insurance can placed in the above mentioned relation regarding mortality profit the following way:

- If the insured person lives there is no change.
- If the insured person dies, then the maturity benefit appears like a new insurance policy at the time of the death, which means that the compounded single premium must be considered "the value of the initial premium reserve (before premium payment) of those who entered the insurance during the year compounded to the end of the year", which is a plus figure. Since its source is a death benefit of exactly the same value, it must be subtracted as "the compounded value of benefit payment" – so altogether we can disregard the separate consideration of filling up of reserves at this time.
- In this case at the end of the year the "year-end premium reserve of the still living" will be the filled up reserve of course.
- The annuity benefit and the payment of the maturity sum are considered normal benefits.

Regarding the mortality profit of the term fix insurance, its benefit is the same as for the everyday people. In case of death the increase of the reserve presents itself only in the change of the "year-end premium reserves of the still living", so it is the simple process that is justified here as well.

18.6. Embedded Value[185]

The calculation of the *embedded value* (EV) does not belong strictly to the technical income statement, but its function is in one sense the same: to get a clearer view of the financial status of the life insurance company. The calculation of the embedded value is more and more common in Hungary and everywhere in the world as well. The EV itself is a method of policy valuation. It tells us the value of an already existing life insurance portfolio. It is useful to be familiar with its value all the time (e.g. manager remuneration can be connected to the increase of the EV), but it is extremely important when the insurance company or one part of the portfolio is for sale.

The generalisation of EV is the *appraisal value,* that can be defined as the sum of the EV and the goodwill, where the goodwill is the future development potential (the value of the portfolio that can be obtained in the future, that the insurance company, the knowledge of the management etc. "has inside itself").

There are many subjective values in the EV, that is why it is the change of the EV and not its actual value that counts.

In case of life insurances the EV gives a more realistic picture of the value of the insurance company than traditional accounting. (The traditional accounting might show a bad company as profitable compared to a good company if surrender rates are high in the bad company, although it is obvious that this is not a good sign for the future.)

The EV can be divided into the following parts:

$$\mathbf{EV = VIF + NAV}$$

where:
VIF = value of in force
NAV = net asset value

The VIF can be calculated with the method of profit testing. The profit test must be performed on a portfolio starting from a given moment. The portfolio is usually not the whole portfolio of the insurance company, but a kind of simpler representation of it (e.g. the individual contracts are grouped according to age or gender).

[185] Based on a lecture held by Tibor Edvi in 2002!

The following formula represents the change of the EV:

$$\Delta EV = VIF*RDR + NAV*RFR + NBC$$

Where:
RDR = risk discount rate
RFR = risk free rate
NBC = *new Business contribution* = NBAV = new business added value

It can be pointed out from this formula that the EV (the value of the company) can be increased mainly with new policies.

The ΔEV itself is not a closed formula, because we supposed that nothing changes. But in reality there can be several differences:

difference between facts and plans (variation) (+/-)

change of suppositions (that must be introduced in the documents attached to the calculation of the EV) (+/-)

The success of the management is usually measured with the changes of the EV purified from all other changes. Such other changes are e.g. tax, mortality, the change of the benchmark yield, so everything that is an environmental factor and does not indicate the performance of the management.

19. THE ACTORS AND RIVALS OF THE LIFE INSURANCE MARKET AND ITS SIGNIFICANCE IN THE NATIONAL ECONOMY

KEY WORDS

Bank insurance
Defined contribution system
HFSA – Hungarian Financial Supervisory Authority
Defined benefit system
Social security
Funded system

19.1. THE ACTORS OF THE LIFE INSURANCE MARKET

The actors of the life insurance market (excluding the clients) can be divided into the following groups:

- suppliers
- rivals
- mediators
- associations
- regulator

The **suppliers** are the insurance companies themselves. The most important insurance companies of the Hungarian market are composite companies (according to their operational permit, though in reality not in every case). At present days the specialised life insurance companies cover only a small part of the market.

The most important **rivals** of life insurance companies are **pension funds**, **banks** and **investment funds**. However it is mainly co-operation and specialisation and not the rivalry that is characteristic of these institutions. It is very common that insurance companies direct financial groups or are members of such a group, which includes the above mentioned different rival companies as well. The bank insurance is a good example of co-operation between insurance companies and banks and pension funds will be discussed in a more detailed way in chapter 19.2. Of course in a wider sense these companies remain rivals of the life insurance companies in spite of the co-operation, and the same way social security is a kind of rival for life insurance as well. If we restrict the group of life insurance companies to the group of joint stock companies dealing with life insurances, then we can say that the life insurance mutuals are also rivals of these companies.

We have already dealt with two types of **mediators,** the independent **brokers** and the **agents** working for one (or sometimes several) insurance companies. Independent financial advisors could be regarded as a type of mediators as well but they haven't appeared on the Hungarian market yet.[186]

Associations are important members of the insurance market, of which two play an important role at present: the Association of Hungarian Insurance Companies (MABISZ) and the Association of Brokers. They play an active part in of giving opinion on Bills regarding the insurance profession, in lobbying for more favourable regulations, operate ethical codex and a committees, examine and spread international experiences, etc.

The **regulator** of the insurance market is naturally the Hungarian state, which performs this task on the one hand by producing regulations, and on the other hand by operating institutions that monitor the compliance of these regulations. The rules are prepared (laws,

[186] On the Hungarian market the special insurance advisors are a similar existing institution, but they usually give advice to insurance companies or deal with claims handling.

government regulations) and published (ministry regulations) primarily by the Ministry of Finance. The compliance of regulations and laws are monitored by the Hungarian Financial Supervisory Authority (HFSA) in case of all financial institutions, among them the insurance companies.

19.2. The Connection Between Life Insurance and the Social Security System and Benefits

Being so significant, and being the functional supplement of the social security pension scheme (and also the health insurance scheme, to a lower degree) and – in a certain sense – its rival as well, the social security pension system – including the institution system of pension funds – is discussed in a separate chapter.

The Hungarian social security system has two subsystems: the pension- and the health insurance subsystem. Life insurance has connections with both subsystems but the importance of these connection is not the same. The connection between life insurance and the social security health insurance subsystem mostly comes from (accident and sickness) insurances taken out as riders to life insurance policies – that would be non-life, but are accounted in the life branch. As these policies are important, but all the same only supplemental elements of the life insurance policy, we'll not discuss the relation of life insurance and social security health insurance in more detail.

The connection with the pension scheme is closer and the two spheres can be compared more easily. The aspects of comparison are:

- goal or function,
- operating system,
- relation between payments in and out,
- the reason for joining,
- profit.

The **goal** of the social security pension scheme and life insurance is almost the same: to provide for the financial safety of old age. However this goal can be achieved in different ways.

There already are certain differences in the main goals. The social security pension scheme tries to be comprehensive and to provide some kind of benefit for every member of society (this goal can never be reached entirely). Life insurance, at least in case of a single company never has this kind of goal. The maximum is to provide this benefit for those members of society who represent a sound demand.

Apart from the common goal and function, life insurance has further functions as well. As we have already discussed them when discussing the different life insurance types, here is a brief list of the functions missing from the social security system (or performed only on a basic level):

- expenses of funeral and covering inheritance tax (in the case of whole life),
- savings for the benefit of a child (e.g. in the case of term fix),
- providing for relatives left behind (term insurance, widow and orphan annuity).

Life insurance operates only as a funded system. Until 1997 the social security pension scheme operated only in the so called pay-as-you-go system, and from 1998 it operates in both systems (if we consider private pension funds to be part of the social security pension scheme, though as institutions there is no connection between them). Let's see what the differences are between the two systems.

In the **funded system** all payments made by the insured in his active period are accumulated on a separate savings account and at retirement the client receives from the accumulated payments an annuity with a capital value equal to the accumulated sum. In other words we can say that everyone receives his own saved money as a pension, and if the risk community would split up, every member would get his own money back (increased with interests).

In the case of the **pay-as-you-go system** the risk community does not accumulate anything, the total of all contribution payments is enough only for covering the pension benefits of those who are retired. If the risk community would split up, no-one would get their contribution payments back, and the persons retired wouldn't receive any pension benefits in the future, as there isn't any kind of reserve accumulated in the system. As the Hungarian social security system operates mainly in a pay-as-you-go system, those who are presently contributors must hope that the system will still operate at the time of their retirement. If it wouldn't operate at that time, they couldn't expect to receive any benefits from anyone.

This means that in contrast to the funded system, the pay-as-you-go system operates basically without reserves[187].

It is common to introduce another pair of categories used for describing pension systems: the concept of **Defined Contribution** and **Defined Benefit** systems. In the first case the contributions paid during the active years and their interests are strictly taken into account, and at retirement the valid pension annuity is calculated based on these. In the second case there are some regulations used for defining the valid benefits at the time of retirement, and contribution payments are adjusted to this. In the case of defined contribution there is a strict individual equivalence, while in the case of defined benefit the contribution payments and the received benefits equal only on the level of the whole risk community. Life insurances are defined by contribution, but the present pay-as-you-go system of the Hungarian social security pension scheme is defined by benefit. The individual pension funds that are part of the social security system also operate in a defined contribution system.

The main goal of the 1997 social security pension system reform is to transform – on the long term, in several decades – one part (about 25%) of the pay-as-you-go system into a funded one. This transformation is gradual and long-term[188]. The organizational units of the funded system are the private pension funds that are similar to the voluntary funds.

If we compare the two systems (the pay-as-you-go and the funded) we can discover further differences as well.

Life insurance can principally not operate in a system other than funded. This is closely connected to its voluntary nature. It would not be worth joining a voluntary life insurance scheme as a contribution payer in a pay-as-you-go system[189], therefore this system can operate only on a compulsory basis, regulated by the state.

There are huge reserves in the funded system that must be invested on the financial market. The investment of this enormous sum of money stimulates the stock exchange continuously and causes capital abundance. In the funded system the financial knowledge of people dealing with these investments becomes more and more important, just as the role of the investing institutions and the financial markets in general.

In the pay-as-you-go system significant reserves are not created, that is why this system does not affect the stock exchange and doesn't stimulate the economy by investing the reserves. Due to the absence of reserves the role of the investment and financial knowledge and the role of the institutions is not so important. However the bureaucracy and the corporations re-distributing the income have a very important role.

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[187] Of course this is only one opinion. The other one is that the pay-as-you-go system has implicit reserves, namely the guarantee of the state. From the point of view of the state this sum is national debt, and since it is not accounted, the debt is implicit as well. That is why if the risk community would split up, the state would repay the implicit national debt – from other income – which means that it pays the pensions "earned" by contribution payment. Everything depends on what the state does.

[188] In other words it can be said that this transformation makes the implicit reserves into explicit ones.

[189] Except if there is an unambiguous state guarantee backing up the system that is also connected to precise accounting. The first pay-as-you-go system of this kind appeared at the end of the 1990s in Sweden. The name of the system is Notional Defined Contribution. Theoretically it can be worth joining the NDC system on a voluntary basis.

The question can be asked: Why was the Hungarian social security pension system operated until the recent past exclusively on a pay-as-you-go basis?

The reason is quite simple and explains why the social security pension system was initially pay-as-you-go type in most countries. The reason is that the pay-as-you-go system can be introduced from one day to the other in the sense that it can immediately provide benefits to those who need it. Of course at the time of introducing this system none of its beneficiaries have any former savings in exchange of the received benefits, although they have "earned the right" for a pension. Actually it is this saved amount – that should back up the payment of the first benefits – that will be missing from the pay-as-you-go system until the end[190], and this is the reason why after several decades of operation it cannot be guaranteed to people who have paid contributions all their lives that they will get their rightful pension if the system should fall down.

If the funded system had been introduced, then the first generation that can receive a complete pension from the system would be the generation which entered the labour market when the system was introduced, as they are the ones who have saved money for a long enough time. So the funded system does not solve the problem of those who are currently in retirement age. That is why at that time the decision was made to introduce the pay-as-you-go system.

The pay-as-you-go system got into a serious depression by the end of the 1990s in Hungary (but not only here). The reason for this is that the system is very sensitive to demographic changes. This system works quite well until there are more people entering the labour market than retired age, as it is the payment of the working that covers the pension of the retired. It still works properly when the average age of the population does not increase. The increase of the average age nowadays increases primarily the time spent in retirement, when the person receives benefits from, and not pays contributions into the social security funds.

Altogether it can be said that the pay-as-you-go system works properly when the average age stagnates or decreases, but when the population is ageing, in the system has to face more and more problems. This is currently the case in Hungary, the population gets older and older and the birth rate is falling.

We have already discussed the **principle of equivalence**, which means – in the case of life insurances – that premium payment is proportional to the risk, and benefits paid are proportional to premium payment. As the outgoing payments depend on chance, we cannot say that the outgoing payments are equal to the incoming payments, but – apart from the interests – we can say that this is true for the expected values.[191]

There is no equivalence in the social security pension scheme, only a so-called quasi-equivalence. The reason of this is the **principle of solidarity,** namely that some people in need receive higher benefits than their contribution payments, which can be done because those who are not in need receive less than their own payment. Because of the competition and the voluntary basis the principle of solidarity is necessarily missing from the operation of life insurance.

The principle of solidarity can be seen at many points of the social security system:

- There is a so-called minimal pension that the pensioner receives whether this sum is "earned" by former contribution payments or not. This is a redistribution of income in favour of those people who had low wages or short time spent in work, or both.

[190] In many countries operating a pay-as-you-go system this amount saved had existed earlier, but has been destroyed in World War II. The pay-as-you-go system was introduced in most countries as the replacement of the former funded system, but the systems haven't been made funded later on. This does not change the validity of the above train of thought.

[191] This use of the principle of equivalence is sometimes called fair calculation, fairness or actuarial fairness. It means that there is no systematic redistribution of income between insured persons. The logic of competition definitely leads to this direction.

- Contribution payments are counted in the pension in a degressive way, which means that the more someone pays in the less he receives proportionally.[192]
- The principle of solidarity works also in favour of women. In spite of the fact that the average life expectancy of women is much higher, so they will spend more time in retirement and consequently will receive more pension than the men, this does not appear in receiving a lower level of pension for the same salary. However in pension schemes operating as funded systems and in case of annuities this is natural[193].

We have already mentioned and pointed out the differences between the two systems, namely that joining the risk community of the life insurance companies is **voluntary,** but the social security pension scheme is **compulsory** by law. It is because of this compulsory feature that it can operate on a pay-as-you-go basis and that the principle of solidarity is taken into consideration.

Currently this difference is one of the main reasons that the social security pension scheme disposes over sums several order higher than all life insurance companies together.

The main business goal of companies selling life insurance products is **making profit**. (As we have mentioned earlier we are not talking about non-profit insurance mutuals, insurance co-operatives, only about joint stock insurance companies.) Making profit is the main motivation of these companies. This can be closely connected to the fact that all of these companies are in private property.

The social security pension scheme is a typical **non-profit** scheme, the motivation of its operation is that it is the state's duty to provide pension for everyone and to satisfy the needs of the insured. The social security system is not in private property, practically it is owned by the insured themselves.

The voluntary pension funds were formed in 1994 as a supplement of the social security scheme. The private pension funds, that were formed following the model of the voluntary ones can be considered to be part of the social security pension system.

Without discussing the differences between pension funds and life insurance companies in detail, let's see some of their similarities and differences. (We use the term pension fund when we are referring to both of funds, but mention the different names when we are speak of the their differences.)

One of the similarities is that both funds and the life insurance companies operate in a funded and defined contribution system.

One similar feature between the voluntary pension funds and life insurance companies is the voluntary feature, which makes them different from the social security pension scheme (and from the private funds, though that was also voluntary for a temporary period). The same can be said about the principle of solidarity, which is missing from not only the voluntary funds but from the private funds as well (apart from the annuities paid by private pension funds, which will probably be unisex, i.e. operating partly on the basis of solidarity).

The owner structure of pension funds and the life insurance companies is different. The pension fund is owned by its members and works on a self-governing basis.

The benefit of the pension funds is mainly the collection and investment of contributions paid by members, and not the risk elements that are characteristic of life insurance

[192] This practice will be ended by law in 2013 and until then its scale will be smaller.

[193] This remark means that the funded system of the social security operating from 1998 is a funded in a limited way, because it contains solidarity elements as well. One of these is that it is forbidden to give different level of pension to the men and women for the same amount of capital though according to the principle of equivalence it would be justified, but this principle was "overwritten" by the principle of solidarity.

companies, e.g. the sum paid out in case of death is significantly higher than the sums paid in.

The pension funds are the rivals of life insurance companies. But apart from the competition there a co-operation between the two spheres can also be imagined, which became “part of the system” in Hungary:

1. Most insurance companies operate so-called “fund service companies” that provide services to the pension funds. These services can be various from the founding of pension funds to administration and fund management. These fund service companies are profit oriented, they do their job in return of fee received from the pension funds. In fact the biggest funds were founded by insurance companies (and by banks in some cases) – though principally it was not allowed by law.
2. The insurance companies have larger risk communities than the voluntary pension funds, that is why for the voluntary funds it is worth to buy some insurance products from insurance companies instead of organizing it for themselves. It is worth for both pension funds to buy the annuities from the insurance company instead of providing it themselves. The annuity is expected to be significant in case of the private pension funds from 2013, and according to the present plans it will mainly be provided by insurance companies.

19.3. The Effect of Life Insurance on the National Economy

Life insurance has direct and indirect positive effects on the national economy. Lets list these effects.

One of the **indirect effects** is that life insurance creates the **feeling of safety** and in some very important cases safety itself. When we emphasize the role of the feeling of safety, we are not think of the insurance providing less than people expect, but of the fact that the insurance itself makes a lot of people more relaxed and happier (which is not a negligible aspect for politics).

The economical effect of safety provided by life insurance – as the economical effect of safety provided by all insurance types – is the fact that in case of an unexpected negative event (death or becoming unable to work) the disadvantageous economical effects will **not spread,** which means that the negative effects will be located to the small circle of the people involved. And there the disadvantageous economical effects will be prevented by the benefit payment of the insurance company. For example if an enterpreneur who has a loan on the enterprise dies, it is certain that this person can be replaced in the enterprise. It can happen that the successors of the enterprise cannot pay the instalments and the enterprise will be bankrupt, that can have an effect on other enterprises as well, so the negative effect can spread. This can be stopped by a life insurance that can cover the instalments from the sum insured, which means that the enterprise won’t be bankrupt if the enterpreneur dies but the successors can terminate it the normal way. (We haven’t mentioned other, more distant positive economical effects of life insurances here.)

The **direct effects** are the following:

1. The companies selling savings type life insurances can be considered from the point of national economy to be companies that are collecting the small capitals of the economy and transferring it into a **big investable capital**. Without this accumulation these small capitals might not get into the production, or with a much lower efficiency than this way, grouped into a large capital together with the other small capitals. That is why life insurance companies are among the largest investment groups in the Western world and their role in stimulating the economy and producing new workplaces is essential.

2. Those business insurance companies that sell savings type life insurance take over certain benefits from the social security system. This is very important in those countries where the social security system has difficulties (as in most European countries, and in Hungary also).

3. The savings type life insurance is nothing but postponed consumption. In the first period until premium payments are more than benefit payments (as is the current situation in Hungary) it has an anti-inflationary effect, since it drives purchasing power away from current consumption, this way decreasing the pressure of inflation.

As a summary we can say that:

1. Life insurance creates workplaces not only in the life insurance business but in businesses connected to the insurance such as banks, financial institutions, health service.
2. The premium reserve collected from the premiums paid by policyholders serves as a basis that is suitable for investing in the national- or private sphere. This aggregation has a positive effect on the state of the national economy.
3. By means of annuity insurance and creating pension funds it provides pension annuity benefits to those who would not get this kind of safety anywhere else.
4. Insurance companies (through paying taxes) provide a significant income to the government. The domestic benefits provided to the insured avoid this money getting abroad.
5. Life insurance companies pay attention to making health services stronger and to encourage the population to better health care. So these kind of social expenses could be decreased.
6. Life insurance has an important role in creating the financial stability of the population. It can be reached by giving the members of society the possibility of minimizing their unexpected financial losses.
7. The society can save a significant sum by organising given social functions in the scope of life insurance.

APPENDIX

MORTALITY RATES AND THEIR ESTIMATION[194]

KEY WORDS

Mortality rate
Mortality intensity
Constant mortality intensity
Estimation of mortality rate
Raw mortality rate
Equalisation

PROBABILITY THEORY APPROACH

The fundamental feature of human life is that nobody knows when it will end, the duration of life can be characterised by a random variable. Let's take an individual of age x years, and denote by T_x the length of the period of time from now until death. ($T_x \in \Re^+$). We denote the distribution function of the T_x probability variable by $F_x(t)$. $F_x(t)$ means the probability that the value of T_x is below t:

$$F_x(t) = P(T_x < t)$$

This value is called the probability of death, or mortality rate, and is denoted by q. The lower right index shows the age of the individual in question, and the lower left index means the period of time that we are looking at.

$${}_t q_x = F_x(t) = P(T_x < t)$$

In insurance mathematics the annual mortality rates have a special role:[195]

$${}_1 q_x = q_x \,,$$

where x is a whole number.

The complement of the probability of death is the probability of being alive, or the survival rate. The survival rate shows the chances of an individual of age x years living through a t interval. We use the letter p to denote the survival rate, and the indexes mean the same thing as in case of the mortality rate. It is obvious that an individual of age x years either survives the t interval or dies during this period, so the two events are exclusive, and the occurrence of one of them is certain:

$${}_t p_x + {}_t q_x = 1$$

Another very important (although quite trivial) relation is:

$${}_{t+s} p_x = {}_s\, p_x \cdot {}_t p_{x+s} \tag{F.1}$$

The meaning of formula (F.1) is very simple: the probability that an individual of age *x* years survives the interval *s+t* equals the probability that he survives the interval *s* and after that also the interval *t* [196].

[194] The following chapter is a good complementary to certain parts of the book, but is not an essential part of it. Although it repeats the material covered by the book in several places, but the repeated parts appear in the book in a totally different context, this way we leave them here so that the message of the appendix remains coherent. The appendix is the work of Ágoston Kolos.
[195] A general agreement is that if the mortality rate refers to one year, we leave the lower left index.

It is important primarily when estimating mortality rates to determine probabilities of death also for a period shorter that one year. The question arises in a way, that if we know the probability of someone dying within one year, then what is the probability of this person dying in the first half of the year or the second half. We would like to point out here a very wide-spread, but incorrect application. If we assume that the probability of death within a year is 0.01, we often hear the statement that the probability of death is 0.005 in the first half of the year as well as in the second half. This is mathematically incorrect. If it would be this way, then the probability of the person surviving the first half of the year would be 0.995, and the probability of a person living at the end o the first half of the year surviving the second half would also be 0.995. This way the probability of death within the year would be $(1\text{-}0{,}995)^2$, which is not equal to 0.01. The difference is small, due to the low death rate, but if we take a higher probability, the difference can be significant.

Defining the probability of death for a period shorter than a year means that we define ${}_tq_x$ as a function of t and q_x. One of the most simple such rules is the "linear mortality". In this case:

$$ {}_t q_x = t \cdot q_x , \tag{F.2} $$

where $0 \le t \le 1$.

Continuing the previous example, if the annual mortality rate is 0.01, then the probability of death in the first half of the year is 0,5·0,001=0,005. The probability of death in the second half of the year can be determined with the aid of formula (F.1) and (F.2):

$$ (1 - q_x) = (1 -_{0,5} q_x)(1 -_{0,5} q_{x+0,5}) , $$

and from this we get:

$$ {}_{0,5} q_{x+0,5} = 1 - \frac{1 - q_x}{1 -_{0,5} q_x} = 1 - \frac{0{,}99}{0{,}995} = 0{,}005025 $$

to six decimals. It can be seen that the mortality rates of the first and the second half of the year are not equal.

Another way of deriving mortality rates of a fraction year is if we suppose that the mortality rate is equal in every interval of length t. Due to reasons discussed later on, this case is called constant mortality intensity [197]. In this case the mortality rate is:

$$ {}_t q_{x+s} = 1 - (1 - q_x)^t . $$

If the probability of death in one year is 0.01, then the probability of death in half a year is: $1 - \sqrt{1 - 0{,}01} =,005013$.

MORTALITY INTENSITY

The mortality rate is a good measure in the hand of actuaries, but it has a big problem: it can only refer to a period of time (an interval). If mortality has to be characterised in at a certain moment of life (i.e. in a point), then we follow a usual mathematical procedure:

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[196] Although we are multiplying together probabilities again, the background of this phenomenon is not that these events are independent (we suggest that the reader tries to determine which two events are independent). In this case multiplying the probabilities is derived from the conditional probabilities.
[197] The mortality intensity will be discussed in the following sub-chapter.

$$\lim_{h \to 0} \frac{{}_h q_{x+t}}{h}$$

We refer to this value as the mortality intensity, and denote it with μ_{x+t}. If the distribution function is differentiable, then we can obtain the mortality intensity by taking its derivative:

$$\mu_{x+t} = \lim_{h \to 0} \frac{{}_h q_{x+t}}{h} = \lim_{h \to 0} \left(\frac{F_x(t+h) - F_x(t)}{1 - F_x(t)} \cdot \frac{1}{h} \right) =$$

$$= \lim_{h \to 0} \frac{F_x(t+h) - F_x(t)}{h} \cdot \frac{1}{1 - F_x(t)} =$$

$$= F_x'(t) \frac{1}{1 - F_x(t)} = \frac{d_t q_x}{dt} \frac{1}{{}_t p_x} = -\frac{d \ln(1 - {}_t q_x)}{dt} \qquad \text{(F.3)}$$

The mortality intensity in case of linear mortality is:

$$\mu_{x+t} = \frac{d_t q_x}{dt} \cdot \frac{1}{1 - {}_t q_x} = \frac{d(t \cdot q_x)}{dt} \cdot \frac{1}{1 - t \cdot q_x} = \frac{q_x}{1 - t \cdot q_x} \qquad \text{(F.4)}$$

The expression (F.4) shows clearly that in case of linear mortality the mortality intensity increases during the year.

The mortality intensity, when constant mortality intensity is supposed:

$$\mu_{x+t} = \frac{d_t q_x}{dt} \cdot \frac{1}{1 - {}_t q_x} = \frac{d(1 - (1 - q_x)^t)}{dt} \cdot \frac{1}{(1 - q_x)^t} = -\ln(1 - q_x)$$

We see that the mortality intensity doesn't change with time, this is where the name came from.

Using the mortality intensity we can determine the mortality rate. Let's start from writing the following relation using formula (F.3):

$$\int_0^t \mu_{x+\tau} d\tau = \int_0^t - \frac{d \ln({}_\tau p_x)}{d\tau} d\tau = -\left[\ln({}_\tau p_x)\right]_0^t = -\ln({}_t p_x) \qquad \text{(F.5)}$$

Formula (F.5) in a rearranged form gives us the relation concerning the mortality rate:

$${}_t q_x = 1 - \exp\left(-\int_0^t \mu_{x+\tau} d\tau \right) \qquad \text{(F.6)}$$

If e.g. the mortality intensity is $\mu_{x+t} = t\mu$, where μ is a fixed constant value, then the mortality rate is:

$$ {}_tq_x = 1-\exp\left(-\int_0^t \tau\mu d\tau\right) = 1-\exp\left(-\frac{t^2}{2}\mu\right) $$

If expression (F.6) is differentiable, then using the μ_{x+t} mortality intensity we can define the density function of the random variable:

$$ f_x(t) = \exp\left(-\int_0^t \mu_{x+\tau} d\tau\right)\mu_{x+t} $$

ESTIMATION OF MORTALITY RATES

Life insurance calculations are based on the so-called annual mortality rates (q_x), where x is a whole number. E.g.: What is the probability of a 20 years old person dying in a year from now on?

Annual mortality rates are estimated from experience data. If the insurer's portfolio is large enough, then it can be estimated from the insurer's own portfolio, and if the available data is not sufficient, then a so-called national mortality table can be used. These tables also come from by estimations.

The estimation of the probability of death can be regarded as quotient estimate, that can be estimated by estimate functions of statistical methods. In practical cases this estimate function cannot be used, because we have a so-called censored observation. If an insured surrenders the policy in the middle of the year, then from that point on we do not know anything about the policy, we do not know if the insured lived until the end of the year or died after surrendering the policy. These cases are called censored observations. The goal is to derive an estimate function that can handle these cases as well.

A number of estimate functions exist for censored observations, we will not try to introduce all methods in detail. We will give a detailed discuss of one particular estimate function, and derive it for a special case.

Let's assume that at a certain age the mortality rate is constant in a given year. If the mortality rate characterising this age and year pair is q_x, then its mortality intensity is μ=-ln(1-q_x). Let there be n number of x aged persons observed until their death, but maximum until one year. Censoring means in this case that we observe maximum for one year. An uncensored observation would be if we would observe every person until death. Let V_i denote the random variable meaning how long we could observe the person of age x, indicated with i. V_i is a random variable with a mixed distribution, that takes the values between 0 and 1 with probability 0 (the value of the density function in point t is $e^{-\mu t}$=exp(ln(1-q_x)t)(-ln(1-q_x))), and takes the value 1 with a positive probability: P(V_i=1)=1-$e^{-\mu}$ =1-exp(-ln(1-q_x).

The distribution of V_i depends on the q_x parameter, we apply a maximum likelihood estimate to this parameter. We have a sample of the random variable V_i with n elements (v_1, v_2, …, v_n). We introduce the variable d_i, that will be of our help. This takes the value 1 if person i dies, and the value 0 if the person i doesn't die. Then the relation

$$ \exp(\ln(1-q_x)v_i)(-\ln(1-q_x))^{d_i} \tag{F.7} $$

describes the density distribution of the variable V_i. If d_i =1, then the relation (F.7) will be the same as the density function of the random variable V_i at the point v_i. If d_i =0, then obviously t_i=1, and the expression is simplified to the form exp(ln(1-q_x)), that gives us the probability that V_i =1. Using this information we can give a maximum likelihood estimate of the parameter q_x:

$$L(q_x) = \prod_i e^{\ln(1-q_x)v_i} \cdot (-\ln(1-q_x))^{d_i} = e^{\ln(1-q_x)v} \cdot (-\ln(1-q_x))^d$$

where, $v = \sum_{i=1}^n v_i$, $d = \sum_{i=1}^n d_i$

$$\ln L(q_x) = \ln(1-q_x) \cdot v + d \cdot \ln(-\ln(1-q_x)),$$

$$\frac{d \ln L(q_x)}{dq_x} = \frac{-v}{1-q_x} + \frac{d}{-\ln(1-q_x)} \cdot \frac{1}{1-q_x} = 0$$

$$\frac{d}{-\ln(1-q_x)} = v$$

$$\hat{q}_{xML} = 1 - e^{\frac{-d}{v}}$$

If we want to give an interval estimate of the mortality rate, then we have to regard $\hat{q}_{xML}$ as the:

$$\hat{q}_{xML} = 1 - e^{\frac{-D}{V}}$$

estimate function, where $V = \sum_{i=1}^n V_i$ and $D = \sum_{i=1}^n D_i$ are random variables. In case of large samples[198] according to the maximum likelihood theory, the variance of the estimate function can be obtained by calculating the second derivative of the logarithm of the likelihood function by the unknown parameter, then we determine the expected value of this expression and take the opposite of the reciprocal value of the expected value.

As a first step we calculate the second derivative of the likelihood function by q_x.

$$\frac{d^2 \ln L(q_x)}{dq_x^2} = \frac{-V}{(1-q_x)^2} - \frac{(1+\ln(1-q_x)) \cdot D}{[(1-q_x)\ln(1-q_x)]^2}$$

Then we have to determine the expected value of the second derivative of the likelihood function. Based on our knowledge of statistics we know that the expected value of the first derivative of the maximum likelihood estimate function (score function) is 0.

[198] When estimating the mortality rates, we start from the portfolio of the insurer, that consists of several hundred or a thousand clients for every age-year pair. In other statistical areas samples of this size are usually considered large. When estimating mortality rates, the probability of death that is to be estimated is quite small, this way we need larger-than-average samples. A sample is considered large, if it contains at least 5 deaths. At age 20 the probability of death can be as small as 0.0005, which means that to have 5 deaths, we would need 10,000 clients on average. Probably even the largest Hungarian insurance companies don't have a portfolio of this size. In case of older ages the portfolio of insurance companies is considered large.

$$E(D) = (-\ln(1-q_x) \cdot E(V) = \mu \cdot E(V)$$

$$E\left(\frac{d^2 \ln L(q_x)}{dq_x^2}\right) = E\left(\frac{-V}{(1-q_x)^2} - \frac{(1+\ln(1-q_x))D}{\left[(1-q_x)\ln(1-q_x)\right]^2}\right) =$$

$$= -\frac{E(V)}{(1-q_x)^2} - \frac{(1+\ln(1-q_x)) \cdot E(D)}{\left[(1-q_x)\ln(1-q_x)\right]^2} =$$

$$= E(V)\left(-\frac{1}{(1-q_x)^2} + \frac{1+\ln(1-q_x)}{(1-q_x)^2 \ln(1-q_x)}\right) =$$

$$= E(V)\left(\frac{-\ln(1-q_x)+1+\ln(1-q_x)}{(1-q_x)^2 \ln(1-q_x)}\right) = E(V)\left(\frac{1}{(1-q_x)^2 \ln(1-q_x)}\right) =$$

We consider v as a good enough estimation of E(V), and substitute q_x with $\hat{q}$.

$$VAR_{\hat{q}} = -\frac{1}{E\left(\frac{d^2 \ln L(q_x)}{dq_x^2}\right)} \approx \frac{d}{v^2}\left(e^{\frac{-d}{v}}\right)^2$$

So to estimate the mortality rate and determine its variance, we only have to determine the variables d and v. The variable d indicates the number of deaths in the observed period (in the examined age-year pair), and the variable v shows how long we could observe all together. This variable has the name “central exposed to risk” in the actuarial professional literature.

The derived estimate function and its variance can also be used if there are surrenders. Surrender is basically censoring (we do not have information about the insured after surrender). The time of surrender is not fixed, it can happen any time within the year. The literature of the subject gave the name “random censoring” to this phenomenon. If e.g. a 40 years old person surrenders the policy at 40.3 years, then in this case d_i=0, since the person didn’t die, and v_i=0.3.

Let’s look at a concrete example of estimating the mortality rate: we observe 10 persons (*a, b, c, d, e, f, g, h, i* and *j*). We have the following information about the observed persons:

a: takes out an insurance at age 20 and is still alive at age 22, at maturity.
b: takes out an insurance at age 20.3 and is still alive at age 22.3, at maturity.
c : takes out an insurance at age 20 that lapses at age 20.4.
d: takes out an insurance at age 20.4 and dies at age 20.8.
e: takes out an insurance at age 20.3 and surrenders the policy at age 21.6.
f: takes out an insurance at age 20.2 and dies at age 21.5.
g: takes out an insurance at age 20.7 and dies at age 22.4.
h: takes out an insurance at age 21.2 and lapses the policy at age 22.2.

i: takes out an insurance at age 20.4, pays up the policy at age 21.3 and after that is still alive at maturity, at age 22,4.
J: takes out an insurance at age 20.3 and dies at age 20.9.

Using this information, let's give an estimate of the annual mortality rates at age 20, 21 and 22!

First we take the annual mortality rate at age 20:
Person *a* reaches age 21, in the case of this person $d_a^{(20)} = 0$ and $v_a^{(20)} = 1$.

Person *b* could only be observed for 0.7 years at age 20, this way $v_b^{(20)} = 0{,}7$. This person also lived until the 21st birthday, so $d_b^{(20)} = 0$.

Person *c* could be observed for $v_c^{(20)} = 0{,}4$ years at age 20. This person leaved the insured group alive, so $d_c^{(20)} = 0$.

Person *d* could be observed for $v_d^{(20)} = 0{,}4$ years at age 20. This person died in this year (and we could "observe" this, in other words the insurer has information about this death), so $d_d^{(20)} = 1$.

In the case of person *e* $v_e^{(20)} = 0{,}7$; $d_e^{(20)} = 0$.

In the case of person *f* $v_f^{(20)} = 0{,}8$; $d_f^{(20)} = 0$ (he lived until his 21st birthday, or didn't die at age 20).

In the case of person *g* $v_g^{(20)} = 0{,}3$; $d_g^{(20)} = 0$.

In the case of person *h* $v_h^{(20)} = 0$; $d_h^{(20)} = 0$.

In the case of person *i* $v_i^{(20)} = 0{,}6$; $d_i^{(20)} = 0$.

In the case of person *j* $v_j^{(20)} = 0{,}6$; $d_j^{(20)} = 1$.

$$v^{(20)} = v_a^{(20)} + v_b^{(20)} + \cdots + v_j^{(20)} = 5{,}5\,; \qquad d^{(20)} = d_a^{(20)} + d_b^{(20)} + \cdots + d_j^{(20)}$$

$$\hat{q}_{20} = \frac{2}{5{,}5} = 0{,}3636\,; \qquad V\hat{A}R_q = 0{,}045960$$

The annual mortality rates at age 21 and 22 are estimated in a similar way. In the following table we take into account how much each person has contributed to the "central exposed to risk" at age 21 and 22:

Person	$v^{(21)}$	$d^{(21)}$	$v^{(22)}$	$D^{(22)}$
a	1	0	0	0
b	1	0	0,3	0
c	0	0	0	0
d	0	0	0	0
e	0,6	0	0	0
f	0,5	1	0	0
g	1	0	0,4	1
h	0,8	0	0,2	0
i	1	0	0,4	0
j	0	0	0	0

Person	$v^{(21)}$	$d^{(21)}$	$v^{(22)}$	$D^{(22)}$
Total	5,9	1	1,3	1

Table F.1.

Using the totals of table F.1

$$\hat{q}_{21} = \frac{1}{5,9} = 0,1695 \qquad V\hat{A}R_q = 0,024249$$

$$\hat{q}_{21} = \frac{1}{1,3} = 0,7692 \qquad V\hat{A}R_q = 0,274183$$

We call the probability of mortality estimation the raw mortality rate. We see that there is high level of variance in the numbers obtained this way, so we have to apply equalisation on these raw mortality rates.

EQUALISATION OF RAW MORTALITY RATES

When equalising the data series, we try to make the random variations of the raw annual mortality rates disappear, but to leave unchanged the tendencies hidden in the data series. If we equalise the data series too much, we also make real phenomena and tendencies disappear.

It is very important that after having equalised the data, we examine (by statistical testing) whether it still suits the initial data set.

Equalising data series and testing this equalisation is not unambiguous and not a simple task, but it doesn't raise problems that concern specifically only the actuarial sciences, so we won't discuss equalisation methods in this book. The interested reader will find ample information on the literature of the subject in the references of the appendix.

As an illustration we'll introduce the equalisation method used by the Hungarian Central Statistical Office:[199]

No equalisation is used for ages 0-3. Due to the large and fast changes, every method would totally distort reality;
in case of ages 4-14 a quartic parabola is fit to the given points;
the values of ages 15-75 are equalised by a mechanical procedure;
the raw mortality rates of ages 76-100 are equalised by an exponential function, or extrapolated in the very old ages due to the lack of sufficient data.

We won't discuss the fitting of the quartic parabola in case of ages 4-14.
The Hungarian mortality tables are equalised between ages 15 and 75 in a two step mechanical procedure[200]. In the first step so-called major points are defined for the ages 5 years apart[201] (x=15, 20, 25, ..., 75):

$$\tilde{q}_x = 1,08 \cdot Z_x - 0,04(Z_{x-5} + Z_{x+5})$$

[199] The Hungarian Central Statistical Office doesn't use the above described method when estimating mortality rates. The description of the method of equalisation can be found in e.g.: Rinágel, pp. 111-114. and Pallós, pp. 25-35.
[200] The equalised mortality rates are denoted by $\tilde{q}$.
[201] The major points are determined by Newton's interpolation formula.

where: $Z_x = \frac{q_{x-2} + q_{x-1} + q_x + q_{x+1} + q_{x+2}}{5}$

In the second step the 4 values between the major points are determined[202].

$$\tilde{q}_{x+n} = \sum_{j=1}^{6} \alpha_{nj} \cdot Z_{x+5(j-3)}$$

where x=15, 20, 25, ...,70; n=1, 2, 3, 4

$$\alpha_{nj} = \begin{pmatrix} 0{,}00256 & -0{,}10560 & 0{,}98080 & 0{,}14560 & -0{,}02400 & 0{,}00064 \\ 0{,}00288 & -0{,}10560 & 0{,}73760 & 0{,}43200 & -0{,}06880 & 0{,}00192 \\ 0{,}00192 & -0{,}06880 & 0{,}43200 & 0{,}73760 & -0{,}10560 & 0{,}00288 \\ 0{,}00064 & -0{,}02400 & 0{,}14560 & 0{,}98080 & -0{,}10560 & 0{,}00256 \end{pmatrix}$$

Above 76 years exponential equalisation is applied based on the Gompertz-Makeham formula, that has the following form:

$$\tilde{q}_x = 1 - e^{a+bc^x} \tag{F.8}$$

The unknown a, b and c constants are calculated the following way:
After taking the logarithm of formula (F.8) and writing it in a more convenient form we get $\ln(1-\tilde{q}_x) = a + bc^x$. If we take the raw value $\hat{q}_x$ instead of the unknown $\tilde{q}_x$, then

$$\ln(1-\hat{q}_x) \approx a + bc^x$$

Let

$$H_1 = \sum_{x=76}^{80} \ln(1-\hat{q}_x) \approx \sum_{x=76}^{80} \left(a + bc^x\right) = 5a + bc^{76}\left(1 + c + ... + c^4\right) \tag{F.9}$$

$$H_2 = \sum_{x=81}^{85} \ln(1-\hat{q}_x) \approx \sum_{x=81}^{85} \left(a + bc^x\right) = 5a + bc^{81}\left(1 + c + ... + c^4\right) \tag{F.10}$$

$$H_3 = \sum_{x=86}^{90} \ln(1-\hat{q}_x) \approx \sum_{x=86}^{90} \left(a + bc^x\right) = 5a + bc^{86}\left(1 + c + ... + c^4\right) \tag{F.11}.$$

(F.9), (F.10) and (F.11) follow from $\frac{H_3 - H_2}{H_2 - H_1} \approx c^5$, that gives the following estimate on the value of c:

$$c = \sqrt[5]{\frac{H_3 - H_2}{H_2 - H_1}}.$$

Having determined the value of c we can also determine the a and b values:

[202] We get the equalised mortality rates based on the Karup-King formula.

$$a = \frac{AC - BD}{15C - B^2} \qquad b = \frac{15D - AB}{15C - B^2}$$

where: $A = H_1 + H_2 + H_3$; $B = \sum_{x=76}^{90} c^x$; $C = \sum_{x=76}^{90} c^{2x}$; $D = \sum_{x=76}^{90} c^x \ln(1 - \hat{q}_x)$;

In case of ages over 90 the probabilities of death are extrapolated based on the function $\tilde{q}_x = 1 - e^{a + bc^x}$.

The national mortality tables contain the mortality rates equalised this way.

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EXPLANATION OF TERMS

Active phase of life
The phase of life approximately from age 20-25 years to 60-70 years, when individuals make a living, provide for others and accumulate for later phases of their life from the income of their own work.

Age of insured
The insurer specifies the *entering age* (or the current age) of the insured by subtracting the insured's birth year from the commencing year (or the current calendar year) of the policy.

Age structure
A diagram illustrating the composition of the population of a country by age and gender.

Annuity
The (generally monthly) sum paid by the insurer regularly to the beneficiary declared in the insurance policy. The annuity can be "life"-annuity, when it is to be paid at most until the death of the insured, or certain annuity, when the payment is guaranteed within a given period (or it can also be the combination of these two).

Application, Proposal
Formally it is always the client (the policyholder) who initiates the signature of the insurance contract by making an application to the insurer to sign an insurance policy (even if the client wouldn't have thought of it on his own, but it was the insurer's agent who persuaded him into making the application). The policy becomes effective when the insurer accepts the application. The application form is the printed document, the client makes the application to the insurance company by filling out this form.

Average age
The average of the ages of a country or group of people at a certain moment.

Beneficiary
The person who receives the benefit defined in the insurance policy from the insurer in case of the insured event. (If the insured event is living until maturity, then the beneficiary is usually the insured.)

Cash flow
The Income and payout of a certain period from the point of view of an individual (institution, person) regarding a certain aspect (e.g. loan, maintaining an enterprise, etc.) all together, represented in chronological order.

Cash flow of the life cycle
All income and outgo of the life cycle arranged in chronological order.

Closed System
Any system that can only have connections to other systems with difficulty, and that cannot, or can only accept elements of other systems with difficulty, and the elements of which cannot connect to other systems.

Commencement date
The insurance policy becomes effective usually the day following the payment (to the account, the cash-desk, or to the agent of the insurer) of the premium advance (the premium of the first insurance period), supposing that the insurer hasn't rejected the application within the period available for underwriting (if there is no agreement stating otherwise).

Computability

The property of a stable system of relations, when its state can be predicted in the long-term relatively precisely and with high certainty.

Current premium reserve

The premium reserve of the insurance has the following parts:

- the premium reserve of the main policy,
- the premium reserve of the investment profit share, and
- the premium reserve of the insurances purchased by the chosen premium increases.

Current sum assured

The current sum assured is the sum assured defined in the policy (initial sum assured) increased by the sum assured of new assurances purchased with premium increase options chosen by the client and/or the sum assured of single premium assurances acquired through investment profit sharing. If the client didn't use the premium increase options or the policy didn't receive an investment profit share (or the insurer handles the investment profit on a separate account), then the current sum assured may be equal to the initial sum assured.

Defined contribution principle

An operating principle of insurance (mostly annuity), which states that the benefit payments received by an individual are made from the earlier accumulations of the individual and interests earned on these, and are proportional to these accumulations and the life-risk of the individual.

Disability adjusted life expectancy

The part of the expected life that the individual spends healthy.

Equivalence

As a principle this means that individuals will receive – above their part of the expenses of maintaining the social institution of "self-care" – in expected value the same amount of financial goods and services from the institution system as the expected value of their solidarity contribution payments made into it.

Family status

The individual's relation to the family (if there is one). If there is no family, than the relation to the institution of family (e.g. single), or the relation to a former family (orphan, widow, widower, divorced, etc…). The position of the individual within the family (e.g. child, husband, wife, etc…).

Financial planning of the life cycle

The projection of the cash flows of the remaining part of the life cycle, and the creation of a consumption and savings strategy that matches the long-term goals.

Foresight

The ability of individuals to take into account future events that are important regarding the life cycle (and mostly its cash flow) as determinants of their current actions.

Frequency of payment

Premium payment can be *regular*:

- monthly (12 times a year),
- quarterly (4 times a year),
- semi-annual (twice a year),
- annual (once a year),

or *single* (when the total premium of the insurance is due at the signature of the policy).

In case of regular premium payment the policyholder can usually change the frequency of premium payment on policy anniversaries.

General Conditions

Insurers generally summarize in separate regulations the policy conditions that concern every single life insurance policy, regardless of insurance type. The General Conditions of different insurance companies are somewhat different – although very similar.

Generation mortality table

A mortality table that follows the actual mortality of a group of people (e.g. the population of a country) born in the same period (e.g. in the same year), contrary to the mortality table, which is an artificial table constructed from the mortality of different generations living at the same time.

Inactive phase of the life

Childhood and young adulthood (until age 20-25 years), and old age, when individuals currently don't have a wage income, or it only covers a small part of the their current consumption.

Income transfer

The flow of income from one person to the other, or one social group to another without an offset.

Indexation techniques

The initial sum insured gradually looses its real value as time passes due to inflation. Indexation techniques serve to subdue the effect of inflation. They are truly effective only if applied together. The indexation techniques are:

- premium increase options and
- investment profit sharing.

Premium increase is obviously only possible in case of regular premium insurances, while the investment profit sharing refers to both regular- and single premium insurances (generally with the exception of term insurance). The client decides whether to make use of the premium increase option or not, while profit sharing is automatically received.

Individual account

This means the requirement that the institution should keep an individual account of the contributions paid and benefits received by every individual.

Initial sum assured, initial sum insured

The sum declared in the insurance policy at the time of its signature, that the insurer used for calculating the initial premium of the policy. The initial sum insured can differ from the current sum insured (see there!) at a given point of the insurance term.

Institutional care

The opposite of "personal" care in this book, i.e. the set of standardized social care solutions that are impersonally organised.

Insurance

The creation of business based risk community, a form of risk transfer.

Insurance benefit

The benefit that the insurer provides in case of the insured event, and/or the expiration of the policy. In case of life insurance this usually means the payment of the sum insured to the beneficiaries immediately after the insured event, or taking over further premium payment until the end of the policy, when the sum insured is paid out.

Insurance consciousness
As a principle this means that individuals make their contribution payments to the providing/self-care institution system consciously, based on a survey of their long-term needs.

Insurance Event, Insured Event
The event, which if incurs, the insurer provides the insurance benefits based on the life insurance policy.
In case of life insurance the *insurance even* can be:

- the death of the insured during the insurance term,
- the insured living at the maturity of the insurance policy.

Insured
The person whose life is connected to the insured event. (It is often the same person as the policyholder.)

Insured Period
The month/quarter/half-year/year counting from the day the policy becomes effective, that an individual premium payment covers. The length of the period consequently depends on the frequency of premium payment.

Life-, accident- and sickness insurance [203]
Grouping the different branches of insurance according to their subject. The subject of this branch is the life, health and well-being of the individual.

Life cycle
The financial aspect of the individual's life from birth until death.

Life expectancy
In any age the number of years that persons of that given age will live in average. A special case of it is life expectancy at birth, that is the remaining life expectancy at age 0.

Life expectancy at birth
The expected (average) age at the time of death of a group of people born at the same time.

Life insurance, life assurance
One of the insurance branches, that gives protection against insured events related to the individual's life (death, living until maturity).

Life insurance contract, life insurance policy
An agreement between the insurer and the policyholder, in which the insurer undertakes the liability against the premium payment of the policyholder of providing the insurance benefits if the insured event occurs. The general conditions of this contracting are regulated by the Civil Code.

Maturity
The date when – if it hasn't been terminated earlier due to death, surrender, or lapse – the insurance policy expires.

Medical examination
Medical examination is necessary so that the insurer can precisely assess the mortality risk of the insured. Since the (mortality) risk of the insured increases on the one had with the (death) sum assured, and on the other hand with the age of the insured, insurers tie the

[203] There is no single term to define these types of insurance, since the term "personal insurance" refers to insurances connected to persons, i.e. apartment-, car- etc. insurances beside the life-, accident- and sickness insurances!

automatic medical examination partly to a certain level of the sum assured and partly to a certain entering age. E.g. it is possible that an insurance company does medical examination every time, if the following age and sum assured limits are exceeded:

Entering age	Initial death sum assured
below 45 years	Above 2 million Forints,
46 to 55 years	Above 1 million Forints,
56 years	In all cases

Naturally there might be cases when the insured doesn't exceed the given limits, but the M.D. of the insurer still finds the medical examination necessary. This might be the case if the medical statement throws light on health problems, the severity of which cannot be assessed solely based on the medical statement.

The expenses of the medical examination are usually covered by the insurer.

Medical Statement

A form that has to be filled out together with the insurance application, containing questions about the insured's health status. Its purpose is that the insurer should get a satisfying idea of the death risk that the insured person represents.

Mortality table

A table containing at least the number of lives by age starting from a given population, but generally also contains the mortality and survival rates.

Non-forfeiture options

The premium reserve of the policy is fundamentally the client's (the policyholder's) money. This way the insurance company has to be able to account for the reserve to the client if the policyholder cannot continue the insurance with the same conditions. Possible non-forfeiture options of life insurance policies are the following: surrender, paying up the policy and policy loan. (The policy conditions specify the concrete non-forfeiture options that a given insurance policy has.)

Open system

Any system that can easily have connections to other similar systems, and that can easily accept elements of other systems, and the elements of which can easily connect to other systems.

Paid-up

Paying up the policy is possible in case of *regular premium insurances* (if policy conditions allow). In this case, the client doesn't pay the premium any more, and the insurer leaves the policy in force without changing its term, and using the available premium reserve as a single premium to purchase an insurance of a lower sum insured.

Pay-as-you-go system

A functioning principle of social security systems popular mainly in Europe, according to which the currently necessary benefit payments of beneficiaries are covered by the current contribution payments of contributors.

Phases of the life cycle

The most important continuous periods of the life cycle. From the economical point of view the life cycle can usually be divided to active-inactive-active phases.

Policy

The printed document issued by the insurance company, that contains the most important elements of the insurance contract. The insurer accepts the application of the policyholder by issuing the policy, i.e. the life insurance contract becomes effective when the policy is issued.

Policy anniversary

Generally the date of the policy becoming effective every year. Currently still a lot of Hungarian insurance companies don't follow this practice, and adjust the policy anniversary to the first day of the month following the date of the policy becoming effective. The commencement of the policy tailored this way is also called "technical commencement date".

Policy conditions

The policy conditions contain those conditions of the insurance contract that only refer to the given type of main policy or insurance rider.

Policyholder

The person who takes out the insurance and generally performs premium payment.

Portability

The characteristic of social security systems (or the lack of this characteristic), that contribution payments and benefits received can be simply changed between countries.

Premium

A fee (premium) depending on the sum assured, the type and term of insurance, the insured's age, gender and medical status, occupation and hobbies, that the policyholder pays for the insurance benefits.

Premiums due

The insurance premium of an insurance period is due in advance on the first day of the given insurance period.

Premium increase options

Most insurers provide the option to clients of increasing the insurance premium (with certain limitations), and consequently also the sum insured every year depending on the inflation rate. Making use of these premium increase options technically means the purchase of new insurance, that have an insurance term equal to the remaining term of the original insurance, and the entering age will be the current age of the insured. In case of premium increase, the premium reserve will be the sum of the premium reserves belonging to the initial sum and to the increased sum(s).

Premium reserve

Every premium that the policyholder pays to the insurer serves two purposes. The greater part is to cover the risks undertaken by the insurer, the insurer pays its liabilities undertaken in the policy from this part (we could say that this is the risk community "member fee"). A smaller part is necessary to cover the insurer's expenses related to the insurance. The greater part is again split into two parts. One part (its value depends on the type of insurance) is paid out to cover the current death benefits the same year as it was collected. But the benefit of the insurer is only expected to be paid in several years (depending on the date of maturity or death). As time passes, the death risk increases and the time of maturity benefit is closer. This way the insurer has to form reserves from the earlier premium payments (from the other part) to cover these insured events occurring later. This reserve is called premium reserve. The value of the premium reserve depends on:

- the entering age of the insured,
- the gender of the insured,
- the insurance term,

- the type of insurance,
- the sum insured,
- the time passed since the commencement of the insurance,

this way the premium reserve can have an infinite number of values.

Private insurance
Insurance not organised by state, mainly business based.

Probability of death = mortality rate
The probability that a person of age x years will not live to be x+1 years old.

Probability of survival
The probability of an individual of age x years living to be x+1 years old.

Profit sharing
If the insurer achieves a yield higher than the technical interest rate when investing the premium reserve, then the policy receives a share from this profit. The insurer divides a part of this as specified in the policy conditions (e.g. 90% of the yield above the technical interest rate) among the individual policies proportional to their premium reserve. The profit share increases directly by a lump sum either the premium reserve, or the balance of an account set up separately for the client. The insurer either transforms the sum of this increase to a sum assured, considering the current age of the insured and the remaining term of the policy (i.e. the former sum insured increases by this value), or simply handles it on the separate account. If the insurer uses the client's share of the investment profit to increase the sum assured, then the investment profit share is regarded as the premium of a single premium insurance.
The value of the premium reserve is important also in this case, since the achieved investment profit is distributed between the individual policies according to the ratio of premium reserves. This also means that it can happen that (since the premium reserve might be zero in the initial period) a policy doesn't receive an investment profit share on the first few occasions. Naturally this is only true for regular premium policies, since in case of single premium policies the premium reserve is significant from the first moment.

Reserve
All types of financial properties accumulated for later use or continuous operation.

Return on investment
The result of investing the premium reserve, that the insurer partly calculates in the premium right from the start (technical interest rate), and partly (in case of a higher return) shares with the policyholder (profit sharing).

Rider
A life, accident or sickness insurance, that cannot be taken out independently, only as a complementary to a life insurance as main policy. Its benefits serve to complement the benefits of the main policy.

Risk
A event occurring randomly and having a negative effect.

Risk commencement date
After the waiting period, or if there is no waiting period, or in cases that are not affected by the waiting period, 0 o'clock of the day following the payment of the premium advance, if the insurance policy later became effective, or would have become effective. I.e. if the insured event occurs after 0 o'clock of the day following the payment of the premium advance, and the policy would have become in force independent of the insured event, then the insurer pays the benefits specified in the policy to the beneficiary.

Risk community

A group organised to ward off or decrease a concrete risk, that operates from payments made by its members, and has the goal of (partly) compensating the negative financial effects in case of those members of the risk community, who have suffered a loss due to this risk.

Those who take out an insurance policy become members of a risk group. In case of life insurance the risk is death and/or living until maturity.

The basic principle of every risk community is that anyone can suffer a loss, but we do not know beforehand who and when. Those who are affected by the risk (and suffer a loss) cannot cope with it on their own. This is why the members of the risk community cover the loss of the effected persons together.

The above statements also mean that in case of life insurance those who live until maturity pay more – disregarding interests – than they receive from the risk community (moreover in some cases they only pay, but do not receive anything). But it is all the same worth joining the risk community, because in return the insured can feel secure.

The insurance company is an institution organising risk communities.

Risk transfer

Handing over the risk that we consider too high, against payment, to someone for whom this risk isn't high.

Rules for investment of life premium reserves

Insurers invest the premium reserve *safely*. This means that they avoid investment forms with higher risk (i.e. investments with a higher expected yield, but an uncertain outcome), and prefer primarily solutions of a modest, but certain yield, e.g. government bonds. The investment options are regulated by law (Act on Insurers and Insurance Activities).

Self-care

Voluntary and conscious long term care of our own safety and security.

Selection Table

A mortality table that contains the mortality rates of a special selected population (primarily a group having a given type of insurance, e.g. term insurance, annuity).

Self-insurance

A risk-handling strategy, when the individuals lean primarily on their own reserves.

Social Security

A compulsory insurance system organised by the state, maintained from contribution payments, to provide primarily pension- and medical benefits. In a wider sense all kinds of state maintained systems (i.e. maintained from taxes) can be called (moreover, here we should call them!) social security systems, although strictly speaking this is not correct.

Solidarity

Solidarity is in the economic sense a kind of redistribution, it is a one-way income transfer from social groups in better financial position to those who are in a worse financial position, who are financially in need.

Structure of cash flow

The incoming and outgoing cash flow grouped according to certain aspects.

Sum assured, Sum insured, Face amount

The sum insured shows the level of insurance benefits if the insured even occurs. Since in case of a life insurance (e.g. an endowment insurance) there might be two insured events, we distinguish the death sum assured and the maturity sum assured (that are not necessarily the same). We also distinguish the initial and the current sum assured (see there).

Surrender

Surrender means that the policyholder terminates the policy and demands the premium reserve from the insurer. The insurer doesn't pay out a certain percent of the premium reserve upon surrender. The purpose of holding back a part of the premium reserve is to compensate the deterioration of the portfolio due to the fact that surrender is more often chosen by the insured representing good risk, than the insured of bad risk. This premium reserve part held back is usually between 1-20%, and decreases as time passes. (The precise regulation varies from company to company.)

The other reason of decreasing the surrender value is that the premium reserve has to be mobilised at surrender, and the insurance company is forced to break up the investment portfolio. This has additional costs.

System of institutions

The set of more or less whole institutions connected to each other, following a united system of goals, and more or less complementing each other – e.g. the institution system of social self-care.

Technical interest rate

Insurers calculate the premium of insurances by supposing that when investing the premium reserve, it will earn an interest of at least 2-4% every year. This supposed fixed (2-4%) interest rate, that varies from insurer to insurer is called technical interest rate. By signing the insurance policy, the insurer guarantees that the policyholder will receive this interest rate.

Insurers choose the technical interest rate between 2 and 4%, because in the long term (and life insurance generally is long term) the yield of investments cannot be predicted. In a consolidated economy with low inflation rates, a 4% interest rate is considered very good. This way no financial institution can take long term commitments above this level.

Naturally the investment yield of the insurer will exceed 2-4% in several years. In these cases the investment profit sharing will come into effect.

The possible maximum value of the technical interest rate for newly signed policies is regulated by an order of the Ministry of Finance. According to the regulation of 2003 the maximum value is 4%.

Term

The period of time starting with the signature of the policy and ending with the maturity date declared in the policy, that can usually be only in whole years.

Transparency

The requirement that the functioning of an institution or a financial product, and the financial affairs of an institution should be visible and understandable without significant effort for those who are interested. It is particularly important that the paths and levels of individual contributions and utilizations of participants can be followed at every moment by the participants.

Underwriting

The insurer examines whether the application covers a normal risk, or due to the possibly significantly worse heath status or occupational (sports, hobby) risk, an increased premium tariff should be used. It is possible that the insurer doesn't take the risk and rejects the application. The base of underwriting is the application, the medical statement, and – in some cases, e.g. higher sum insured – the data provided by a medical examination. In some cases – e.g. very rare occupation or dangerous hobby – the insurer might also request the filling out of another questionnaire.

In case of extremely high sum insured – to prevent insurance fraud – the insurer might ask separate questions, that try to determine if there is some hidden purpose of signing the application. Such questions might refer to the relation of the insured and the beneficiary, the financial status of the policyholder, etc.

Unearned premium

If in case of regular premium insurances the insurer calculates the premium tariff based on monthly premium payment, but the policyholder chooses a premium payment less frequent (quarterly, semi-annual, annual), then the premium that applies to the whole months remaining from the insurance period is called “unearned premium”, since the insurer has not yet “earned” this part of the premium. If the client has unearned premium when the insured event occurs, then the insurer pays this back together with the benefit payment.

Waiting period

It is very common that insurers define a waiting period, within which there is no benefit payment. This period is usually half a year. If the insured event happens within this period, the insurer doesn’t pay the sum insured to the beneficiary (apart from certain exceptions, e.g. accidents), only the premium reserve.

If the policy became effective following a medical examination, then insurers usually do not apply a waiting period.

Zillmerization

In case of regular premium payment a lot of insurers make the premium reserve of the policy zero in the commencing period of the policy (the first 1-4 years, depending on the entering age of the insured and the insurance term), so it only becomes positive, a significant value compared to the paid premiums later during the term. The reason of this is that the greater part of the insurer’s expenses connected to the policy arise immediately when it is signed. To cover these, the insurer uses the total premium of the first 1-4 years, so this does not fill the reserve. The excess premium used in the first years is given back from the premium loading of the later years to fill the reserve (and during that period the increase of the premium reserve will be faster than if the insurer didn’t use this technique).

This type of reserving method is called zillmerization. It has an effect – among others – on the profit sharing and on non-forfeiture options (see there!). This way the investment profit share and – in case of surrender – the surrender value of the policy is zero or a low sum in the first years.

NOTATION AND THE MOST IMPORTANT RELATIONS USED IN THE BOOK

i: technical interest rate, or generally nominal interest rate
T: capital
- $T_n = T_0 \cdot (1 + n \cdot i)$, in case of simple interest calculation,
- $T_n = T_0 \cdot (1 + i)^n$, in case of compound interest calculation.

v: $\frac{1}{1+i}$ discount factor
k: depending on the formula:
- inflation rate,
- the annual increasing rate of an annuity with increasing payment,
- the index rate of an indexed life insurance, the last year in which the part of the zillmer premium above the cover of the risk of the given year goes totally to cover z.

r real interest rate $r = \frac{1+i}{1+k} - 1$
PV: Present Value $PV = \frac{T_n}{(1+i)^n} = T_n \cdot v^n$
RV: Real Value $RV = \frac{T_n}{(1+k)^n}$
x: the entering age of the insured
ω: the highest age observed in the population
l_x: the number of living at age x from a starting population of 100,000
d_x: the number of those living to be age x, but dying before their $(x+1)^{th}$ birthday in the same population ($d_x = l_x - l_{x+1}$)
q_x: mortality rate, probability of death: the probability that someone living to be x years old doesn't live to be (x+1) years old ($q_x = d_x/l_x$)
p_x: survival rate, probability of survival: the complement of the mortality rate ($p_x = 1 - q_x$)
n: insurance term
m: depending on the formula: the premium term or the deferred phase
t: the number of whole years passed since the commencement of the insurance
g: the length of the guarantee period
h: in case of endowment insurance the ratio of the death sum assured to the maturity sum assured
p: the number of premium or annuity payments in a year
D_x: the discounted number of living (commutation number) $D_x = l_x \cdot v^x$
C_x: the discounted number of dead (commutation number) $C_x = d_x \cdot v^{x+1}$ or $C_x = d_x \cdot v^{x+\frac{1}{2}}$
N_x: the discounted value of the total remaining number of years of all living at age x (commutation number) $N_x = D_x + D_{x+1} + D_{x+2} + ... + D_\omega$
M_x: the discounted sum of all deaths starting from age x (commutation number) $M_x = C_x + C_{x+1} + ... + C_\omega$
S_x: commutation number $S_x = N_x + N_{x+1} + ... + N_\omega$
R_x: commutation number $R_x = M_x + M_{x+1} + ... + M_\omega$
D_{xy}: the two-life version of commutation number D $D_{xy} = D_x \cdot l_y$

N_{xy}: the two-life version of commutation number N $N_{xy} = D_{xy} + D_{x+1,y+1} + ...$

ä: single net premium of 1 Ft yearly annuity in advance

a: single net premium of 1 Ft yearly annuity in arrears

$ä_x$: single net premium of 1 Ft yearly lifetime annuity paid in advance in case of an insured with entering age x $ä_x = \frac{N_x}{D_x}$.

${}_{m|}ä_x$ single net premium of 1 Ft yearly lifetime annuity paid in advance, after a deferred period of m years, in case of an insured with entering age x ${}_{m|}ä_x = \frac{N_{x+m}}{D_x}$

a_x: single net premium of 1 Ft yearly lifetime annuity paid in arrears in case of an insured with entering age x $a_x = {}_1ä_x = ä_x - 1$

$ä_x^{(p)}$ single net premium of 1 Ft yearly lifetime annuity paid in advance in p number of payments a year, in case of an insured with entering age x. According to an approximation formula: $ä_x^{(p)} = ä_x - \frac{p-1}{2p}$

$ä_{x:\overline{n}|}$: single net premium of 1 Ft yearly temporary annuity paid in advance in case of an insured with entering age x, and paid for a term of n years. $ä_{x:\overline{n}|} = ä_x - {}_{n|}ä_x = \frac{N_x - N_{x+n}}{D_x}$

$ä_{x:\overline{n}|}^{(p)}$: single net premium of 1 Ft yearly temporary annuity paid in advance in case of an insured with entering age x, paid for a term of n years in p number of payments a year. According to an approximation formula: $ä_{x:\overline{n}|}^{(p)} = ä_{x:\overline{n}|} - \frac{p-1}{2p} \cdot \left(1 - A_{x:\overline{n}|}^{\;\;1}\right)$,

${}_{m|}F_n$: single net premium of 1 Ft yearly certain temporary annuity paid in advance, with m years deferred phase and n years term (after the deferred phase!) ${}_{m|}F_n = v^m \cdot \frac{1 - v^n}{1 - v}$

${}_{m|}F$: single net premium of 1 Ft yearly infinite certain annuity with a deferred phase of m years, paid in advance ${}_{m|}F = \frac{v^m}{1 - v}$

${}^Aä_x^g$: single net premium of 1 Ft yearly lifetime annuity paid in advance, with a guarantee period of g years, counting from the commencement of the annuity. ${}^Aä_x^g = {}_{0|}F_g + {}_{g|}ä_x$

${}^Bä_x^g$: single net premium of 1 Ft yearly lifetime annuity paid in advance, with a guarantee period of g years, counting from the death of the insured. ${}^Bä_x^g = {}_{0|}F_g + v^g \cdot ä_x$

$ä_{xy}$: single net premium of 1 Ft yearly joint life annuity paid in advance in case of insured with entering age x and y $ä_{xy} = \frac{N_{xy}}{D_{xy}}$

$(Iä)_x$: single net premium of a lifetime annuity paid in advance with 1 Ft payment in the first year, and increasing with k Forints every year $(Iä)_x = \frac{(1-k) \cdot N_x + k \cdot S_x}{D_x}$

A: single net premium of 1 Forint sum assured

$A_{x:\overline{n|}}$: single net premium of an arbitrary type of insurance with 1 Forint sum assured, an insured of entering age x and a term of n years

$A^{1}_{x:\overline{n|}}$: single net premium of a term insurance with 1 Forint sum assured, an insured of entering age x and a term of n years $A^{1}_{x:\overline{n|}} = \frac{M_x - M_{x+n}}{D_x}$

$A_{x:\overline{n|}}^{\;\;1}$: single net premium of a pure endowment insurance with 1 Forint sum assured, insured of entering age x and y, and a term of n years $A_{x:\overline{n|}}^{\;\;1} = \frac{D_{x+n}}{D_x}$

$A^{1}_{xy:\overline{n|}}$: single net premium of a joint life term insurance with 1 Forint sum assured, insured of entering age x and y, and a term of n years $A^{1}_{xy:\overline{n|}} = v \cdot \ddot{a}_{xy:\overline{n|}} - {}_1\ddot{a}_{xy:\overline{n+1|}}$

$A_{xy:\overline{n|}}^{\;\;1}$: single net premium of a joint life pure endowment insurance with 1 Forint sum assured, insured of entering age x and y, and a term of n years $A_{xy:\overline{n|}}^{\;\;1} = \frac{D_{(x+n)(y+n)}}{D_{xy}}$

P: regular annual net premium of 1 Forint sum assured $P = \frac{A}{\ddot{a}_{x:\overline{m|}}}$

$P^{1}_{x:\overline{m|}}$: regular annual net premium of a term insurance with 1 Forint sum assured and a premium term of m years (and an insurance term of n years) $P^{1}_{x:\overline{m|}} = \frac{A^{1}_{x:\overline{n|}}}{\ddot{a}_{x:\overline{m|}}}$

$P_{x:\overline{m|}}^{\;\;1}$: regular annual net premium of a pure endowment insurance with 1 Forint sum assured and a premium term of m years (and an insurance term of n years) $P_{x:\overline{m|}}^{\;\;1} = \frac{A_{x:\overline{n|}}^{\;\;1}}{\ddot{a}_{x:\overline{m|}}}$

A_n: single net premium of a term fix insurance[204] with a term of n years

$P^{b}_{x:\overline{n|}}$: annual gross premium of an arbitrary type of insurance with 1 Forint sum assured, an insured with entering age x and an insurance term of n years

$\lambda_{x:\overline{n|}}$: the ratio of the premium loading and the annual net premium of an arbitrary type of insurance with an insured of entering age x and an insurance term of n years

V: the premium reserve

$\underline{V}_t$: the value of the premium reserve on the t[th] policy anniversary, before premium payment

$\overline{V}_t$: the value of the premium reserve on the t[th] policy anniversary, after premium payment

S: the sum assured

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[204] In the Hungarian insurance profession the view that single premium term fix insurance doesn't really exist is quite popular. This would truly be a strange design, since it is actually a long term deposit with fixed interest rate, that is usually not regarded as insurance. This way I agree with the fact that insurers will probably not launch such a product in the practice, but still I state that it exists. (Bein-Bogyó-Havas have also seriously derived its formula just as other types of insurances!) Just think about the theoretical deduction of regular premium insurances from single premium ones! In this case we have to define the single premium version to get the regular premium term fix (the existence of which is never questioned!) Or we can think of the profit sharing of term fix, that might be changed to a single premium insurance! In this case – in a certain sense – this concept becomes an existing design even in practice.

SAD: standard death sum assured. It is 1 in case of term insurance, 0 in case of pure endowment insurance and something else in all other cases (but probably falls between 0 and 1)

SAM: standard pure endowment sum assured. It is 0 in case of term insurance, 1 in case of pure endowment insurance and 1 in case of pure endowment with premium refund and endowment

SAT: standard term fix sum assured. It is generally 0, but 1 in case of term fix insurance.

SAP: standard premium refund sum assured. It is generally 0, but 1 in case of pure endowment with premium refund.

AA: standard annual annuity sum. It can have the value 0 (generally) or 1.

SAR: sum at risk. The current premium reserve subtracted from the current sum assured at the moment of death. A negative value in case of pure endowment insurance

gy: premium frequency (gy = 1, 2, 4 or 12)

k_{gy}: correction factor if the premium is not annual

z: the percentage of the sum assured, that shows how much more we use from the first premium than the expense loading. (zillmer-percentage)

P_1: the net premium of the first year, that remains from the gross premium after the first year's expenses

PZ: the net premium of all other years, that already contains the repayment of zillmerization (it is also called reserve-premium or zillmer premium)

BP: the gross premium (it is the same as P^b)

VP: the original premium loading

FVP: the continuous premium loading

p: the net premium necessary for covering the first year's risks

e $PZ - P_{x+1:n-1}$, if this is positive

h $p - P_1$, if this is positive

p_i: the net premium part in the i^{th} year of the term necessary to cover the risks of the given year ($p_1 = p$) (not to be mistaken for the survival rate! We use a different letter in the index to avoid confusion!)

h_i: the sum in the i^{th} year of the term remaining from the original z that has not yet been recovered from the PZ of the given year (naturally compounded, etc., as it arises in the given year) ($h_1 = h$)

z_i: in the i^{th} year of the term the part of PZ of the given year that is paid to cover z (naturally compounded, etc., as it arises in the given year)

THE HUNGARIAN MALE AND FEMALE POPULATION MORTALITY TABLES AND COMMUTATION NUMBERS OF YEAR 1998, WITH 3.5% TECHNICAL INTEREST RATE

MALE POPULATIN MORTALITY TABLE

Kor (x)	lx	dx	qx	vx	Cx	Dx	Mx	Nx	Rx	Sx
0	100 000	1 082	1,08%	1,00000	1 045,41	100 000,00	12 509	2 587 457	658 320	57 070 481
1	98 918	100	0,10%	0,96618	93,35	95 572,95	11 464	2 487 457	645 811	54 483 024
2	98 818	49	0,05%	0,93351	44,20	92 247,66	11 370	2 391 884	634 347	51 995 568
3	98 769	43	0,04%	0,90194	37,47	89 083,98	11 326	2 299 636	622 976	49 603 684
4	98 726	28	0,03%	0,87144	23,58	86 034,01	11 289	2 210 552	611 650	47 304 048
5	98 698	22	0,02%	0,84197	17,90	83 101,07	11 265	2 124 518	600 361	45 093 496
6	98 676	20	0,02%	0,81350	15,72	80 272,99	11 247	2 041 417	589 096	42 968 978
7	98 656	21	0,02%	0,78599	15,95	77 542,72	11 232	1 961 144	577 849	40 927 561
8	98 635	21	0,02%	0,75941	15,41	74 904,56	11 216	1 883 601	566 617	38 966 417
9	98 614	23	0,02%	0,73373	16,31	72 356,15	11 200	1 808 697	555 402	37 082 815
10	98 591	26	0,03%	0,70892	17,81	69 893,01	11 184	1 736 341	544 201	35 274 119
11	98 565	27	0,03%	0,68495	17,87	67 511,67	11 166	1 666 448	533 017	33 537 778
12	98 538	29	0,03%	0,66178	18,54	65 210,80	11 148	1 598 936	521 851	31 871 331
13	98 509	29	0,03%	0,63940	17,92	62 987,06	11 130	1 533 725	510 703	30 272 395
14	98 480	27	0,03%	0,61778	16,12	60 839,15	11 112	1 470 738	499 573	28 738 670
15	98 453	43	0,04%	0,59689	24,80	58 765,67	11 096	1 409 899	488 462	27 267 932
16	98 410	51	0,05%	0,57671	28,42	56 753,63	11 071	1 351 133	477 366	25 858 033
17	98 359	60	0,06%	0,55720	32,30	54 806,01	11 042	1 294 380	466 295	24 506 900
18	98 299	70	0,07%	0,53836	36,41	52 920,36	11 010	1 239 574	455 253	23 212 520
19	98 229	79	0,08%	0,52016	39,70	51 094,37	10 974	1 186 653	444 242	21 972 946
20	98 150	88	0,09%	0,50257	42,73	49 326,84	10 934	1 135 559	433 269	20 786 293
21	98 062	92	0,09%	0,48557	43,16	47 616,05	10 891	1 086 232	422 335	19 650 734
22	97 970	94	0,10%	0,46915	42,61	45 962,69	10 848	1 038 616	411 443	18 564 502
23	97 876	97	0,10%	0,45329	42,48	44 365,78	10 806	992 653	400 595	17 525 887
24	97 779	100	0,10%	0,43796	42,31	42 823,01	10 763	948 287	389 790	16 533 233
25	97 679	108	0,11%	0,42315	44,15	41 332,57	10 721	905 464	379 026	15 584 946
26	97 571	117	0,12%	0,40884	46,22	39 890,70	10 677	864 132	368 306	14 679 481
27	97 454	128	0,13%	0,39501	48,85	38 495,52	10 630	824 241	357 629	13 815 350
28	97 326	143	0,15%	0,38165	52,73	37 144,89	10 582	785 746	346 999	12 991 108
29	97 183	159	0,16%	0,36875	56,65	35 836,05	10 529	748 601	336 417	12 205 363
30	97 024	179	0,18%	0,35628	61,62	34 567,56	10 472	712 765	325 888	11 456 762
31	96 845	198	0,20%	0,34423	65,85	33 336,99	10 411	678 197	315 416	10 743 997
32	96 647	219	0,23%	0,33259	70,37	32 143,80	10 345	644 860	305 006	10 065 800
33	96 428	243	0,25%	0,32134	75,45	30 986,44	10 274	612 716	294 661	9 420 940
34	96 185	274	0,28%	0,31048	82,19	29 863,14	10 199	581 730	284 387	8 808 224
35	95 911	314	0,33%	0,29998	91,01	28 771,08	10 117	551 867	274 188	8 226 494
36	95 597	367	0,38%	0,28983	102,77	27 707,14	10 026	523 096	264 071	7 674 627
37	95 230	429	0,45%	0,28003	116,07	26 667,41	9 923	495 389	254 045	7 151 531
38	94 801	496	0,52%	0,27056	129,66	25 649,54	9 807	468 721	244 123	6 656 143
39	94 305	565	0,60%	0,26141	142,70	24 652,51	9 677	443 072	234 316	6 187 421
40	93 740	627	0,67%	0,25257	153,01	23 676,14	9 534	418 419	224 639	5 744 350
41	93 113	683	0,73%	0,24403	161,04	22 722,49	9 381	394 743	215 104	5 325 931
42	92 430	734	0,79%	0,23578	167,21	21 793,06	9 220	372 020	205 723	4 931 188
43	91 696	784	0,85%	0,22781	172,56	20 888,89	9 053	350 227	196 502	4 559 167
44	90 912	837	0,92%	0,22010	178,00	20 009,94	8 881	329 339	187 449	4 208 940
45	90 075	900	1,00%	0,21266	184,92	19 155,28	8 703	309 329	178 568	3 879 601
46	89 175	969	1,09%	0,20547	192,37	18 322,60	8 518	290 173	169 866	3 570 273
47	88 206	1 045	1,18%	0,19852	200,44	17 510,63	8 325	271 851	161 348	3 280 100
48	87 161	1 124	1,29%	0,19181	208,30	16 718,04	8 125	254 340	153 023	3 008 249
49	86 037	1 201	1,40%	0,18532	215,04	15 944,40	7 917	237 622	144 898	2 753 909
50	84 836	1 272	1,50%	0,17905	220,05	15 190,17	7 702	221 678	136 981	2 516 287
51	83 564	1 335	1,60%	0,17300	223,14	14 456,44	7 482	206 487	129 280	2 294 609
52	82 229	1 390	1,69%	0,16715	224,48	13 744,43	7 258	192 031	121 798	2 088 122
53	80 839	1 446	1,79%	0,16150	225,63	13 055,17	7 034	178 287	114 540	1 896 091

Kor (x)	lx	dx	qx	vx	Cx	Dx	Mx	Nx	Rx	Sx
54	79 393	1 505	1,90%	0,15603	226,89	12 388,06	6 808	165 231	107 506	1 717 804
55	77 888	1 576	2,02%	0,15076	229,56	11 742,25	6 581	152 843	100 698	1 552 573
56	76 312	1 654	2,17%	0,14566	232,77	11 115,61	6 352	141 101	94 116	1 399 729
57	74 658	1 737	2,33%	0,14073	236,19	10 506,94	6 119	129 986	87 764	1 258 628
58	72 921	1 823	2,50%	0,13598	239,50	9 915,45	5 883	119 479	81 645	1 128 643
59	71 098	1 908	2,68%	0,13138	242,19	9 340,64	5 643	109 563	75 762	1 009 164
60	69 190	1 991	2,88%	0,12693	244,18	8 782,58	5 401	100 222	70 119	899 601
61	67 199	2 067	3,08%	0,12264	244,93	8 241,41	5 157	91 440	64 718	799 378
62	65 132	2 139	3,28%	0,11849	244,89	7 717,79	4 912	83 198	59 561	707 939
63	62 993	2 207	3,50%	0,11449	244,13	7 211,91	4 667	75 481	54 649	624 740
64	60 786	2 274	3,74%	0,11062	243,03	6 723,90	4 423	68 269	49 982	549 259
65	58 512	2 339	4,00%	0,10688	241,53	6 253,49	4 180	61 545	45 559	480 991
66	56 173	2 403	4,28%	0,10326	239,75	5 800,49	3 938	55 291	41 379	419 446
67	53 770	2 460	4,58%	0,09977	237,13	5 364,59	3 699	49 491	37 440	364 154
68	51 310	2 510	4,89%	0,09640	233,77	4 946,05	3 462	44 126	33 741	314 663
69	48 800	2 550	5,23%	0,09314	229,46	4 545,02	3 228	39 180	30 280	270 537
70	46 250	2 583	5,58%	0,08999	224,57	4 161,86	2 998	34 635	27 052	231 357
71	43 667	2 599	5,95%	0,08694	218,32	3 796,54	2 774	30 473	24 054	196 722
72	41 068	2 601	6,33%	0,08400	211,10	3 449,84	2 555	26 677	21 280	166 248
73	38 467	2 591	6,74%	0,08116	203,18	3 122,07	2 344	23 227	18 724	139 571
74	35 876	2 576	7,18%	0,07842	195,17	2 813,31	2 141	20 105	16 380	116 344
75	33 300	2 557	7,68%	0,07577	187,18	2 523,00	1 946	17 292	14 239	96 239
76	30 743	2 541	8,27%	0,07320	179,72	2 250,50	1 759	14 769	12 293	78 948
77	28 202	2 484	8,81%	0,07073	169,75	1 994,68	1 579	12 518	10 534	64 179
78	25 718	2 425	9,43%	0,06834	160,11	1 757,48	1 409	10 523	8 955	51 661
79	23 293	2 360	10,13%	0,06603	150,55	1 537,93	1 249	8 766	7 545	41 138
80	20 933	2 289	10,93%	0,06379	141,08	1 335,38	1 099	7 228	6 296	32 372
81	18 644	2 209	11,85%	0,06164	131,55	1 149,13	958	5 893	5 197	25 144
82	16 435	2 117	12,88%	0,05955	121,81	978,73	826	4 744	4 240	19 251
83	14 318	2 013	14,06%	0,05754	111,91	823,82	704	3 765	3 414	14 507
84	12 305	1 893	15,38%	0,05559	101,68	684,06	592	2 941	2 709	10 743
85	10 412	1 758	16,88%	0,05371	91,23	559,25	491	2 257	2 117	7 802
86	8 654	1 608	18,58%	0,05190	80,63	449,10	399	1 698	1 626	5 545
87	7 046	1 442	20,47%	0,05014	69,86	353,29	319	1 249	1 227	3 847
88	5 604	1 267	22,61%	0,04845	59,30	271,49	249	895	908	2 599
89	4 337	1 082	24,95%	0,04681	48,93	203,00	190	624	659	1 703
90	3 255	898	27,59%	0,04522	39,24	147,20	141	421	470	1 079
91	2 357	719	30,50%	0,04369	30,35	102,99	101	274	329	659
92	1 638	552	33,70%	0,04222	22,52	69,15	71	171	227	385
93	1 086	404	37,20%	0,04079	15,92	44,30	49	101	156	215
94	682	279	40,91%	0,03941	10,62	26,88	33	57	108	113
95	403	181	44,91%	0,03808	6,66	15,35	22	30	75	56
96	222	110	49,55%	0,03679	3,91	8,17	15	15	53	26
97	112	60	53,57%	0,03555	2,06	3,98	12	7	37	11
98	52	30	57,69%	0,03434	1,00	1,79	9	3	26	4
99	22	14	63,64%	0,03318	0,45	0,73	8	1	16	1
100	8	8	100,00%	0,03206	8,00	0,26	8	0	8	0

FEMALE POPULATION MORTALITY TABLE

Kor (x)	lx	dx	qx	vx	Cx	Dx	Mx	Nx	Rx	Sx
0	100 000	845	0,85%	1,00000	816,43	100 000,00	9 115	2 687 586	554 630	63 074 549
1	99 155	72	0,07%	0,96618	67,21	95 801,93	8 299	2 587 586	545 515	60 386 963
2	99 083	37	0,04%	0,93351	33,37	92 495,04	8 232	2 491 784	537 216	57 799 377
3	99 046	20	0,02%	0,90194	17,43	89 333,82	8 198	2 399 289	528 984	55 307 593
4	99 026	26	0,03%	0,87144	21,89	86 295,44	8 181	2 309 955	520 786	52 908 304
5	99 000	24	0,02%	0,84197	19,52	83 355,34	8 159	2 223 660	512 605	50 598 348
6	98 976	20	0,02%	0,81350	15,72	80 517,04	8 140	2 140 305	504 446	48 374 688
7	98 956	15	0,02%	0,78599	11,39	77 778,52	8 124	2 059 788	496 306	46 234 384
8	98 941	12	0,01%	0,75941	8,80	75 136,94	8 112	1 982 009	488 182	44 174 596
9	98 929	11	0,01%	0,73373	7,80	72 587,27	8 104	1 906 872	480 070	42 192 587
10	98 918	12	0,01%	0,70892	8,22	70 124,83	8 096	1 834 285	471 966	40 285 715
11	98 906	14	0,01%	0,68495	9,26	67 745,24	8 088	1 764 160	463 870	38 451 431
12	98 892	17	0,02%	0,66178	10,87	65 445,07	8 078	1 696 415	455 782	36 687 271
13	98 875	19	0,02%	0,63940	11,74	63 221,09	8 068	1 630 970	447 704	34 990 856
14	98 856	16	0,02%	0,61778	9,55	61 071,44	8 056	1 567 749	439 636	33 359 886
15	98 840	24	0,02%	0,59689	13,84	58 996,67	8 046	1 506 677	431 581	31 792 138
16	98 816	25	0,03%	0,57671	13,93	56 987,77	8 032	1 447 680	423 534	30 285 461
17	98 791	27	0,03%	0,55720	14,54	55 046,72	8 018	1 390 693	415 502	28 837 780
18	98 764	28	0,03%	0,53836	14,56	53 170,70	8 004	1 335 646	407 484	27 447 087
19	98 736	30	0,03%	0,52016	15,08	51 358,09	7 989	1 282 475	399 480	26 111 442
20	98 706	31	0,03%	0,50257	15,05	49 606,27	7 974	1 231 117	391 490	24 828 966
21	98 675	31	0,03%	0,48557	14,54	47 913,71	7 959	1 181 511	383 516	23 597 849
22	98 644	31	0,03%	0,46915	14,05	46 278,89	7 945	1 133 597	375 557	22 416 338
23	98 613	31	0,03%	0,45329	13,58	44 699,86	7 931	1 087 318	367 612	21 282 741
24	98 582	32	0,03%	0,43796	13,54	43 174,69	7 917	1 042 618	359 681	20 195 423
25	98 550	35	0,04%	0,42315	14,31	41 701,14	7 904	999 444	351 764	19 152 804
26	98 515	39	0,04%	0,40884	15,41	40 276,64	7 889	957 743	343 861	18 153 361
27	98 476	45	0,05%	0,39501	17,17	38 899,23	7 874	917 466	335 972	17 195 618
28	98 431	52	0,05%	0,38165	19,17	37 566,62	7 857	878 567	328 098	16 278 152
29	98 379	60	0,06%	0,36875	21,38	36 277,07	7 837	841 000	320 241	15 399 585
30	98 319	68	0,07%	0,35628	23,41	35 028,94	7 816	804 723	312 404	14 558 585
31	98 251	78	0,08%	0,34423	25,94	33 820,98	7 793	769 694	304 588	13 753 862
32	98 173	88	0,09%	0,33259	28,28	32 651,33	7 767	735 873	296 795	12 984 168
33	98 085	98	0,10%	0,32134	30,43	31 518,90	7 738	703 222	289 028	12 248 295
34	97 987	110	0,11%	0,31048	33,00	30 422,62	7 708	671 703	281 290	11 545 073
35	97 877	125	0,13%	0,29998	36,23	29 360,84	7 675	641 280	273 582	10 873 370
36	97 752	143	0,15%	0,28983	40,04	28 331,73	7 639	611 919	265 907	10 232 090
37	97 609	163	0,17%	0,28003	44,10	27 333,61	7 599	583 588	258 268	9 620 170
38	97 446	184	0,19%	0,27056	48,10	26 365,18	7 555	556 254	250 669	9 036 583
39	97 262	206	0,21%	0,26141	52,03	25 425,50	7 507	529 889	243 115	8 480 329
40	97 056	231	0,24%	0,25257	56,37	24 513,67	7 455	504 463	235 608	7 950 440
41	96 825	257	0,27%	0,24403	60,60	23 628,34	7 398	479 950	228 153	7 445 976
42	96 568	284	0,29%	0,23578	64,70	22 768,72	7 338	456 321	220 755	6 966 026
43	96 284	312	0,32%	0,22781	68,67	21 934,06	7 273	433 553	213 418	6 509 705
44	95 972	341	0,36%	0,22010	72,52	21 123,66	7 204	411 619	206 145	6 076 152
45	95 631	368	0,38%	0,21266	75,61	20 336,82	7 132	390 495	198 941	5 664 534
46	95 263	392	0,41%	0,20547	77,82	19 573,49	7 056	370 158	191 809	5 274 039
47	94 871	412	0,43%	0,19852	79,02	18 833,76	6 978	350 585	184 753	4 903 880
48	94 459	435	0,46%	0,19181	80,61	18 117,85	6 899	331 751	177 775	4 553 296
49	94 024	459	0,49%	0,18532	82,19	17 424,55	6 819	313 633	170 876	4 221 545
50	93 565	492	0,53%	0,17905	85,12	16 753,13	6 736	296 209	164 057	3 907 912
51	93 073	531	0,57%	0,17300	88,76	16 101,48	6 651	279 455	157 321	3 611 703
52	92 542	576	0,62%	0,16715	93,02	15 468,23	6 563	263 354	150 669	3 332 248
53	91 966	625	0,68%	0,16150	97,52	14 852,13	6 470	247 886	144 107	3 068 894
54	91 341	674	0,74%	0,15603	101,61	14 252,36	6 372	233 034	137 637	2 821 008
55	90 667	721	0,80%	0,15076	105,02	13 668,79	6 270	218 781	131 265	2 587 975
56	89 946	761	0,85%	0,14566	107,10	13 101,54	6 165	205 112	124 995	2 369 193
57	89 185	794	0,89%	0,14073	107,96	12 551,39	6 058	192 011	118 829	2 164 081
58	88 391	831	0,94%	0,13598	109,17	12 018,98	5 950	179 459	112 771	1 972 070
59	87 560	876	1,00%	0,13138	111,19	11 503,37	5 841	167 440	106 821	1 792 611
60	86 684	937	1,08%	0,12693	114,92	11 003,17	5 730	155 937	100 980	1 625 170
61	85 747	1 013	1,18%	0,12264	120,03	10 516,17	5 615	144 934	95 250	1 469 233

Kor (x)	lx	dx	qx	vx	Cx	Dx	Mx	Nx	Rx	Sx
62	84 734	1 098	1,30%	0,11849	125,71	10 040,52	5 495	134 418	89 635	1 324 299
63	83 636	1 193	1,43%	0,11449	131,96	9 575,27	5 369	124 377	84 140	1 189 881
64	82 443	1 295	1,57%	0,11062	138,40	9 119,51	5 237	114 802	78 770	1 065 504
65	81 148	1 402	1,73%	0,10688	144,77	8 672,72	5 099	105 682	73 533	950 702
66	79 746	1 508	1,89%	0,10326	150,45	8 234,66	4 954	97 010	68 434	845 020
67	78 238	1 609	2,06%	0,09977	155,10	7 805,74	4 804	88 775	63 480	748 010
68	76 629	1 719	2,24%	0,09640	160,10	7 386,68	4 649	80 969	58 676	659 235
69	74 910	1 844	2,46%	0,09314	165,93	6 976,79	4 488	73 583	54 028	578 265
70	73 066	1 991	2,72%	0,08999	173,10	6 574,93	4 323	66 606	49 539	504 683
71	71 075	2 153	3,03%	0,08694	180,86	6 179,48	4 149	60 031	45 217	438 077
72	68 922	2 320	3,37%	0,08400	188,30	5 789,66	3 969	53 851	41 067	378 046
73	66 602	2 492	3,74%	0,08116	195,42	5 405,57	3 780	48 062	37 099	324 194
74	64 110	2 670	4,16%	0,07842	202,29	5 027,36	3 585	42 656	33 318	276 133
75	61 440	2 850	4,64%	0,07577	208,63	4 655,06	3 383	37 629	29 734	233 476
76	58 590	3 168	5,41%	0,07320	224,07	4 289,01	3 174	32 974	26 351	195 847
77	55 422	3 259	5,88%	0,07073	222,71	3 919,90	2 950	28 685	23 177	162 874
78	52 163	3 352	6,43%	0,06834	221,32	3 564,64	2 727	24 765	20 227	134 189
79	48 811	3 446	7,06%	0,06603	219,83	3 222,78	2 506	21 200	17 500	109 424
80	45 365	3 535	7,79%	0,06379	217,88	2 893,96	2 286	17 978	14 994	88 224
81	41 830	3 613	8,64%	0,06164	215,16	2 578,22	2 068	15 084	12 708	70 246
82	38 217	3 673	9,61%	0,05955	211,34	2 275,87	1 853	12 505	10 640	55 163
83	34 544	3 707	10,73%	0,05754	206,08	1 987,58	1 642	10 229	8 787	42 657
84	30 837	3 707	12,02%	0,05559	199,11	1 714,28	1 436	8 242	7 145	32 428
85	27 130	3 662	13,50%	0,05371	190,04	1 457,20	1 236	6 528	5 710	24 186
86	23 468	3 564	15,19%	0,05190	178,70	1 217,88	1 046	5 070	4 473	17 658
87	19 904	3 406	17,11%	0,05014	165,00	998,00	868	3 853	3 427	12 588
88	16 498	3 183	19,29%	0,04845	148,99	799,25	703	2 855	2 559	8 735
89	13 315	2 899	21,77%	0,04681	131,10	623,23	554	2 055	1 856	5 881
90	10 416	2 558	24,56%	0,04522	111,77	471,05	423	1 432	1 303	3 826
91	7 858	2 175	27,68%	0,04369	91,82	343,35	311	961	880	2 394
92	5 683	1 770	31,15%	0,04222	72,20	239,92	219	618	569	1 433
93	3 913	1 368	34,96%	0,04079	53,91	159,61	147	378	350	815
94	2 545	997	39,17%	0,03941	37,96	100,30	93	218	203	437
95	1 548	677	43,73%	0,03808	24,91	58,94	55	118	110	219
96	871	423	48,56%	0,03679	15,04	32,04	30	59	55	101
97	448	241	53,79%	0,03555	8,28	15,92	15	27	25	42
98	207	122	58,94%	0,03434	4,05	7,11	7	11	10	16
99	85	55	64,71%	0,03318	1,76	2,82	3	4	4	5
100	30	30	1	0,03206	0,93	0,96	1	1	1	1

DISABILITY ADJUSTED LIFE EXPECTANCY IN WHO MEMBER COUNTRIES[205]

Rank	WHO Member	Average	Male	Female
1	Japan	74,5	71,9	77,2
2	Australia	73,2	70,8	75,5
3	France	73,1	69,3	76,9
4	Sweden	73,0	71,2	74,9
5	Spain	72,8	69,8	75,7
6	Italy	72,7	70,0	75,4
7	Greece	72,5	70,5	74,6
8	Switzerland	72,5	69,5	75,5
9	Monaco	72,4	68,5	76,3
10	Andorra	72,3	69,3	75,2
11	San Marino	72,3	69,5	75,0
12	Canada	72,0	70,0	74,0
13	Netherlands	72,0	69,6	74,4
14	United Kingdom	71,7	69,7	73,7
15	Norway	71,7	68,8	74,6
16	Belgium	71,6	68,7	74,6
17	Austria	71,6	68,8	74,4
18	Luxembourg	71,1	68,0	74,2
19	Iceland	70,8	69,2	72,3
20	Finland	70,5	67,2	73,7
21	Malta	70,5	68,4	72,5
22	Germany	70,4	67,4	73,5
23	Israel	70,4	69,2	71,6
24	United States of America	70,0	67,5	72,6
25	Cyprus	69,8	68,7	70,9
26	Dominica	69,8	67,2	72,3
27	Ireland	69,6	67,5	71,7
28	Denmark	69,4	67,2	71,5
29	Portugal	69,3	65,9	72,7
30	Singapore	69,3	67,4	71,2
31	New Zealand	69,2	67,1	71,2
32	Chile	68,6	66,0	71,3
33	Cuba	68,4	67,4	69,4
34	Slovenia	68,4	64,9	71,9
35	Czech Republic	68,0	65,2	70,8
36	Jamaica	67,3	66,8	67,9
37	Uruguay	67,0	64,1	69,9
38	Croatia	67,0	63,3	70,6
39	Argentina	66,7	63,8	69,6
40	Costa Rica	66,7	65,2	68,1
41	Armenia	66,7	65,0	68,3
42	Slovakia	66,6	63,5	69,7
43	Saint Vincent and the Grenadines	66,4	65,0	67,8
44	Georgia	66,3	63,1	69,4
45	Poland	66,2	62,3	70,1
46	Yugoslavia	66,1	64,2	68,1
47	Panama	66,0	64,9	67,2
48	Antigua and Barbuda	65,8	63,4	68,3
49	Grenada	65,5	62,4	68,5
50	United Arab Emirates	65,4	65,0	65,8
51	Republic of Korea	65,0	62,3	67,7
52	Venezuela	65,0	62,9	67,1
53	Barbados	65,0	62,4	67,6
54	Saint Lucia	65,0	62,4	67,6
55	Mexico	65,0	62,4	67,6
56	Bosnia and Herzegovina	64,9	63,4	66,4
57	Trinidad and Tobago	64,6	62,8	66,4
58	Saudi-Arabia	64,5	65,1	64,0
59	Brunei	64,4	63,4	65,4
60	Bulgaria	64,4	61,2	67,7
61	Bahrain	64,4	63,9	64,9
62	Hungary	64,1	60,4	67,9
63	Lithuania	64,1	60,6	67,5
64	Macedonia	63,7	61,8	65,6
65	Azerbaijan	63,7	60,6	66,7
66	Qatar	63,5	64,2	62,8
67	Cook Islands	63,4	62,2	64,5
68	Kuwait	63,2	63,0	63,4
69	Estonia	63,1	58,1	68,1
70	Ukraine	63,0	58,5	67,5
71	Paraguay	63,0	60,7	65,3
72	Oman	63,0	61,8	64,1
73	Turkey	62,9	64,0	61,8
74	Columbia	62,9	60,3	65,5

[205] This can be found on the home page of the WHO (www.who.org): Healthy Life Expectancy Rankings - World Health Organization Disability Adjusted Life Expectancy (=DALE). The Economist discusses the subject in the 2000. June 10. issue 23, pp. 114-115. – Life expectancy: Quality counts.

Rank	WHO Member	Average	Male	Female
75	Tonga	62,9	61,4	64,3
76	Sri Lanka	62,8	59,3	66,3
77	Suriname	62,7	60,2	65,2
78	Mauritius	62,7	59,0	66,3
79	Dominican Republic	62,5	62,1	62,9
80	Romania	62,3	58,8	65,8
81	China	62,3	61,2	63,3
82	Latvia	62,2	57,1	67,2
83	Belarus	61,7	56,2	67,2
84	Algeria	61,6	62,5	60,7
85	Niue	61,6	61,0	62,2
86	Saint Kitts and Nevis	61,6	58,7	64,4
87	El Salvador	61,5	58,6	64,5
88	Republic of Moldova	61,5	58,5	64,5
89	Malaysia	61,4	61,3	61,6
90	Tunisia	61,4	62,0	60,7
91	Russian Federation	61,3	56,1	66,4
92	Honduras	61,1	60,0	62,3
93	Ecuador	61,0	59,9	62,1
94	Belize	60,9	58,5	63,3
95	Lebanon	60,6	61,2	60,1
96	Iran	60,5	61,3	59,8
97	Samoa	60,5	58,7	62,3
98	Guyana	60,2	57,1	63,3
99	Thailand	60,2	58,4	62,1
100	Uzbekistan	60,2	58,0	62,3
101	Jordan	60,0	60,7	59,3
102	Albania	60,0	56,5	63,4
103	Indonesia	59,7	58,8	60,6
104	Micronesia	59,6	58,7	60,6
105	Peru	59,4	58,0	60,8
106	Fiji	59,4	57,7	61,1
107	Libya	59,3	59,7	58,9
108	Seychelles	59,3	56,4	62,1
109	Bahamas	59,1	56,7	61,6
110	Morocco	59,1	58,7	59,4
111	Brazil	59,1	55,2	62,9
112	Palau	59,0	57,4	60,7
113	Philippines	58,9	57,1	60,7
114	Syria	58,8	58,8	58,9
115	Egypt	58,5	58,6	58,3
116	Vietnam	58,2	56,7	59,6
117	Nicaragua	58,1	56,4	59,9
118	Cape Verde	57,6	54,6	60,6
119	Tuvalu	57,4	57,1	57,6
120	Tajikistan	57,3	55,1	59,4
121	Marshall Islands	56,8	56,0	57,6
122	Kazakhstan	56,4	51,5	61,2
123	Kyrgyzstan	56,3	53,4	59,1
124	Pakistan	55,9	55,0	56,8
125	Kiribati	55,3	53,9	56,6
126	Iraq	55,3	55,4	55,1
127	Solomon Islands	54,9	54,5	55,3
128	Turkmenistan	54,3	51,9	56,7
129	Guatemala	54,3	52,1	56,4
130	Maldives	53,9	54,4	53,3
131	Mongolia	53,8	51,3	56,3
132	Sao Tome and Principe	53,5	52,1	54,8
133	Bolivia	53,3	52,5	54,1
134	India	53,2	52,8	53,5
135	Vanuatu	52,8	51,3	54,4
136	Nauru	52,5	49,8	55,1
137	Democratic People's Republic of Korea	52,3	51,4	53,1
138	Bhutan	51,8	51,4	52,2
139	Myanmar	51,6	51,4	51,9
140	Bangladesh	49,9	50,1	49,8
141	Yemen	49,7	49,7	49,7
142	Nepal	49,5	49,4	49,5
143	Gambia	48,3	47,2	49,4
144	Gabon	47,8	46,6	49,0
145	Papua New Guinea	47,0	45,5	48,5
146	Comoros	46,8	46,1	47,5
147	Laos	46,1	45,0	47,1
148	Cambodia	45,7	43,9	47,5
149	Ghana	45,5	45,0	46,0
150	Congo	45,1	44,3	45,9
151	Senegal	44,6	43,5	45,6
152	Equatorial Guinea	44,1	42,8	45,4
153	Haiti	43,8	42,4	45,2
154	Sudan	43,0	42,6	43,5
155	Cote'd Ivoire	42,8	42,2	43,3
156	Cameroon	42,2	41,5	43,0
157	Benin	42,2	41,9	42,6
158	Mauritania	41,4	40,2	42,5
159	Togo	40,7	40,0	41,4
160	South-Africa	39,8	38,6	41,0

Rank	**WHO Member**	Average	Male	Female
161	Chad	39,4	38,6	40,2
162	Kenya	39,3	39,0	39,6
163	Nigeria	38,3	38,1	38,4
164	Swaziland	38,1	37,8	38,4
165	Angola	38,0	37,0	38,9
166	Djibouti	37,9	37,7	38,1
167	Guinea	37,8	37,0	38,5
168	Afghanistan	37,7	36,7	38,7
169	Eritrea	37,7	38,5	36,9
170	Bissau-Guinea	37,2	36,8	37,5
171	Lesotho	36,9	36,6	37,2
172	Madagascar	36,6	36,5	36,8
173	Somalia	36,4	35,9	36,9
174	Democratic Republic of Congo	36,3	36,4	36,2
175	Central African Republic	36,0	35,6	36,5
176	Tanzania	36,0	35,9	36,1
177	Namibia	35,6	35,8	35,4
178	Burkina Faso	35,5	35,3	35,7
179	Burundi	34,6	34,6	34,6
180	Mozambique	34,4	33,7	35,1
181	Liberia	34,0	33,8	34,2
182	Ethiopia	33,5	33,5	33,5
183	Mali	33,1	32,6	33,5
184	Zimbabwe	32,9	33,4	32,4
185	Rwanda	32,8	32,9	32,7
186	Uganda	32,7	32,9	32,5
187	Botswana	32,3	32,3	32,2
188	Zambia	30,3	30,0	30,7
189	Malawi	29,4	29,3	29,4
190	Niger	29,1	28,1	30,1
191	Sierra Leone	25,9	25,8	26,0

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